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A laser gyro with a four-mirror square resonator: quantitative estimation of the dependence of the synchronisation zone parameters of the frequencies of counterpropagating waves on the active-medium gain

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Abstract. **For a laser gyro with a four-mirror square resonator (with a perimeter of 20 cm) we have calculated, on the basis of the previously developed [see Bondarenko E.A. *Quantum Electron.*, 41, 824 (2011)] model, the dependence of the parameters of the synchronisation zone of the frequencies of counterpropagating waves on the active-medium gain. The results obtained are in qualitative agreement with known experimental data for gyroscopes with three-mirror resonators.**

Keywords: *laser gyroscope, ring gas laser, frequency locking of counterpropagating waves.*

1. Introduction

Among main types of laser gyros widely used in practice, we can single out the device based on a ring gas He–Ne laser (the ratio of the isotope concentrations, ^{20}Ne : ^{22}Ne = 1:1) with a flat N-mirror ($N = 3, 4$) resonator ensuring generation of linearly polarised radiation in the sagittal plane. The laser, usually operating at 0.6328 μm, is pumped by a dc parallel discharge obtained by a common cathode and two anodes [1–3].

According to relations (5.55)–(5.57) from [3] and to expressions (6.45)–(6.47) from [4], when the currents are balanced in the discharge arms, the resonator is fine tuned to the centre of the emission line and the losses are identical, the system of equations describing the dynamics of the dimensionless intensities I_j ($j = 1, 2$) and the phase difference ψ of counterpropagating waves of such a laser gyro can be written as

$$\dot{I}_1 = (\alpha - \beta I_1 - \theta I_2) I_1 - 2r_2\sqrt{I_1 I_2}\cos(\psi + \varepsilon_2),$$
$$\dot{I}_2 = (\alpha - \beta I_2 - \theta I_1) I_2 - 2r_1\sqrt{I_1 I_2}\cos(\psi - \varepsilon_1), \quad (1)$$
$$\dot{\psi} = M\Omega + r_2\sqrt{I_2/I_1}\sin(\psi + \varepsilon_2) + r_1\sqrt{I_1/I_2}\sin(\psi - \varepsilon_1).$$

In deriving these equations it was taken into account that the wave with $j = 1$ propagates in the direction of the laser gyro rotation. In system (1) α, β, θ are the Lamb coefficients that characterise the properties of the active medium; $M = (1 + K_a)M_g$ is the scale factor of the laser gyro, primarily determined by the geometrical component $M_g = 8\pi S/(\lambda L)$ and also taking into account the properties of the medium through a small parameter K_a (L is the perimeter of the axial contour; S is the covered area); Ω is the angular velocity of the device rotation in the inertial space; r_j and ε_j are the moduli and arguments of complex integral coefficients $r_j\exp(i\varepsilon_j)$ of the linear coupling of counterpropagating waves, characterising their interaction through backscattering, absorption and transmission of radiation on the mirrors. (The relations for calculating the parameters α, β, θ of system (1) can be found, for example, in [5], and the parameter K_a – in [6]. An empirical formula for calculating K_a is derived in [3]. In addition, a set of expressions to estimate the parameters $\alpha, \beta, \theta, K_a, r_j, \varepsilon_j$ is given in [7]. These expressions are applicable for the case when the laser gyro operates at total pressures of the He–Ne mixture from 1 to 5–6 Torr, and its cavity has the shape of an equilateral triangle or a square.)

In our paper [8], based on the analysis of (1) we obtained the formulas for calculating the parameters of the synchronisation zone of the frequencies of counterpropagating electromagnetic waves generated in the laser gyro. These parameters are the coordinates $\Omega_{(-)}$ and $\Omega_{(+)}$ of the left and right boundaries of the synchronisation zone on the axis of the angular velocity Ω, the coordinate of its centre $\Omega_{(0)} = (\Omega_{(+)} + \Omega_{(-)})/2$ and the half-width of this zone $\Omega_s = (\Omega_{(+)} - \Omega_{(-)})/2$. The relations obtained in [8] supplement the results of earlier theoretical studies [3, 9–16] and have the form

$$\Omega_{(\pm)} = \pm\frac{\sqrt{r_p^2 + \mu^2 r_m^2 \pm 2\mu(r_2^2 - r_1^2)}}{\sqrt{1-\mu^2}\,M},$$
$$\Omega_{(0)} = \frac{\sqrt{r_p^2 + \mu^2 r_m^2 + 2\mu(r_2^2 - r_1^2)} - \sqrt{r_p^2 + \mu^2 r_m^2 - 2\mu(r_2^2 - r_1^2)}}{2\sqrt{1-\mu^2}\,M}, \quad (2)$$
$$\Omega_s = \frac{\sqrt{r_p^2 + \mu^2 r_m^2 + 2\mu(r_2^2 - r_1^2)} + \sqrt{r_p^2 + \mu^2 r_m^2 - 2\mu(r_2^2 - r_1^2)}}{2\sqrt{1-\mu^2}\,M}.$$

In view of the condition $|r_2 - r_1| \ll (r_1 + r_2)/2$ (see, for example, [3]) implemented in practice, expressions (2) can be written as

$$\Omega_{(\pm)} \approx \Omega_{(0)} \pm \Omega_s,$$
$$\Omega_{(0)} = \frac{\mu(r_2^2 - r_1^2)}{\sqrt{(1-\mu^2)(r_p^2 + \mu^2 r_m^2)}\,M}, \quad (3)$$
$$\Omega_s = \frac{\sqrt{r_p^2 + \mu^2 r_m^2}}{\sqrt{1-\mu^2}\,M},$$

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where

$$r_{\rm p} = \sqrt{r_1^2 + r_2^2 + 2r_1r_2\cos\varepsilon_{12}}\,;$$
$$r_{\rm m} = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\varepsilon_{12}}\,;\quad \varepsilon_{12} = \varepsilon_1 + \varepsilon_2,$$
$$\mu = \frac{2r_1r_2\sin\varepsilon_{12}}{\alpha_{\rm m}r_{\rm p}}\quad (|\mu| < 1); \tag{4}$$
$$\alpha_{\rm m} = \alpha_{\rm p}\frac{1-h}{1+h};\quad \alpha_{\rm p} = \alpha = \frac{c}{L}(g - \Gamma);\quad h = \frac{\theta}{\beta}.$$

Here $r_{\rm p}$ and $r_{\rm m}$ are combinations of the parameters of the linear coupling of counterpropagating waves; $\alpha_{\rm p}$ and $\alpha_{\rm m}$ are, respectively, the inverse relaxation times of the sum and difference of the intensities of counterpropagating waves; g is the unsaturated linear gain of the active medium; Γ is the resonator losses per trip; h is the parameter depending on the total pressure of the He–Ne mixture [17]; μ is the quantity characterising the effect of the active-medium gain on the parameters of the synchronisation zone. Note that expressions (2) and (3) are valid under the condition of weak coupling of counterpropagating waves, which suggests that in the entire range of working discharge currents used in laser gyros, the ratios $r_{\rm p}/\alpha_{\rm p}$ and $r_{\rm m}/\alpha_{\rm m}$ are much smaller than unity. In modern devices operating at sufficiently large excesses of the pump over the threshold [3], the above condition is usually satisfied.

Based on the analysis of expressions (2)–(4) in [8], the following conclusions were drawn:

(i) in the general case of the asymmetric ($r_1 \neq r_2$) linear coupling of the counterpropagating waves, the left and right boundaries of the synchronisation zone of the laser gyro are located at different distances from the coordinate origin: $\Omega_{(+)} \neq -\Omega_{(-)}$. As a result, the centre of the region is shifted along the axis of the angular velocity Ω by the finite quantity $\Omega_{(0)} \neq 0$;

(ii) with increasing active-medium gain g, the shift $\Omega_{(0)}$ of the centre of the synchronisation zone and its half-width $\Omega_{\rm s}$ decrease, approaching asymptotically the established finite values

$$\Omega_{(0)}^{\rm asymp} = 0,\quad \Omega_{\rm s}^{\rm asymp} = \frac{r_{\rm p}}{M} = \frac{\sqrt{r_1^2 + r_2^2 + 2r_1r_2\cos\varepsilon_{12}}}{M}. \tag{5}$$

Restrictions on the amount of work [8] did not allow us to present the results of quantitative assessment of the dependence of the quantities $\Omega_{(+)}$, $\Omega_{(-)}$, $\Omega_{(0)}$, $\Omega_{\rm s}$ on the active-medium gain g for a particular laser gyro. Therefore, the purpose of this paper is to carry out such an assessment for a widely used laser gyro with a four-mirror square cavity and to compare (qualitatively) the obtained results with experimental data, well-known from [18–21], typical of three-mirror cavity gyros.

2. Description of a laser gyro and derivation of relations for the calculation of its parameters

Following [3], as an example we will choose a laser gyro with a four-mirror square cavity having a nominal length of the arm $l = 50$ mm and a perimeter $L = 4l = 200$ mm. According to [3], such a device is characterised by the half-width of the synchronisation zone, $\Omega_{\rm s} \approx 0.05$ deg s^{-1}. The angular resolution of the laser gyro is 2.61″, and the geometric scale factor is $M_{\rm g} = 496459$. The gyroscope operates at a total pressure of the He–Ne mixture of 6.5 Torr.

Using expressions (3), (4) we perform quantitative assessment of the values of $\Omega_{(+)}$, $\Omega_{(-)}$, $\Omega_{(0)}$, $\Omega_{\rm s}$ for this particular laser gyro, provided that the parameter of the relative excitation $N_{\rm rel} = g/\Gamma$ varies from 2 to 8 [3], which corresponds to a change in the linear active-medium gain g from 2Γ to 8Γ. In order not to present (with comments) cumbersome formulas for the calculation of the small parameter $K_{\rm a}$, as well as expressions for the estimates of β and θ, we assume $M = M_{\rm g}$, and in addition, set $h = 0.652$.

2.1. Relation for the calculation of the parameter Γ

In order to make use of expressions (3) and (4), we must first calculate the total loss Γ for a given laser gyro. We assume that the resonator of the device is formed by two flat signal mirrors (M_1, M_2) and two spherical mirrors (M_3, M_4) with a radius of curvature $R = 1000$ mm mounted on piezocorrectors (mirrors are numbered clockwise). For flat mirrors M_1 and M_2 we have specified the energy parameters, i.e., the integral coefficient of light scattering, $K^{\rm f}_{\rm scat}$, into the full solid angle 4π sr; absorption losses, $\Gamma^{\rm f}_{\rm absorp}$; and useful transmission losses, $\Gamma^{\rm f}_{\rm transm}$. For spherical mirrors M_3 and M_4 we have specified the integral light scattering coefficient, $K^{\rm s}_{\rm scat}$, and absorption losses, $\Gamma^{\rm s}_{\rm absorp}$. They are $K^{\rm f}_{\rm scat} = 5\times10^{-6}$, $\Gamma^{\rm f}_{\rm absorp} = 55\times10^{-6}$, $\Gamma^{\rm f}_{\rm transm} = 60\times10^{-6}$, $K^{\rm s}_{\rm scat} = 10\times10^{-6}$, $\Gamma^{\rm s}_{\rm absorp} = 50\times10^{-6}$. Then, neglecting small diffraction losses due to the presence of an aperture diaphragm in the laser gyro cavity, the desired formula for the calculation of Γ can be written in the form

$$\Gamma = 2(K^{\rm f}_{\rm scat} + \Gamma^{\rm f}_{\rm absorp} + \Gamma^{\rm f}_{\rm transm}) + 2(K^{\rm s}_{\rm scat} + \Gamma^{\rm s}_{\rm absorp}). \tag{6}$$

With the given parameters of the mirrors we find from (6) that $\Gamma = 360\times10^{-6}$.

2.2. Relations for the calculation of the parameters r_1, r_2 and ε_{12}

Now it is necessary to derive expressions for calculating r_1, r_2 and ε_{12}. These expressions must ensure the possibility of modelling a situation in which the laser gyro under study has an asymmetry ($r_1 \neq r_2$) of the linear coupling of the counterpropagating waves. The asymmetry of the wave coupling in this device can be realised, for example, by moving identically and controllably the spherical mirrors in opposite directions: the mirror M_4 moves out of the resonator to a distance w (such a direction of the movement is assumed positive), and the mirror M_3 – on the contrary – moves into the cavity to exactly the same distance. Note that the perimeter of the axial contour, L, [22, 23] of the laser gyro resonator remains unchanged and equal to its initial value $4l$; however, the geometrical shape of the contour is changing: from the square one, it almost turns into diamond-shaped, elongated along the diagonal connecting the mirrors M_2 and M_4. (Recall that the axial contour of the laser gyro cavity is a longitudinal axis of symmetry of the Gaussian beam with the TEM$_{00}$ mode, which determines the centre of the light spot of the beam at any arbitrary cross section.)

As applied to this laser gyro, to calculate complex integral coefficients $r_j\exp(i\varepsilon_j)$ of the linear coupling between the counterpropagating waves we have the expression

$$\frac{L}{c}r_j\exp(i\varepsilon_j) = a_{\rm f}\left\{\exp\left[i\left(\frac{\pi}{2} - \chi_{\rm f} \pm \varphi_1\right)\right] + \exp\left[i\left(\frac{\pi}{2} - \chi_{\rm f} \pm \varphi_2\right)\right]\right\}$$
$$+ a_{\rm s}\left\{\exp\left[i\left(\frac{\pi}{2} - \chi_{\rm s} \pm \varphi_3\right)\right] + \exp\left[i\left(\frac{\pi}{2} - \chi_{\rm s} \pm \varphi_4\right)\right]\right\}$$

$$+ b_{\rm f}[\exp(\pm{\rm i}\varphi_1) + \exp(\pm{\rm i}\varphi_2)]$$

$$+ b_{\rm s}[\exp(\pm{\rm i}\varphi_3) + \exp(\pm{\rm i}\varphi_4)], \qquad (7)$$

which describes the result of summation of integrated local coupling coefficients of these waves with respect to all four mirrors. (Hereafter the upper arithmetic signs in the formulas correspond to $j = 1$, and the lower ones – to $j = 2$.)

Expression (7) differs from the similar-in-structure relations, well-known from [1–4, 10, 12, 14, 16, 21, 24–38], by the fact that in addition to the integral light scattering coefficient of each of the mirrors, mirror losses due to absorption and transmission are also taken into account. The second feature of this expression is that it predicts a substantially different (than as it follows, for example, from [16, 25, 29]) dependence of r_1, r_2 [and, hence, $\Omega_{(+)}$, $\Omega_{(-)}$, $\Omega_{(0)}$, $\Omega_{\rm s}$] on the radius of curvature of spherical mirrors (which is in qualitative agreement with the experimental data, well-known from [39], obtained for a three-mirror laser gyro).

Consider expression (7) in more detail. In the right-hand side of (7) there appear two groups of parameters. The parameters of the first group – $a_{\rm f}$, $\chi_{\rm f}$, $b_{\rm f}$ and $a_{\rm s}$, $\chi_{\rm s}$, $b_{\rm s}$ – characterise the individual properties of flat and spherical mirrors, respectively. The parameters of the second group – the phase angles φ_n ($n = 1, 2, 3, 4$) – describe the influence of the identical oppositely directed controllable movements of the spherical mirrors.

To calculate the parameters of the first group in the right-hand side of (7), we use the phenomenological formulas that have been proposed in [7]:

$$a_{\rm f} = \frac{1}{2}\theta_{\rm f}\sqrt{K^{\rm f}_{\rm scat}}, \quad \chi_{\rm f} = \arcsin\sqrt{K^{\rm f}_{\rm scat}},$$

$$b_{\rm f} = \frac{1}{2}\theta_{\rm f}(\Gamma^{\rm f}_{\rm absorp} + \Gamma^{\rm f}_{\rm transm}),$$

$$a_{\rm s} = \frac{1}{2}\theta_{\rm s}\sqrt{K^{\rm s}_{\rm scat}}, \quad \chi_{\rm s} = \arcsin\sqrt{K^{\rm s}_{\rm scat}},$$

$$b_{\rm s} = \frac{1}{2}\theta_{\rm s}\Gamma^{\rm s}_{\rm absorp}, \quad \theta_{\rm f} = w_{\rm f}/L, \quad w_{\rm f} = \sqrt{w^{(x)}_{\rm f}w^{(y)}_{\rm f}}, \qquad (8)$$

$$\theta_{\rm s} = w_{\rm s}/L, \quad w_{\rm s} = \sqrt{w^{(x)}_{\rm s}w^{(y)}_{\rm s}},$$

$$w^{(z)}_{\rm f} = \left(\frac{2\lambda l}{\pi}\right)^{1/2}\left[\frac{(4 - 7\zeta + 2\zeta^2)^2}{4 - (2 - 8\zeta + 3\zeta^2)^2}\right]^{1/4},$$

$$w^{(z)}_{\rm s} = \left(\frac{2\lambda l}{\pi}\right)^{1/2}\left[\frac{(4 - 3\zeta)^2}{4 - (2 - 8\zeta + 3\zeta^2)^2}\right]^{1/4}.$$

When using the two last expressions to estimate the quantities $w^{(z)}_{\rm f}$, $w^{(z)}_{\rm s}$, it is needed to follow the rule: if the superscript is $z = x$, then $\zeta = \xi = pl$, where $p = 2(\sqrt{2}/R)$; if the superscript is $z = y$, then $\zeta = \eta = ql$, where $q = \sqrt{2}/R$. Here p and q are the optical powers of the spherical mirrors, respectively, in axial and sagittal planes; and ξ and η are the small dimensionless parameters introduced for brevity.

In formulas (8) $a_{\rm f}$ and $a_{\rm s}$ are the moduli of local complex dimensionless coefficients of the counterpropagating wave coupling through backscattering of radiation, respectively, on flat and spherical mirrors; $\chi_{\rm f}$ and $\chi_{\rm s}$ are the 'angles of scattering losses' on these mirrors; $b_{\rm f}$ are the moduli of local complex dimensionless coefficients of the counterpropagating wave coupling through absorption and transmission of radiation by flat mirrors; $b_{\rm s}$ are the moduli of local complex dimensionless coefficients of the wave coupling through absorption by spherical mirrors; $w_{\rm f}$ and $w_{\rm s}$ are the effective half-widths of the Gaussian beam of the working laser gyro mode in its cross sections, where the flat and spherical mirrors are, respectively, placed; $w^{(x)}_{\rm f}$, $w^{(x)}_{\rm s}$ and $w^{(y)}_{\rm f}$, $w^{(y)}_{\rm s}$ are the half-widths of the Gaussian beam in the axial plane xz and sagittal plane yz in the above cross sections; $\theta_{\rm f}$ and $\theta_{\rm s}$ are half the angles at which one can see the light spots (of diameter $2w_{\rm f}$ and $2w_{\rm s}$) of a Gaussian beam on the surface of flat and spherical mirrors, provided that they are observed from the centres of the same mirrors at a distance equal to L, in a situation when the axial contour of the laser gyro cavity is expanded in a straight line. With the given parameters of the mirrors expression (8) yields the following numerical estimates: $p = 0.0028$ mm^{-1}, $q = 0.0014$ mm^{-1}, $\xi = 0.14$, $\eta = 0.07$, $a_{\rm f} = 1.15\times10^{-6}$, $a_{\rm s} = 1.72\times10^{-6}$, $\chi_{\rm f} = 461''$, $\chi_{\rm s} = 652''$, $b_{\rm f} = 5.91\times10^{-8}$, $b_{\rm s} = 2.72\times10^{-8}$, $w_{\rm f} = 0.205$ mm, $w_{\rm s} = 0.218$ mm, $w^{(x)}_{\rm f} = 0.186$ mm, $w^{(x)}_{\rm s} = 0.202$ mm, $w^{(y)}_{\rm f} = 0.227$ mm, $w^{(y)}_{\rm s} = 0.235$ mm, $\theta_{\rm f} = 212''$, $\theta_{\rm s} = 225''$.

To calculate the parameters of the second group in the right-hand side of (7), we use the relation

$$\varphi_n = \frac{4\pi}{\lambda}S_n, \qquad (9)$$

where S_n ($n = 1, 2, 3, 4$) is the distance measured along the axial contour (clockwise) between the reference plane (located at the origin of the coordinates) and the centre of the mirror M_n. The origin of the coordinates is chosen on the surface of the mirror M_1 at a point, where the centre of the light spot of a Gaussian beam is located (at this point the axial contour touches the surface of the mirror M_1 and is reflected from it).

To estimate S_n ($n = 1, 2, 3, 4$), we use the formula

$$S_n = -t_n\sin\theta_n + \sum_{m=1}^{n}L^{(m)}_{m-1}, \qquad (10)$$

where $L^{(1)}_0 = 0$ and $L^{(m)}_{m-1}$ is the length of the laser gyro cavity arm between the mirrors M_{m-1} and M_m (which is the distance measured along the axial contour between the centres of light spots of the Gaussian beam on the surfaces of these mirrors); t_n is the displacement of the centre of the light spot of the Gaussian beam at the surface of the mirror M_n relative to its centre (which is measured in the axial plane to the right); θ_n is half the angle between the arms of the laser gyro resonator at the mirror M_n (in our case, $\theta_{\rm n} = \pi/4$). It follows from (10) that

$$S_1 = -(\sqrt{2}/2)t_1, \quad S_2 = -(\sqrt{2}/2)t_2 + L^{(2)}_1,$$

$$S_3 = -(\sqrt{2}/2)t_3 + L^{(2)}_1 + L^{(3)}_2, \qquad (11)$$

$$S_4 = -(\sqrt{2}/2)t_4 + L^{(2)}_1 + L^{(3)}_2 + L^{(4)}_3.$$

The methods for calculating the quantities $L^{(m)}_{m-1}$ and t_n for plane N-mirror misaligned (i.e., with displaced mirrors) laser gyro resonators of arbitrary (flat) form, containing, in the general case, plane-parallel plates in the arms, have been proposed in [40] and [41], respectively. Based on these techniques, as applied to this laser gyro resonator for the situation under study, when the spherical mirrors move identically and controllably in opposite directions to a distance w, for the mentioned values we can obtain the expressions

$$L_1^{(2)} = L_3^{(4)} = l, \quad L_2^{(3)} = l + \sqrt{2}\frac{\xi}{8-3\xi}w,$$
$$t_1 = t_2 = -\frac{4-\xi}{8-3\xi}w, \quad t_3 = t_4 = \frac{4}{8-3\xi}w, \tag{12}$$

taking into account these expressions we derive from (11)

$$S_1 = (\sqrt{2}/2)\frac{4-\xi}{8-3\xi}w, \quad S_2 = l + (\sqrt{2}/2)\frac{4-\xi}{8-3\xi}w,$$
$$S_3 = 2l - (\sqrt{2}/2)\frac{4-2\xi}{8-3\xi}w, \quad S_4 = 3l - (\sqrt{2}/2)\frac{4-2\xi}{8-3\xi}w. \tag{13}$$

In order to simplify the final formulas, we assume that an integer of wavelengths λ fits the length l of each arm of the laser gyro cavity. Then, in (13) the quantities l can be omitted and the relations for φ_n

$$\varphi_1 = \varphi_2 = \varphi_f, \quad \varphi_3 = \varphi_4 = \varphi_s, \tag{14}$$

can be written with the help of (9), where

$$\varphi_f = 2\pi\frac{4-\xi}{8-3\xi}\frac{\sqrt{2}w}{\lambda}, \quad \varphi_s = -2\pi\frac{4-2\xi}{8-3\xi}\frac{\sqrt{2}w}{\lambda} \tag{15}$$

are the phase angles, which depend both on the oppositely directed movements w of the spherical mirrors and on the parameter ξ, proportional to their optical power p in the axial plane.

In view of (7), (8), (14), (15) the expressions for the quantities r_j, ε_j $(j = 1, 2)$ take the form

$$r_j = 2\frac{c}{L}\sqrt{A_j^2 + B_j^2}, \quad \varepsilon_j = \frac{\pi}{2} - \arctan\frac{A_j}{B_j}, \tag{16}$$

$$A_j = a_f\sin(\chi_f \mp \varphi_f) + a_s\sin(\chi_s \mp \varphi_s) + b_f\cos\varphi_f + b_s\cos\varphi_s,$$
$$B_j = a_f\cos(\chi_f \mp \varphi_f) + a_s\cos(\chi_s \mp \varphi_s) \pm b_f\sin\varphi_f \pm b_s\sin\varphi_s. \tag{17}$$

Then, by substituting (17) into (16) we obtain the following expanded relations for the calculation of the parameters r_j and ε_{12}:

$$r_j = r_j(\varphi) = 2\frac{c}{L}\{a_f^2 + a_s^2 + b_f^2 + b_s^2 + 2[a_f b_f\sin\chi_f + a_s b_s\sin\chi_s + b_f b_s\cos\varphi + a_f a_s\cos(\chi_f - \chi_s \mp \varphi) + a_f b_s\sin(\chi_f \mp \varphi) + a_s b_f\sin(\chi_s \pm \varphi)]\}^{1/2}, \tag{18}$$

$$\varepsilon_{12} = \varepsilon_{12}(\varphi) = \pi - \arctan\frac{N(\varphi)}{D(\varphi)}, \tag{19}$$

$$N(\varphi) = a_f^2\sin 2\chi_f + a_s^2\sin 2\chi_s + 2(a_f b_f\cos\chi_f + a_s b_s\cos\chi_s) + 2[a_f a_s\sin(\chi_f + \chi_s) + a_f b_s\cos\chi_f + a_s b_f\cos\chi_s]\cos\varphi,$$

$$D(\varphi) = a_f^2\cos 2\chi_f + a_s^2\cos 2\chi_s - 2(a_f b_f\sin\chi_f + a_s b_s\sin\chi_s) - b_f^2 - b_s^2 + 2[a_f a_s\cos(\chi_f + \chi_s) - a_f b_s\sin\chi_f - a_s b_f\sin\chi_s - b_f b_s]\cos\varphi,$$

where

$$\varphi = \varphi_f - \varphi_s = 2\pi(\sqrt{2}w/\lambda) \tag{20}$$

is the phase angle, which depends only on the oppositely directed movements w of the spherical mirrors.

It follows from the analysis of expressions (18)–(20) that the values of r_1, r_2 and ε_{12} are periodic functions of w with a period $w_{period} = (\sqrt{2}/2)\lambda$. The latter means that the parameters $\Omega_{(+)}$, $\Omega_{(-)}$, $\Omega_{(0)}$, Ω_s of the synchronisation zone of the frequencies of the counterpropagating waves of the laser gyro under study will also be periodic functions of w with the specified period. In addition, the analysis of (18), (20) implies that in this device the linear coupling of the counterpropagating waves will be symmetrical ($r_1 = r_2$) only in two particular cases: $w = 0$, $\varphi = 0$ (the maximum degree of coupling) and $w = (\sqrt{2}/4)\lambda$, $\varphi = \pi$ (the minimum degree of coupling).

The relations (6), (8), (18)–(20) obtained in this section for calculating the parameters Γ, r_1, r_2, ε_{12} of the laser gyro under consideration allow us to proceed to quantitative assessment of the dependence of $\Omega_{(+)}$, $\Omega_{(-)}$, $\Omega_{(0)}$, Ω_s on the active-medium gain g.

3. Quantitative estimation of the dependence of the synchronisation zone parameters of the counterpropagating wave frequencies of the laser gyro on the active-medium gain

3.1. Symmetric linear coupling of counterpropagating waves

Consider first the particular case of symmetric ($r_1 = r_2$) linear coupling of counterpropagating waves provided that it manifests itself to the maximum extent. Such a situation would occur in the case of the initial geometry of the laser gyro cavity, when $w = 0$ and $\varphi = 0$. In this case, we derive from expressions (18), (19) [7]

$$r_1 = r_2 = r_j(0) = 2\frac{c}{L}\{a_f^2 + a_s^2 + 2[a_f a_s\cos(\chi_f - \chi_s) + (a_f\sin\chi_f + a_s\sin\chi_s)(b_f + b_s)] + (b_f + b_s)^2\}^{1/2}, \tag{21}$$

$$\varepsilon_{12} = \varepsilon_{12}(0) = \pi - \arctan\frac{N(0)}{D(0)}, \tag{22}$$

$$N(0) = a_f^2\sin 2\chi_f + a_s^2\sin 2\chi_s + 2[a_f a_s\sin(\chi_f + \chi_s) + (a_f\cos\chi_f + a_s\cos\chi_s)(b_f + b_s)],$$

$$D(0) = a_f^2\cos 2\chi_f + a_s^2\cos 2\chi_s + 2[a_f a_s\cos(\chi_f + \chi_s) - (a_f\sin\chi_f + a_s\sin\chi_s)(b_f + b_s)] - (b_f + b_s)^2,$$

which, under the given above parameters of the mirrors, yield the following numerical estimates: $r_1 = r_2 = 8616$ s^{-1}, $(L/c)r_1 = (L/c)r_2 = 5.74\times10^{-6}$, $\varepsilon_{12} = 176.24°$.

The dependence of the quantity Ω_s on the relative excitation parameter $N_{rel} = g/\Gamma$ (at $\Gamma = 360\times10^{-6}$), constructed on the basis of expressions (3), (4), (6), (8) with account for the values of r_1, r_2 and ε_{12} found by formulas (21), (22), is shown in Fig. 1. One can see that with increasing active-medium gain $g = \Gamma N_{rel}$ from the minimum $(2\times360\times10^{-6})$ to the maximum $(8\times360\times10^{-6})$ value, the half-width Ω_s of the synchronisation zone of the counterpropagating wave frequencies of the laser gyro under consideration decreases (approximately by the hyperbolic law), asymptotically approaching an established finite value $\Omega_s^{asymp} = 0.065$ deg s^{-1} (the lower horizontal dashed line), which was calculated by formula (5). This behaviour of Ω_s is in qualitative agreement with experimental data of [18, 19] and, in particular, [20] (see Fig. 1 in [20]), which were obtained for laser gyros with three-mirror equilateral cavities.

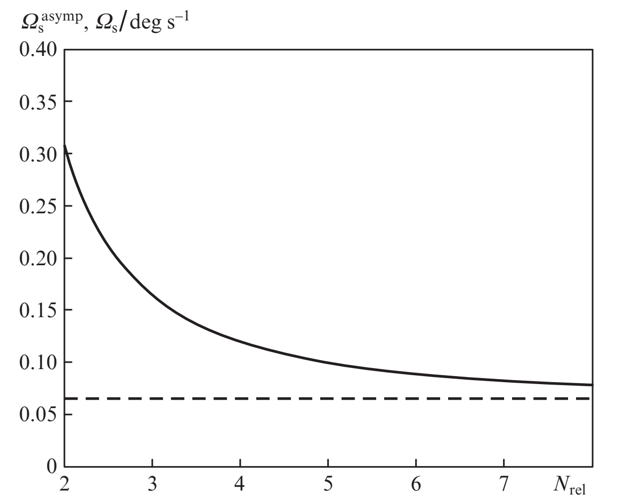

Figure 1. Dependence of the quantity Ω_s on N_{rel} at $w = 0$. The horizontal dashed line is the asymptote Ω_s^{asymp}.

3.2. Asymmetric linear coupling of counterpropagating waves

Consider now the general case of asymmetric ($r_1 \neq r_2$) linear coupling of the counterpropagating waves. This situation in the laser gyro will take place at $w \neq 0$ and $w \neq (\sqrt{2}/4)\lambda$. Figure 2 shows the dependences plotted on the basis of formulas (3) taking into account relations (4)–(6), (8), (18)–(20). These figures illustrate the dependence of the parameters $\Omega_{(+)}$, $\Omega_{(-)}$, $\Omega_{(0)}$, Ω_s, and Ω_s^{asymp} (lower dashed curve) on the value of w, varying in the range $0 - w_{period}$. The dependences in Fig. 2a correspond to the minimum value of the relative excitation parameter $N_{rel} = 2$, when the active-medium gain g is $2\times360\times10^{-6}$. The dependences in Fig. 2b correspond to the maximum value of the parameter $N_{rel} = 8$, when the active-medium gain is $8\times360\times10^{-6}$.

The analysis of the dependences shown in Fig. 2 implies the following: (i) in the general case of the asymmetric linear coupling of the counterpropagating waves of the laser gyro under study, the left and right boundaries of the synchronisation zone of the frequencies of these waves are located (relative to the origin of coordinates) at different distances, so that the centre of the zone is shifted along the axis of the angular velocity Ω; (ii) the coordinates of these boundaries and the coordinate of the centre of the zone are periodic functions of the identical oppositely directed controllable movements of the spherical mirrors. These two facts agree qualitatively with the experimental data [21] (see Fig. 6 in [21]), which were obtained for a laser gyro with a three-mirror equilateral cavity. In addition, it also follows from the analysis of the dependences that with increasing active-medium gain, the shift of the centre of the synchronisation zone and its half-width decrease.

4. Conclusions

In this paper we have considered a laser gyro with a four-mirror square cavity having a perimeter of 20 cm. Using expressions (3), (4) given in [8] and additional relations (6), (8) (18)–(22) obtained in Section 2, we have presented a quantitative assessment of the dependence of the parameters $\Omega_{(+)}$, $\Omega_{(-)}$, $\Omega_{(0)}$, Ω_s of the synchronisation zone of the counterpropagating wave frequencies on the active-medium gain g for the laser gyro in question. This estimate is in qualitative agreement with well-known experimental data [18–21] obtained for three-mirror cavity gyroscopes.

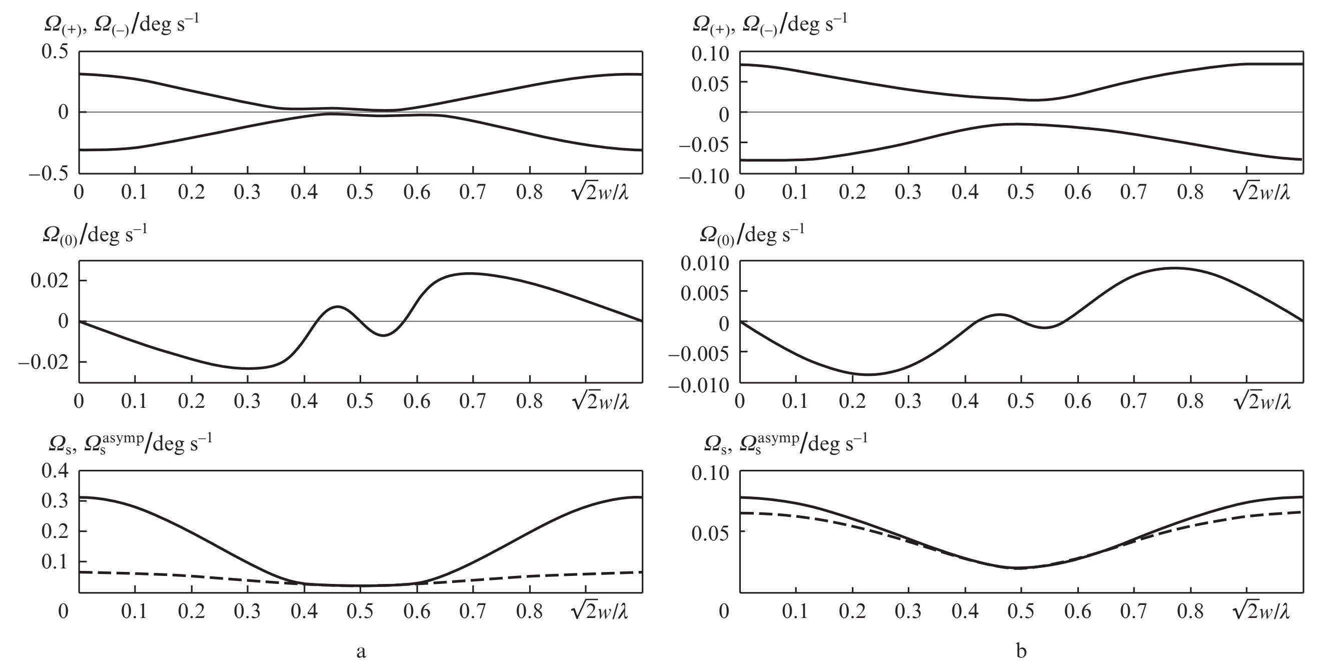

Figure 2. Dependences of the quantities $\Omega_{(+)}$, $\Omega_{(-)}$, $\Omega_{(0)}$, Ω_s (solid curves) and Ω_s^{asymp} (dashed curves) on w at $N_{rel} = 2$ (a) and 8 (b).

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