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Light pressure on a solid body immersed in a liquid medium

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Abstract. **We have solved the problem about the force with which an electromagnetic pulse in a liquid (or gaseous) medium at rest affects a solid body (also at rest) immersed in it. We have shown that under certain conditions (relating to the characteristics of the medium and the pulse shape), the formula for the force exerted per unit area of a body surface is obtained from the Landau–Lifshitz equations for static fields in the same way as, according to Pitaevskii, the field stress tensor is obtained from the static field stress tensor with the dispersion taken into account. The formula for the force acting on the wall, from which an incident quasi-monochromatic plane wave with a given intensity is reflected, differs from the corresponding formula for the case when the body is in a vacuum by the factor $\pm n_1$, where n_1 is the refractive index, and the upper (lower) sign corresponds to a positive (negative) group velocity of the wave in the medium.**

Keywords: *light pressure, dispersion, negative group velocity.*

1. The solution is known for the force acting on a wall of a flat solid surface – when a plane electromagnetic wave is incident on it in vacuum (see problem 1 in [1, § 47]). Let us introduce the following notations: $\boldsymbol{S}^{(0)}$ is a vector of the energy flux density of the incident wave; $\boldsymbol{N}$ is a unit vector along the normal to the surface of the wall, directed deep inside the body. If the coordinate axes x, y, z are chosen so that $N_x = N_y = 0$, $N_z = 1$ and $S_x^{(0)} \geqslant 0$, $S_y^{(0)} = 0$, $S_z^{(0)} \geqslant 0$, then the components of the force, $\tilde{\boldsymbol{P}}$, exerted per unit area of a body surface will be determined by the formulas

$$\tilde{P}_x = \frac{S_z^{(0)}}{c}(1 - R)\sin\theta_0,\ \tilde{P}_y = 0,\ \tilde{P}_z = \frac{S_z^{(0)}}{c}(1 + R)\cos\theta_0, \quad (1)$$

where R is the reflectance and θ_0 is the angle of incidence $[S_x^{(0)} = S^{(0)}\sin\theta_0,\ S_y^{(0)} = 0,\ S_z^{(0)} = S^{(0)}\cos\theta_0]$.

We know of no studies in which a similar problem is solved for the case when the wave propagates in a medium (liquid or gas). We only mention papers [2, 3]: Veselago [2] considers the case of normal incidence of a wave on a perfectly reflective body, and in [3] he studies the case of normal incidence of a wave on a body, completely absorbing radiation. A detailed discussion of these works will be presented below (in section 6).

2. Consider a solid body fully immersed in a liquid (or gaseous) medium and held at rest (with the force of gravity taken into account) by extraneous (with respect to the liquid) forces (for example, using threads on which the solid body is suspended). We shall assume that the electromagnetic field in the medium can be considered quasi-monochromatic [4, § 80]:

$$\boldsymbol{E}(\boldsymbol{r},t) = \mathrm{Re}\boldsymbol{E}_0(\boldsymbol{r},t)\exp(-\mathrm{i}\omega t),$$
$$\boldsymbol{H}(\boldsymbol{r},t) = \mathrm{Re}\boldsymbol{H}_0(\boldsymbol{r},t)\exp(-\mathrm{i}\omega t), \quad (2)$$

where ω is the field frequency; the 'amplitudes' of the electric ($\boldsymbol{E}_0$) and magnetic ($\boldsymbol{H}_0$) fields are slowly [compared to $\exp(-\mathrm{i}\omega t)$] varying functions of time: if τ_0 is the time characterising the rate of change in $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$, the parameter

$$\frac{1}{\omega\tau_0} \ll 1. \quad (3)$$

The force $\boldsymbol{P}$ exerted per unit area of the surface of a solid body at rest is determined by the momentum flux or the stress tensor [4, § 16; 5]:

$$P_i = -\sigma_{ij}N_j, \quad (4)$$

Hereafter the twice repeated indices $i, j, k = x, y, z$ imply summation. The expression for the stress tensor in an isotropic medium with the dispersion taken into account was obtained by Pitaevskii [4, § 81; 6]:

$$\sigma_{ij} = -P_0\delta_{ij} + \frac{1}{4\pi}\left\{\varepsilon(\omega)\langle E_iE_j\rangle - \frac{1}{2}\left[\varepsilon - \rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T\right]\langle E^2\rangle\delta_{ij}\right\}$$
$$+ \frac{1}{4\pi}\left\{\mu(\omega)\langle H_iH_j\rangle - \frac{1}{2}\left[\mu - \rho\left(\frac{\partial\mu}{\partial\rho}\right)_T\right]\langle H^2\rangle\delta_{ij}\right\}, \quad (5)$$

where the angle brackets denote averaging over the field period; P_0 is the pressure that would be in the medium in the absence of the field at the given values of the density ρ and temperature T; $\varepsilon(\omega)$ and $\mu(\omega)$ are the dielectric permittivity and magnetic permeability of the medium, respectively.

A remarkable feature of the Pitaevskii tensor (5) consists in the fact that, in contrast to the expression for the field energy [4, § 80], it [in the zero approximation in the parameter (3)] does not contain derivatives $\partial\varepsilon/\partial\omega$, $\partial\mu/\partial\omega$ and is formally obtained from the corresponding tensor for the statistical field by using the replacement

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$$\varepsilon \to \varepsilon(\omega),\ E_iE_j \to \langle E_iE_j\rangle,\ \mu \to \mu(\omega),\ H_iH_j \to \langle H_iH_j\rangle. \quad (6)$$

Landau and Lifshitz [4, §16, 35] derived the formulas for the total force and the total moment of the force with which a statistical field (with constant temperature and density) acts on a solid body. As a force exerted per unit area these formulas employ [see formula (4)]

$$\tilde{P}_i = -\tilde{\sigma}_{ij}N_j,\ \tilde{\sigma}_{ij} = \frac{1}{4\pi}\Big[\varepsilon\Big(E_iE_j - \frac{1}{2}E^2\delta_{ij}\Big) + \mu\Big(H_iH_j - \frac{1}{2}H^2\delta_{ij}\Big)\Big]. \quad (7)$$

We can assume (this was indicated to us by Pitaevskii) that in the case of a quasi-monochromatic electromagnetic field (2), the force $\tilde{\boldsymbol{P}}$ is given by (7), in which it is necessary only to make a replacement (6). Let us show that under certain conditions, this assumption is justified.

3. As in [4, §16], we believe that the liquid is at rest in the presence of the electromagnetic field, i.e., the total force exerted per unit volume of a liquid is $\boldsymbol{f} = 0$. The force $\boldsymbol{f}$ is represented as a sum [4, §75, 81]:

$$\boldsymbol{f} = -\nabla P_0 + \rho\boldsymbol{g} + \boldsymbol{f}^{\mathrm{H}} + \boldsymbol{f}^{\mathrm{A}} + \boldsymbol{f}^{\mathrm{d}}, \quad (8)$$

where $\boldsymbol{g}$ is the acceleration of gravity;

$$f^{\mathrm{H}} = -\frac{1}{8\pi}(\langle E^2\rangle\nabla\varepsilon + \langle H^2\rangle\nabla\mu) + \frac{1}{8\pi}\nabla\rho\Big[\Big(\frac{\partial\varepsilon}{\partial\rho}\Big)_T\langle E^2\rangle + \Big(\frac{\partial\mu}{\partial\rho}\Big)_T\langle H^2\rangle\Big] \quad (9)$$

is the Helmholtz power;

$$f^{\mathrm{A}} = \frac{\varepsilon\mu - 1}{4\pi c}\frac{\partial}{\partial t}\langle \boldsymbol{E}\times\boldsymbol{H}\rangle \quad (10)$$

is the Abraham force; $\boldsymbol{f}^{\mathrm{d}}$ is the force associated with dispersion, which for a non-magnetic medium ($\mu = 1$) was investigated in [4, §81; 7–9]. For an arbitrary medium (where not only $\partial\varepsilon/\partial\omega \neq 0$, but also $\partial\mu/\partial\omega \neq 0$) we can only assume that the force $\boldsymbol{f}^{\mathrm{d}}$ (as the Abraham force $\boldsymbol{f}^{\mathrm{A}}$) is of the order of $|\boldsymbol{E}_0|^2/(c\tau_0)$. We will restrict ourselves below to these fields (2), for which $c\tau_0 \gg L_0$, where L_0 is the distance characterising the functions $|\boldsymbol{E}_0(\boldsymbol{r}, t)|$ and $|\boldsymbol{H}_0(\boldsymbol{r}, t)|$. For these fields the Abraham force $\boldsymbol{f}^{\mathrm{A}}$ and the force $\boldsymbol{f}^{\mathrm{d}}$ are negligible compared with the Helmholtz force $\boldsymbol{f}^{\mathrm{H}}$.

As in [4, §16], we assume below that the liquid is homogeneous in composition and in thermal equilibrium ($\nabla T = 0$). Therefore,

$$\nabla\varepsilon = \Big(\frac{\partial\varepsilon}{\partial\rho}\Big)_T\nabla\rho,\ \nabla\mu = \Big(\frac{\partial\mu}{\partial\rho}\Big)_T\nabla\rho. \quad (11)$$

With (11) taken into account, expression (8) for the force is simplified and the equation $\boldsymbol{f} = 0$ is reduced to the form

$$\nabla P_0 = \rho\nabla\Big\{\boldsymbol{gr} + \frac{1}{8\pi}\Big[\Big(\frac{\partial\varepsilon}{\partial\rho}\Big)_T\langle E^2\rangle + \Big(\frac{\partial\mu}{\partial\rho}\Big)_T\langle H^2\rangle\Big]\Big\}. \quad (12)$$

Equation (12) is easily solved [4, §16] if the density of the liquid is considered constant ($\nabla\rho = 0$):

$$P_0(\boldsymbol{r}, t) = p(\boldsymbol{r}) + \frac{\rho}{8\pi}\Big[\Big(\frac{\partial\varepsilon}{\partial\rho}\Big)_T\langle E^2\rangle + \Big(\frac{\partial\mu}{\partial\rho}\Big)_T\langle H^2\rangle\Big], \quad (13)$$

where

$$p(\boldsymbol{r}) = p_0 + \rho\boldsymbol{gr} \quad (14)$$

is the pressure in the absence of the field and the constant $p_0 = p(0)$.

Equation (12) is solved in the other extreme case – a rarefied gas. For this medium [4, §15]

$$\varepsilon(\omega) = 1 + 4\pi n\alpha(\omega),\ \mu(\omega) = 1 + 4\pi n\beta(\omega), \quad (15)$$

where n is the number of molecules per unit volume; $\alpha(\omega)$ and $\beta(\omega)$ are the electric and magnetic polarisabilities of a molecule. Equation (12) with account for expressions (15) and the equation of state (Clapeyron equation) $P_0 = nT$ (the Boltzmann constant is set equal to unity) is reduced to the form:

$$T\nabla n = nm\boldsymbol{g} + \frac{1}{2}n\nabla[\alpha(\omega)\langle E^2\rangle + \beta(\omega)\langle H^2\rangle], \quad (16)$$

where m is the mass of the molecule. The density n of molecules depends on the field. Within the framework of the linear electrodynamics ($\boldsymbol{D} \propto \boldsymbol{E}$, $\boldsymbol{B} \propto \boldsymbol{H}$), in (15) and, therefore, in the second term on the right hand side of (16) we should replace the density n by the density of molecules in the absence of the field $n_0(\boldsymbol{r})$ [4, §15]. After that, the solution of equation (16) takes the form

$$n(\boldsymbol{r}, t) = n_0(\boldsymbol{r})\Big[1 + \frac{\alpha(\omega)\langle E^2\rangle + \beta(\omega)\langle H^2\rangle}{2T}\Big], \quad (17)$$

$$n_0(\boldsymbol{r}) = n_0(0)\exp\Big(\frac{m\boldsymbol{gr}}{T}\Big). \quad (18)$$

For the pressure $P_0 = nT$ we obtain the same expression (13), but now the pressure $p(\boldsymbol{r})$ is determined not by (14) but by the formula

$$p(\boldsymbol{r}) = n_0(\boldsymbol{r})T = p_0\exp\Big(\frac{m\boldsymbol{gr}}{T}\Big),\ p_0 = n_0(0)T. \quad (19)$$

Substitution of P_0 (13) into (5) reduces the tensor to the form

$$\sigma_{ij} = -p(\boldsymbol{r})\delta_{ij} + \tilde{\sigma}_{ij},$$

$$\tilde{\sigma}_{ij} = \frac{1}{4\pi}\Big\{\varepsilon(\omega)\Big[\langle E_iE_j\rangle - \frac{1}{2}\langle E^2\rangle\delta_{ij}\Big] + \mu(\omega)\Big[\langle H_iH_j\rangle - \frac{1}{2}\langle H^2\rangle\delta_{ij}\Big]\Big\}, \quad (20)$$

where the tensor $\tilde{\sigma}_{ij}$ is actually derived from the tensor (7) by using the replacement (6). The first term in the expression for σ_{ij} in (20) makes [in view of (4)] the contribution $p(\boldsymbol{r})\boldsymbol{N}$, which after integration over the entire surface of the solid body results in a force equal, as it should be, to the Archimedean force $-M\boldsymbol{g}$, where M is the mass of a liquid (gas) in the volume of a solid body.

4. Consider a quasi-monochromatic plane wave in a liquid at rest. We repeat the calculations, which are presented in [4, §83, 86], but without imposing, however, any restriction on $\mu(\omega)$. For a quasi-monochromatic plane wave the functions $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ in (2) can be written in the form [4, §103]

$$\boldsymbol{E}_0(\boldsymbol{r},t) = \boldsymbol{E}_{00}(\boldsymbol{r},t)\exp(\mathrm{i}\boldsymbol{kr}),$$
$$\boldsymbol{H}_0(\boldsymbol{r},t) = \boldsymbol{H}_{00}(\boldsymbol{r},t)\exp(\mathrm{i}\boldsymbol{kr}), \tag{21}$$

where the amplitudes $\boldsymbol{E}_{00}$ and $\boldsymbol{H}_{00}$ are slowly [compared to $\exp(\mathrm{i}\boldsymbol{kr})$] varying functions of the coordinates:

$$\frac{1}{kL_0} \propto \frac{\lambda}{L_0} \ll 1 \tag{22}$$

($\lambda = 2\pi/k$ is the wavelength). Forces $\boldsymbol{f}^{\mathrm{A}}$ and $\boldsymbol{f}^{\mathrm{d}}$ in (8) can be ignored, unless the ratio of the small parameters (3) and (22) is

$$\frac{(\omega\tau_0)^{-1}}{(kL_0)^{-1}} \propto \frac{L_0}{c\tau_0} \ll 1, \text{ or } \tau_0 \gg L_0/c.$$

In the zeroth approximation in the small parameters (3) and (22), the amplitudes $\boldsymbol{E}_{00}$ and $\boldsymbol{H}_{00}$, the frequency ω and the wave vector $\boldsymbol{k}$ are related by [4, §83]

$$\boldsymbol{H}_{00} = \frac{c}{\omega\mu(\omega)}\boldsymbol{k}\times\boldsymbol{E}_{00},\ \boldsymbol{E}_{00} = -\frac{c}{\omega\varepsilon(\omega)}\boldsymbol{k}\times\boldsymbol{H}_{00}, \tag{23}$$

$$\boldsymbol{k}^2 = \frac{\omega^2}{c^2}\varepsilon(\omega)\mu(\omega). \tag{24}$$

For real $\varepsilon(\omega)$ and $\mu(\omega)$ the vector $\boldsymbol{k}$ can be real [then the field $E, H \sim \exp(\mathrm{i}\boldsymbol{kr})$ does not decay], only if at a given frequency ω, we have $\varepsilon(\omega) > 0$ and $\mu(\omega) > 0$, or $\varepsilon(\omega) < 0$ and $\mu(\omega) < 0$ [10, 11]. If the decay is negligible, it is convenient to introduce the unit vector

$$\boldsymbol{l} = \boldsymbol{k}/k,\ k = n(\omega)\omega/c,\ n(\omega) = \sqrt{\varepsilon(\omega)\mu(\omega)}$$

($\omega > 0$) [4, §83] and write (23) in the form

$$\boldsymbol{H}_{00} = \pm\sqrt{\frac{\varepsilon(\omega)}{\mu(\omega)}}\boldsymbol{l}\times\boldsymbol{E}_{00},\ \boldsymbol{E}_{00} = \pm\sqrt{\frac{\mu(\omega)}{\varepsilon(\omega)}}\boldsymbol{l}\times\boldsymbol{H}_{00}; \tag{25}$$

hereafter the upper (lower) sign corresponds to $\varepsilon(\omega)$, $\mu(\omega) > 0$ [$\varepsilon(\omega), \mu(\omega) < 0$].

For the energy density we obtain the expression [4, §83]

$$W = \frac{1}{16\pi\mu\omega}\frac{\partial(\omega^2\varepsilon\mu)}{\partial\omega}|\boldsymbol{E}_{00}|^2 = \frac{1}{16\pi\varepsilon\omega}\frac{\partial(\omega^2\varepsilon\mu)}{\partial\omega}|\boldsymbol{H}_{00}|^2, \tag{26}$$

and for the vector of the energy flux density we obtain the expression [10, 11]

$$\langle\boldsymbol{S}\rangle = \frac{c}{4\pi}\langle\boldsymbol{E}\times\boldsymbol{H}\rangle = \pm\frac{c}{8\pi}\sqrt{\frac{\varepsilon}{\mu}}|\boldsymbol{E}_{00}|^2\boldsymbol{l} = \pm\frac{c}{8\pi}\sqrt{\frac{\mu}{\varepsilon}}|\boldsymbol{H}_{00}|^2\boldsymbol{l}. \tag{27}$$

The expression for the group velocity is derived from (24):

$$\boldsymbol{V}_{\mathrm{gr}} = \frac{\partial\omega}{\partial\boldsymbol{k}} = \frac{c}{\partial(n\omega)/\partial\omega}\boldsymbol{l} = \frac{2\omega\varepsilon\mu}{\partial(\omega^2\varepsilon\mu)/\partial\omega}\boldsymbol{V}_{\mathrm{ph}}, \tag{28}$$

where the phase velocity of the wave is

$$\boldsymbol{V}_{\mathrm{ph}} = \boldsymbol{l}\frac{\omega}{k} = \boldsymbol{l}\frac{c}{n}. \tag{29}$$

It follows from (26) – (28) that, as expected [4, §80],

$$\langle\boldsymbol{S}\rangle = W\boldsymbol{V}_{\mathrm{gr}}. \tag{30}$$

The tensor $\tilde{\sigma}_{ij}$ in (20) for a quasi-monochromatic plane wave (2), (21) is reduced to the form

$$\tilde{\sigma}_{ij} = \frac{1}{8\pi}\mathrm{Re}\Big[\varepsilon(\omega)\Big(E^*_{00i}E_{00j} - \frac{1}{2}|E_{00}|^2\delta_{ij}\Big) + \mu(\omega)\Big(H^*_{00i}H_{00j} - \frac{1}{2}|H_{00}|^2\delta_{ij}\Big)\Big]. \tag{31}$$

We will express the amplitude $\boldsymbol{H}_{00}$ in (31) through $\boldsymbol{E}_{00}$ according to (25) and use the well-known formula [1, §86], relating the product of two unit antisymmetric tensors of rank 3, e_{ijk}, with a unit symmetric tensor of rank 2, δ_{ij}. After simple calculations, we obtain

$$\tilde{\sigma}_{ij} = -\frac{1}{8\pi}\varepsilon(\omega)|E_{00}|^2 l_i l_j. \tag{32}$$

In view of (27) the tensor (32) can be presented in the form:

$$\tilde{\sigma}_{ij} = \tilde{\sigma}_{ji} = \sigma^{\mathrm{PR}}_{ij},$$

where

$$\sigma^{\mathrm{PR}}_{ij} = -\frac{1}{\omega}k_i\langle S_j\rangle \tag{33}$$

is the stress tensor introduced by Polevoi and Rytov [12] (see also [13]).

5. To obtain formulas similar to (1) for the case of vacuum, it is necessary to consider the quasi-monochromatic plane wave in a liquid at rest, the wave being incident on an immobile wall. By $\boldsymbol{E}$ and $\boldsymbol{H}$ in (20) is meant, of course, the total field strength in the liquid near the wall, i.e., the intensities of the incident and reflected waves. We choose the coordinate axes in the same way as in [4, §86], and in Section 1. We assume that the wave attenuation at the frequency ω in the 1st medium (liquid or gas) is negligible. Then, the expressions for the unit vectors and $\boldsymbol{l}_0$ and $\boldsymbol{l}_1$ along the wave vector $\boldsymbol{k}_0$ and $\boldsymbol{k}_1$ of the incident and reflected waves and for the wave vector $\boldsymbol{k}_2$ (not necessarily real) of the refracted wave in the 2nd medium (solid body) will have the form:

$$l_{0x} = \pm\sin\theta_0,\ l_{0y} = 0,\ l_{0z} = \pm\cos\theta_0,$$
$$l_{1x} = \pm\sin\theta_0,\ l_{1y} = 0,\ l_{1z} = \mp\cos\theta_0, \tag{34}$$
$$k_{2x} = \pm\frac{\omega}{c}n_1\sin\theta_0,\ k_{2y} = 0,\ k^2_{2z} = \frac{\omega^2}{c^2}(\varepsilon_2\mu_2 - n_1^2\sin^2\theta_0),$$

where $n_1 = \sqrt{\varepsilon_1(\omega)\mu_1(\omega)}$.

Consider first the case when the incident wave is polarised perpendicular to the plane of incidence. Then, [see (23), (25) and (34)]

$$E^{(0)}_{00i} = E^{(0)}_{00}\delta_{iy},\ E^{(1)}_{00i} = E^{(1)}_{00}\delta_{iy},\ E^{(2)}_{00i} = E^{(2)}_{00}\delta_{iy},$$

$$H^{(0)}_{00x} = -\cos\theta_0\sqrt{\frac{\varepsilon_1}{\mu_1}}E^{(0)}_{00},\ H^{(0)}_{00y} = 0,\ H^{(0)}_{00z} = \sin\theta_0\sqrt{\frac{\varepsilon_1}{\mu_1}}E^{(0)}_{00},$$

$$H^{(1)}_{00x} = \cos\theta_0\sqrt{\frac{\varepsilon_1}{\mu_1}}E^{(1)}_{00},\ H^{(1)}_{00y} = 0,\ H^{(1)}_{00z} = \sin\theta_0\sqrt{\frac{\varepsilon_1}{\mu_1}}E^{(1)}_{00}, \quad (35)$$

$$H^{(2)}_{00x} = -\frac{\kappa_2}{\mu_2}E^{(2)}_{00},\ H^{(2)}_{00y} = 0,\ H^{(2)}_{00z} = \pm\frac{n_1}{\mu_2}\sin\theta_0 E^{(2)}_{00},$$

where $\kappa_2 = ck_{2z}/\omega$, and $\mathrm{Im}\,\kappa_2 > 0$, in accordance with the fact that the wave decays deep into the solid body [4, §86]. The boundary conditions (continuity of the functions E_y and H_x at the liquid – solid body interface) have the form

$$E^{(0)}_{00} + E^{(1)}_{00} = E^{(2)}_{00},\ \sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_0\left(E^{(0)}_{00} - E^{(1)}_{00}\right) = \frac{\kappa_2}{\mu_2}E^{(2)}_{00}. \quad (36)$$

These boundary conditions make it possible to find the reflectivity $r_\perp = E^{(1)}_{00}/E^{(0)}_{00}$ with respect to the amplitude:

$$r_\perp = \frac{\mu_2\sqrt{\varepsilon_1/\mu_1}\cos\theta_0 - \kappa_2}{\mu_2\sqrt{\varepsilon_1/\mu_1}\cos\theta_0 + \kappa_2}. \quad (37)$$

Using (27) it is easy to show that the power reflectivity has the form

$$R_\perp = \frac{|\langle S_{1z}\rangle|}{|\langle S_{0z}\rangle|} = |r_\perp|^2. \quad (38)$$

Similarly, we can consider the case when the incident wave is polarised in the plane of incidence [4, §86]. In this case [see (23), (25) and (34)]

$$H^{(0)}_{00i} = H^{(0)}_{00}\delta_{iy},\ H^{(1)}_{00i} = H^{(1)}_{00}\delta_{iy},\ H^{(2)}_{00i} = H^{(2)}_{00}\delta_{iy},$$

$$E^{(0)}_{00x} = -\sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_0 H^{(0)}_{00},\ E^{(0)}_{00y} = 0,\ E^{(0)}_{00z} = -\sqrt{\frac{\mu_1}{\varepsilon_1}}\sin\theta_0 H^{(0)}_{00},$$

$$E^{(1)}_{00x} = -\sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_0 H^{(1)}_{00},\ E^{(1)}_{00y} = 0,\ E^{(1)}_{00z} = -\sqrt{\frac{\mu_1}{\varepsilon_1}}\sin\theta_0 H^{(1)}_{00}, \quad (39)$$

$$E^{(2)}_{00x} = \frac{\kappa_2}{\varepsilon_2}H^{(2)}_{00},\ E^{(2)}_{00y} = 0,\ E^{(2)}_{00z} = \mp\frac{n_1}{\varepsilon_2}\sin\theta_0 H^{(2)}_{00}.$$

The boundary conditions (continuity of the functions H_y and E_x) have the form

$$H^{(0)}_{00} + H^{(1)}_{00} = H^{(2)}_{00},\ \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_0\left(H^{(0)}_{00} - H^{(1)}_{00}\right) = \frac{\kappa_2}{\varepsilon_2}H^{(2)}_{00}. \quad (40)$$

From them we obtain the reflectivity $r_\parallel = H^{(1)}_{00}/H^{(0)}_{00}$ with respect to the amplitude:

$$r_\parallel = \frac{\varepsilon_2\sqrt{\mu_1/\varepsilon_1}\cos\theta_0 - \kappa_2}{\varepsilon_2\sqrt{\mu_1/\varepsilon_1}\cos\theta_0 + \kappa_2}; \quad (41)$$

the power reflectivity

$$R_\parallel = \frac{|\langle S_{1z}\rangle|^2}{|\langle S_{0z}\rangle|^2} = |r_\parallel|^2. \quad (42)$$

The desired force $\tilde{P}_i$ is obtained from the first formula in (7) for $N_i = \delta_{iz}$ and formula (20):

$$\tilde{P}_i = \frac{1}{8\pi}\varepsilon_1(\omega)\left[\frac{1}{2}|\boldsymbol{E}^{(01)}|^2\delta_{iz} - \mathrm{Re}\left(E_i^{(01)*}E_z^{(01)}\right)\right]$$
$$+\frac{1}{8\pi}\mu_1(\omega)\left[\frac{1}{2}|\boldsymbol{H}^{(01)}|^2\delta_{iz} - \mathrm{Re}\left(H_i^{(01)*}H_z^{(01)}\right)\right], \quad (43)$$

where $\boldsymbol{E}^{(01)} = \boldsymbol{E}^{(0)}_{00} + \boldsymbol{E}^{(1)}_{00}$ and $\boldsymbol{H}^{(01)} = \boldsymbol{H}^{(0)}_{00} + \boldsymbol{H}^{(1)}_{00}$. Simple calculations, which use formulas (35), (37), (38) and (39), (41), (42) lead to the following results:

$$\tilde{P}_{\perp x} = \frac{\varepsilon_1}{8\pi}|E^{(0)}_{00}|^2(1 - R_\perp)\sin\theta_0\cos\theta_0,\ \tilde{P}_{\perp y} = 0,$$
$$\tilde{P}_{\perp z} = \frac{\varepsilon_1}{8\pi}|E^{(0)}_{00}|^2(1 + R_\perp)\cos^2\theta_0, \quad (44)$$

$$\tilde{P}_{\parallel x} = \frac{\mu_1}{8\pi}|H^{(0)}_{00}|^2(1 - R_\parallel)\sin\theta_0\cos\theta_0,\ \tilde{P}_{\parallel y} = 0,$$
$$\tilde{P}_{\parallel z} = \frac{\mu_1}{8\pi}|H^{(0)}_{00}|^2(1 + R_\parallel)\cos^2\theta_0. \quad (45)$$

In view of (27), (29) and (30), expressions (44) and (45) can be expressed in one form for both polarisations

$$\tilde{P}_x = \pm n_1(\omega)\frac{\langle S_{0z}\rangle}{c}(1 - R)\sin\theta_0,\ \tilde{P}_y = 0,$$
$$\tilde{P}_z = \pm n_1(\omega)\frac{\langle S_{0z}\rangle}{c}(1 + R)\cos\theta_0, \quad (46)$$

where, as usual, the upper (lower) sign corresponds to the case when the group velocity of the wave in a medium (liquid or gas) is positive, i.e., parallel to the phase velocity (negative, i.e., antiparallel to the phase velocity).

Expressions (46) differ from expressions (1) for the vacuum only by the factor $\pm n_1(\omega)$. Note that in the same way as in (1), in (46) there are no terms corresponding to the interference of incident and reflected waves.

If a sold body has the shape of a plate whose thickness in the z direction is so great that the wave refracted in it completely decays, then the total force exerted by a light pulse on the body is obtained by integrating the force (46) with respect to the face of the plate onto which the light pulse is incident.

Note that the force (46) can be expressed through the stress tensor (33) related to the incident wave by the expression:

$$\tilde{P}_i = -(1 - R)\sigma^{\mathrm{PR}(0)}_{iz}\ (i = x, y),\ \tilde{P}_z = -(1 + R)\sigma^{\mathrm{PR}(0)}_{zz}. \quad (47)$$

6. In Section 1 we have already mentioned papers [2, 3], concerning only the force $\tilde{P}_z$ at normal incidence ($\theta_0 = 0$). Veselago [2] considers the wave incident normally on a perfectly reflective body. The author proceeds from the following assumptions: (i) the direction of the force with which the incident and reflected waves act on the body is the same as the direction of the momentum of the incident wave, and (ii) the momentum (or rather the momentum density) of the wave coincides with the direction of the wave vector $\boldsymbol{k}$ and phase velocity V_{ph} of the wave*. Therefore, when the group velocity in the medium is positive ($\boldsymbol{k}_0 \uparrow\uparrow \langle \boldsymbol{S}_0 \rangle$), 'light pressure' takes place, and when the group velocity is negative ($\boldsymbol{k}_0 \uparrow\downarrow \langle \boldsymbol{S}_0 \rangle$), 'light pressure' is replaced by 'light attraction'. The initial assumptions are wrong: the power is determined not by the momentum but by its change per unit time, or the momentum flux (or, by the stress tensor differing from it in the sign); the direction of the momentum density of the field coincides with that of the energy flux density (differs from $\langle \boldsymbol{S} \rangle$ by the factor $1/c^2$, see [4, §75]). However, the sign of the force $\tilde{P}_z$ predicted in [2] is correct [see (46) for $\theta_0 = 0$ and $R = 1$].

Veselago [3] also considers the case of normal incidence of the wave on a body completely absorbing radiation. The author rejects both assumptions, which he used in his previous work [2]. He agrees that the force is determined not by the momentum but by the momentum flux. The momentum flux density of one (only incident) wave is written in [3] without any justification as the ratio of the energy flux density to the phase velocity of the wave. Veselago apparently does not notice that this ratio coincides with the component $-\sigma_{zz}^{\text{PR}(0)}$ of the Polevoi–Rytov tensor [see (47) at $\theta_0 = 0$ and $R = 0$], which he discusses at the end of his article.

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* Veselago [2] refers to [12]. As for papers [12, 13], we note only that their authors believe that in an isotropic medium the group velocity of the wave $\boldsymbol{V}_{\text{gr}}$ has always the same direction as the phase velocity $\boldsymbol{V}_{\text{ph}}$ ($\boldsymbol{V}_{\text{gr}} \uparrow\uparrow \boldsymbol{V}_{\text{ph}}$), referring to the first edition of book [4] and not noticing that the existing proving is presented from the very beginning in [4, §84] for media with $\mu = 1$.