

LIGHT SCATTERING

PACS numbers: 42.25.Gy; 87.64.Cc; 42.66.Lc
DOI: 10.1070/QE2012v042n04ABEH014767

Scattering of light passing through a statistically rough interface between media with different refractive indices after laser correction of vision

A.V. Semchishen, V.N. Seminogov, V.A. Semchishen

Abstract. **Forward scattering of light passing through large-scale irregularities of the interface between two media having different refractive indices is considered. An analytical expression for the ratio of intensities of directional and diffusion components of scattered light in the far-field zone is derived. It is theoretically shown that the critical depth of possible interface relief irregularities, starting from which the intensity of the diffuse component in the passing light flow becomes comparable with the directional light component, responsible for the image formation on the eye retina, is 3–4 μm, with the increase in the refractive index in the postoperational zone taken into account. These profile depth values agree with the experimentally measured ones and may affect the contrast sensitivity of vision.**

Keywords: *ablation, photorefractive surgery, scattering, rough surface.*

1. Introduction

Modern techniques of photorefractive surgery correcting the refraction abnormalities allow high-probability restoration of visual acuity up to the baseline of preoperational best corrected visual acuty under photopic conditions (day-light vision). However, clinical observations and experimental data show that after such operations the contrast acuity of vision may be lower than before the operation with lens correction, particularly, under the mesopic (twilight vision) and scotopic (night vision) conditions of illumination. The reduction of the contrast acuity of vision is more or less present in all patients and is due neither to surgical implications, nor to higher-order aberrations induced [1]. Therefore, there are other factors affecting the contrast acuity of vision, one of which being, apparently, the roughness of the operation zone surface, or the interface, as they commonly refer it in modern ophthalmology. This roughness results from the action of a mechanical microkeratome or femtosecond laser radiation used to form the corneal flap, as well as the radiation of an excimer laser that forms the cornea ablation profile, required to correct the abnormalities of refraction.

Earlier [2, 3] we reported the estimates of the effect of irregularities of the surface of the stromal part of the eye cornea, appearing at the interface and having the larger size compared with the wavelength of light, on the quality of the image produced on the retina. It was shown that such large-scale irregularities may affect the acuity of vision even under high-contrast photopic conditions. In the present paper we consider forward scattering of light passing through a statistically rough interface between two media having different values of the refractive index. We derived an analytical expression for the ratio of directional component intensity to that of the diffuse one in the far-field zone. We also calculated the critical values of the irregularity size parameters of the interface relief, at which the intensity of the diffuse component in the light flow passed through the interface becomes greater than the intensity of the directional light component, responsible for producing the image on the eye retina.

A.V. Semchishen, V.N. Seminogov, V.A. Semchishen Institute of Laser and Information Technologies, Russian Academy of Sciencies, Pionerskaya ul. 2, 142190 Troitsk, Moscow region, Russia;
e-mail: tonymob@yandex.ru

Received 10 November 2011; revision received 27 January 2012
Kvantovaya Elektronika **42** (4) 345–349 (2012)
Translated by V.L. Derbov

2. Formulation of the problem

At present the most rigorous of the existing methods that allow description of light scattering by surfaces not only with a small-scale ($h \ll \lambda$), but also with a large-scale ($h \gg \lambda$) relief is the Kirchhoff method, based on the idea of replacing the real rough surface at each point with a tangent plane. The refraction and reflection of the incident wave by the tangent plane is assumed to obey the Snell and Fresnel laws. The method is applicable to surfaces with large-scale irregularities ($l_c \gg \lambda$), in which $\sigma \ll l_c/\lambda$, where $\sigma = h/l_c$ is the mean-square slope, h is the mean-square depth of the surface irregularities, λ is the wavelength of incident radiation in vacuum, l_c is the correlation length that determines the characteristic size of irregularities along the interface between the media. Based on this approach the theories of electromagnetic wave scattering by an ideally-conducting surface were developed for both periodic [4] and stochastic [5, 6] irregularities of the interface between two media. The theory of scattering due to reflection from a statistically rough surface, based on the Kirchhoff method, is thoroughly discussed in books [7, 8]. Except our papers [9, 10] we do not know other papers where this method was used to study scattering of plane waves caused by refraction at a statistically rough interface.

In Refs [9, 10] the Kirchhoff method and the integral Stratton–Chu relations [11] were employed to develop a vector analytical theory to describe the refraction of arbitrarily polarised light beam with an arbitrary distribution of intensity in the cross section, normally incident onto an interface between two dielectrics possessing a large-scale roughness. The mathematical essence of the Kirchhoff method is presented and the conditions of its validity are formulated, which explicitly determine the limitations imposed on the statistical parameters of the surface. General analytical expressions are derived for the mean intensity of the directional and diffuse components of the scattered radiation, valid both in the Fraunhofer and the Fresnel zones. The spatial distribution of the mean intensity of the directional component and the spatial evolution of the distribution of mean intensity of the diffuse component are investigated at weak ($h \ll \lambda$) and strong ($h \gg \lambda$) phase fluctuations, respectively. In the present paper we consider a particular problem of light passing the rough boundary of the eye cornea interface after the vision correction by means of ablation of the stromal part of the cornea using the excimer ArF laser (λ = 193 nm).

We make use of a reduced optical scheme of a human eye, in which the cornea and the lens are replaced with a single lens having the focal length 17 mm. The interface zone is assumed to be localised in front of the lens, as shown in Fig. 1. The rough interface between the first dielectric medium with the refractive index $n_1 > 0$ and the second medium with the refractive index $n_2 > 0$ is described by the equation $z = f(x, y)$. Let this surface be irradiated with a light beam having the radius r_0 and the uniform intensity I_0. Our goal is to determine the ratio of intensities of the directional and diffuse components of the scattered light in the far-field zone, since it is equivalent to the ratio of these quantities in the focal plane of a reduced eye.

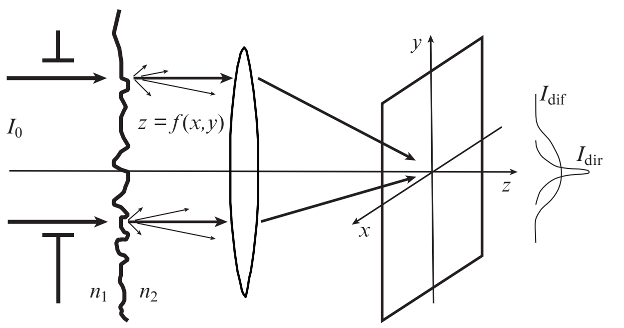

Figure 1. Reduced optical scheme of the eye.

Let us assume that the rough surface is spatially homogeneous and statistically isotropic, and the probability density distributions for the deviation of the surface $z = f(x,y)$ from the mean plane $\overline{f(x,y)} = 0$ (the bar over the function symbol denotes statistical averaging) obey the Gaussian law. In this case the probability density $w(f_x, f_y)$ for the distribution of the surface slopes $f_x = \partial f(x, y)/\partial x$ и $f_y = \partial f(x, y)/\partial y$ possesses random character and is determined by the equation

$$w(f_x, f_y)\,{\rm d}f_x{\rm d}f_y = \frac{1}{2\pi\sigma^2}\exp\left(-\frac{f_x^2 + f_y^2}{2\sigma^2}\right){\rm d}f_x{\rm d}f_y,$$

where $\sigma^2 = \overline{f_x^2} = \overline{f_y^2}$.

The detailed mathematical description of the problem is presented in [9, 10]. The physical nature of the considered optical approximation is the following. Each point of the surface acts as a source of a continuum of microscopic beams, refracted following the Fresnel and Snell laws by the surface irregularities having the Gaussian statistics of the local slopes. Microbeams, scattered in one direction, are focused by the eye into a definite point of the retina. In our scheme this is a point in the focusing lens focal plane. Hence, the illuminance at this point is a result of mutual action of microbeams, scattered in one direction from all regions of the illuminated irregular surface.

In an eye having ideal optical quality of the cornea surface the light beam, passing through the pupil, is focused into a small spot on the retina in correspondence with the laws of diffraction. In the real eye the intensity distribution in the light spot on the retina is determined by aberrations. In the presence of irregularities on the interface between the media having different refractive indices a certain fraction of the radiation is scattered, thus reducing the contrast of the point image. To solve the formulated problem it is sufficient to calculate the intensity $\overline{I(0,0,z_{\rm p})}$ in the far-field zone at a certain point $z = z_{\rm p}$ = const belonging to the semispace $z > f(x,y)$ (Fig. 1).

3. Calculation of intensity ratio of the directional and diffuse scattered light components

Consider the light propagation through a rough surface from the medium with the refractive index n_1 into the medium with the refractive index n_2. Assume that the correlation radius $l_{\rm c}$, the vacuum wavelength of light λ, and the radius (determined by the pupil size) r_0 of the light beam, incident on the surface, satisfy the inequality $\lambda \ll l_{\rm c} \ll r_0$.

To evaluate the effect of the rough interface boundary on the quality of the image on the retina it will be sufficient to determine the intensity ratio of scattered and directional components of the light on the visual axis, i.e., at a single point in the centre of the yellow spot of the retina. Under these conditions one can derive analytical expressions for the light energy of the diffuse component after passing through the interface, as well as for the intensities of the directional and diffuse components of the scattered light. From Eqn (44) of Ref. [9] it follows that in the far-field zone ($z \gg r_0^2/\lambda$) the intensity of the directional component at the observation point ($x = y = 0$, z = const) can be presented in the form

$$I_{\rm dir}(x = 0, y = 0, z) = \frac{n_2^2 k_0^2 F}{4\pi^2 z^2} I_0(\pi r_0^2)^2 \exp(-\sigma_{\rm ph}^2), \qquad (1)$$

where $k_0 = 2\pi/\lambda$; $F = 4n_1n_2/(n_1 + n_2)^2$ is the Fresnel transmission coefficient in the case of a plane surface; $\sigma_{\rm ph}^2 = k_0^2 \times (n_1 - n_2)^2 h^2$; $\sigma_{\rm ph}$ is the mean-square phase fluctuation of the wave, passed through the rough surface.

In correspondence with Eqn (45) from Ref. [9], the expression for the ratio of the directional component energy $E_{\rm dir}$ to the energy of the incident radiation E_0 has the form

$$\frac{E_{\rm dir}}{E_0} = F\exp\left[-4\pi^2(n_1 - n_2)^2\left(\frac{h}{\lambda}\right)^2\right]. \qquad (2)$$

The intensity distribution for the diffuse component at the observation point ($x = y = 0$, z = const) in the far-field zone, following from the general formula (36) of Ref. [9], has the form

$$I_{\rm dif}(x = 0, y = 0, z) = \frac{n_2^2 k_0^2 l_{\rm c}^2 F}{2\pi z^2} I_0(\pi r_0^2)\exp\left(-\sigma_{\rm ph}^2\right)\sum_{\gamma=1}^{\infty}\frac{\sigma_{\rm ph}^{2\gamma}}{\gamma\gamma!}. \quad (3)$$

From the distributions (1) and (3) we find that the ratio of intensities of the directional and diffuse components of the scattered field in the far-field zone can be expressed as

$$\frac{I_{\rm dir}}{I_{\rm dif}} = \frac{1}{2}\left(\frac{r_0}{l_{\rm c}}\right)^2 \frac{1}{\Gamma(\sigma_{\rm ph}^2)}, \quad \Gamma(\sigma_{\rm ph}^2) = \sum_{\gamma=1}^{\infty}\frac{(\sigma_{\rm ph}^2)^\gamma}{\gamma\gamma!}. \qquad (4)$$

Let us introduce the notation $\varphi = \sigma_{\rm ph}^2$. It is easy to see that $\Gamma(\varphi = 0) = 0$ and

$$\frac{{\rm d}\Gamma(\varphi)}{{\rm d}\varphi} = \sum_{\gamma=1}^{\infty} \frac{\varphi^{\gamma-1}}{\gamma!} = \frac{1}{\varphi}\sum_{\gamma=1}^{\infty} \frac{\varphi^{\gamma}}{\gamma!} = \frac{1}{\varphi}\left(\sum_{\gamma=1}^{\infty} \frac{\varphi^{\gamma}}{\gamma!} + 1 - 1\right)$$

$$= \frac{1}{\varphi}\left(\sum_{\gamma=0}^{\infty} \frac{\varphi^{\gamma}}{\gamma!} - 1\right) = \frac{{\rm e}^{\varphi} - 1}{\varphi}. \quad (5)$$

From Eqn (5), taking the boundary condition $\Gamma(\varphi = 0) = 0$ into account, we get

$$\Gamma(\varphi) = \int_0^{\varphi} \frac{{\rm e}^t - 1}{t} {\rm d}t. \quad (6)$$

Expanding the exponential function in powers of t, it is easy to see that Eqn (6) coincides with the expression for the function $\Gamma(\varphi)$ in Eqn (4).

At small $\sigma_{\rm ph}^2$ we have $\Gamma(\sigma_{\rm ph}^2) = \sigma_{\rm ph}^2$. In this case Eqn (4) yields the expression

$$\frac{I_{\rm dir}}{I_{\rm dif}} = \frac{1}{2}\left(\frac{r_0}{l_{\rm c}}\right)^2 \frac{1}{\sigma_{\rm ph}^2} = \frac{1}{8\pi^2(n_1 - n_2)^2}\left(\frac{r_0}{l_c}\right)^2\left(\frac{h}{\lambda}\right)^{-2}. \quad (7)$$

Let us determine the asymptotic behaviour of the ratio (7) at high h. Using Eqn (6) and the L'Hospital rule, one can prove that at $\varphi > 2$ the following expansion is valid:

$$\Gamma(\varphi) = \frac{{\rm e}^{\varphi}(1 - {\rm e}^{-\varphi})}{\varphi}\left(1 + \frac{1}{\varphi} + \frac{2!}{\varphi^2} + \frac{3!}{\varphi^3} + \ldots\right). \quad (8)$$

Substituting Eqn (8) into the expression (4), we get

$$\frac{I_{\rm dir}}{I_{\rm dif}} = \frac{1}{2}\left(\frac{r_0}{l_{\rm c}}\right)^2 \frac{\sigma_{\rm ph}^2 {\rm e}^{-\sigma_{\rm ph}^2}}{(1 - {\rm e}^{-\sigma_{\rm ph}^2})} =$$

$$= \frac{2\pi^2(n_1 - n_2)^2 \exp[-4\pi^2(n_1 - n_2)^2(h/\lambda)^2]}{1 - \exp[-4\pi^2(n_1 - n_2)^2(h/\lambda)^2]}\left(\frac{r_0}{l_{\rm c}}\right)^2\left(\frac{h}{\lambda}\right)^2. \quad (9)$$

4. Results and discussion

When correcting the vision, the roughness of the interface surface appears as a result of the mutual action of excimer laser radiation and mechanical or laser instruments used to cut off the eye corneal flap in the method of laser keratomileusis in situ (LASIK), or in the process of ablation of the cornea surface in the case of photorefractive keratectomy (PRK). Because of the arising post-operation irregularities a fraction of the radiation incident on the eye is diffusively scattered when passing through the cornea rough surface and produces a flare on the retina that does not carry information about the object. The remaining part of the radiation passes through the fragments of the surface with $l_{\rm c} = 0$ without scattering and produces a sharp image of the object. On the retina of such an eye one will observe a sharp image of the object against the flare background produced by the scattered light. The distribution of the light passed through the interface between the diffuse and directional components, as well as the intensity ratio of these components at the point (0, 0, z = const), depend on the depth of the surface relief. In the course of the regenerative process the surface irregularities of the cornea stromal part, induced in the region of laser ablation when using the PRK method, are covered with epithelial cells, which rather repeat the stroma surface relief and possess almost similar refractive index ($n_2 = 1.376$). In this method of correction the scattering occurs at the interface between the epithelium and the lacrimal film whose refractive index is $n_1 =$ 1.33. When the LASIK method is used, the light flux after the operation passes through the rough surfaces of the cut-off flap and the zone of the cornea stromal part, ablated by the laser radiation, and is partially scattered by these surfaces. The space between the flap and the stromal bed with the refractive index $n_2 = 1.376$, is most probably filled with the intraocular fluid, whose refractive index is $n_1 = 1.336$.

All calculations are performed for an ideal eye, and no higher-order aberrations, individual in each patient, are taken into account. Then the distribution function for the energy in the image of a remote point source at the retina of the eye is determined only by the diffraction of light passing through the pupil. In a real eye the contribution of aberrations is sufficiently high [12], and when the pupil diameter exceeds 4 mm the energy distribution function, produced by a distant point source on the retina, is completely determined by the high-order aberrations. Hence, in the calculations presented below we assume the pupil radius to be $r_0 = 2$ mm, and the correlation length $l_{\rm c} = 100$ μm.

Figure 2 presents the theoretically calculated dependence (2) of the ratio of light energy $E_{\rm dir}$, carried by the directional component, to the energy of the radiation E_0, incident on the coarse surface, on the ratio h/λ for $\Delta n = n_2 - n_1 = 0.04$. It is seen that for the light with $\lambda = 0.5$ μm the scattering by irregularities with the relief depth of 3 – 4 μm leads to such an angle redistribution of the light energy that the directional component fraction appears to be no greater than 10% of the incident energy.

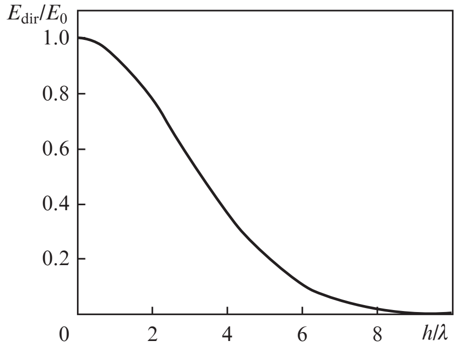

Figure 2. Ratio of the energy $E_{\rm dir}$ carried by the directional component to the energy E_0 of the radiation, incident on the rough surface, vs. the ratio h/λ for $\Delta n = 0.04$

Using Eqns (4) and (6) we calculated the dependence of the ratio of the maximal intensity of the directional component $I_{\rm dir}^{\rm max}$ to that of the diffuse component $I_{\rm dif}^{\rm max}$ in the far-field zone (in our case on the eye retina) on h/λ (Fig. 3). The choice of the values $\Delta n = 0.04$ and 0.08 is caused by the increase of the refractive index of the cornea surface ablated by the radiation of the excimer laser ($\lambda = 193$ nm). As shown in Ref. [13], the refractive index of the ablated eye stroma surface depends on the exposure radiation dose and may be as large as 1.41. Its increase leads to the reduction of the critical size of the irregularities. Thus, for $\Delta n = 0.04$ (difference of refractive index values before the operation) the diffusive

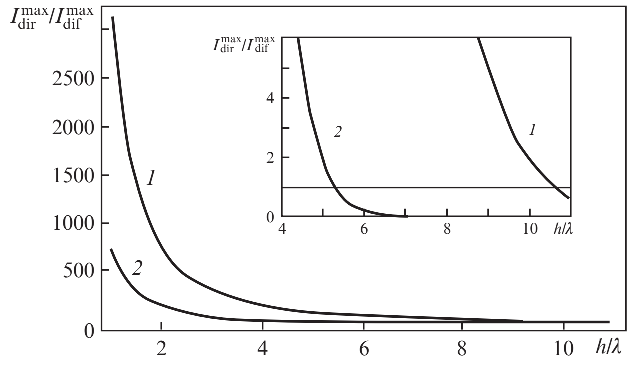

Figure 3. Dependences of the ratio $I_{dir}^{max}/I_{dif}^{max}$ in the far-field zone on h/λ, plotted using Eqns (4) and (6) at $r_0 = 2$ mm, $l_c = 100$ μm, $\Delta n = 0.04$ (*1*) and 0.08 (*2*). The inset shows a magnified fragment of the plot.

component becomes essential ($I_{dir} = I_{dif}$) at the irregularity depth $h > 5$ μm. After the vision correcting operation Δn may reach 0.08, and the irregularities having the depth $h > 3$ μm may affect the contrast sensitivity of vision.

Table 1 summarises the surface relief depth values, measured when using the mechanical microkeratome (Nidek) and femtosecond lasers Intralase (repetition rate 15 and 30 kHz) and Da Vinci [14]. It is seen that the depths of the irregularities at the stroma surface are nearly the same (2–3 μm) after the use of both the microkeratome and the femtosecond laser. The measurements show that in the most advanced surgical setups the laser ablation causes additional roughness with $h \approx$ 2–4 μm. Therefore, we can safely assume the depth of characteristic stroma irregularities after the vision correction operation to be 3–4 μm. As shown by histological studies and electron microscopy of the cornea sections, after cutting off the flap and repositioning it back onto the stromal bed a cavity arises, i.e., the flap does not tightly adjoin the stromal part of the cornea. After the laser ablation this gap may only be enlarged. Due to the biomechanical disintegration of the cornea, the flap does not move as a unit with the corneal bed, which also affects its gapping. Most probably, in a favourable case this gap will be filled with a liquid having similar optical parameters as the intraocular one. The analysis of the roughness appearing on the stroma surface after ablation yields the correlation length $l_c \leqslant 100$ μm.

From the dependences shown in Figs 2 and 3 it is seen that when the roughness depth is $h \geqslant 10\lambda$, practically all the energy of the incident radiation appears to be concentrated in the diffuse component. The ratio of the intensity of the directional component that produces sharp image at the retina to the intensity of the diffuse component is close to unity. Let us calculate the width of the distribution $2\rho_{dif}$ of the diffuse component intensity at the level exp(–2) at the retina surface using the formula $2\rho_{dif} = (4h/l_c)(n_2/n_1 - 1)f$ [10], where f is the focal length of the lens. Assume the focal length of the reduced eye to be 17 mm and the ratio $h/l_c = 0.05$. Then it is easy to calculate that, depending on Δn, the diameter of the diffuse spot on the retina will be 100–200 μm. The surface relief depth, at which the scattered component becomes essential and the directional component may be neglected, is determined by the condition $I_{dif} \geqslant I_{dir}$: $2\pi^2(n_2 - n_1)^2(h/\lambda)^2 \geqslant \ln(r_0/l_c)$ [10]. This condition is valid for an ideal aberration-free eye.

Table 1. Depth of roughness produced by microkeratomes of different types [14].

Manufacturer	Depth of surface roughness/μm	
	Mean	Minimal
Nidek (MK2000)	2.6±1.5	1.1±1.1
Intralase (FS15)	3.1±0.7	2.1±0.6
Intralase (FS30)	2.8±0.6	1.9±0.4
Da Vinchi	3.0±0.3	2.0±0.4

5. Conclusion

Thus, in the present paper it is established that the roughness a few micrometres deep, produced in the laser ablation zone, may affect the contrast sensitivity of the vision of patients after photo-refraction vision-correcting operations, particularly under the condition of low contrast and low illumination, when the pupil is large and the image of a remote point on the retina is determined only by higher-order aberrations, for which the point-spreading function is much broader than the one caused by diffraction.

The reflection from the interface between the media is practically zero and, therefore, the interface cannot be observed in reflected light using, e.g., a slit lamp. Indeed, the Fresnel reflection coefficient for a surface separating media with different refractive indices may be calculated using the formula $R = [1 - 4n_2n_1/(n_2 + n_1)^2]$, i.e., in our case $R < 0.03\%$.

A few words should be said about what will happen after the vision correction using the PRK method. In this case the rough surface of the stromal part of the cornea is contiguous with the lacrimal film rather than with the intraocular liquid ($n_1 = 1.336$). In this sense there is no difference between the PRK and LASIK methods. In the process of re-epithelisation (epithelium restoration) the influence of roughness on the acuity of vision will weaken, and after six months after the operation, when the rehabilitation processes will be completed, the vision of the patient may return to the preoperation level of contrast sensitivity. The process of smoothing the cornea surface up to the optical quality is determined by the simple nature of the re-epithelisation process [15]. On the contrary, after the LASIK correction of vision the quality of surface of the stromal part of cornea and the yawning flap remains practically changeless and the contrast sensitivity does not reach its preoperation level.

References

1. Pallikaris I.G., Kymionis G.D., Panagopoulou S.I. *J. Cataract Refract. Surg.*, **28**, 1737 (2002).
2. Semchishen V.A., Mrochen M. *Vestnik oftalmologii*, **120** (1), 43 (2004).
3. Mrochen M., Semchishen V. *J. Refract. Surg.*, **19** (5), 597 (2003).
4. Brekhovskikh L.M. *Zh. Eksp. Teor. Fiz.*, **23**, 275 (1952) [*Sov. Phys. JETP*, **23**, 289 (1952)].
5. Beckmann P., Spizzichino A. *The Scattering of Electromagnetic Waves from Rough Surfaces* (Oxford: Pergamon Press, 1963).
6. Isakovich M.A. *Zh. Eksp. Teor. Fiz.*, **23**, 305 (1952) [*Sov. Phys. JETP*, **23**, 305 (1952)].
7. Rytov S.A., Kravtsov Yu.A., Tatarskii V.I. *Vvedeniye v statisticheskuyu radiofiziku. Chast II. Sluchainyye polya* (Introduction to Statistical Radiophysics. Part II. Random fields) (Moscow: Nauka, 1978) p. 429.
8. Bass F.G., Fuks I.M. *Rasseyaniye voln na statisticheski nerovnoy poverkhnosti* (Scattering of Waves by a Statistically Rough Surface) (Moscow: Nauka, 1972).
9. Seminogov V.N. *Laser Phys.*, **12** (9), 1239 (2002).
10. Seminogov V.N., Semchishen V.A., Panchenko V.Ya., Seiler T., Mrochen M. *Laser Phys.*, **12** (11), 1333 (2002).

11. Stratton J.A. *Electromagnetic Theory* (New York, McGraw Hill, 1941).
12. Semchishen V.A., Mrochen M., Seiler T. *Refraktsionnaya khirurgiya i oftalmologiya*, **3** (1), 5 (2003).
13. Patel S., Flio J.L., Artola A. *J. Cataract Refract. Surg.*, **34**, 1077 (2008).
14. Ripken T., Bernau W., Oberheide U., Schumacher S., Knorz M., Kermani O., Lubatschowski H. *Proc. 19th Congress of German Ophthalmic Surgeons* (Nuremberg, 2006).
15. Huang D., Tang M., Shekhar R. *Am. J. Ophthalmol.*, **135**, 267 (2003).