

SOLITONS

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Formation and optical control of dissipative vortex solitons in hollow-core optical fibres filled with a cold atomic gas

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Abstract. **We consider the problem of formation of optical vortex solitons in a medium of three-level atoms, taking into account the local field effects. The principal possibility of formation of optical solitons controlled by an optical pump wave under the off-resonance Raman conditions of the lambda-type interaction scheme is predicted in an optically dense medium, i.e., in a hollow-core photonic-crystal optical fibre filled with a gas of cold atoms ^{87}Rb.**

Keywords: *nonlinear atom–optical interaction, optically dense medium, vortex solitons, gas-filled fibres, optic control.*

5. Introduction

The study of the possible control of formation of stable spatial optical structures in different media [1] is of considerable interest in connection with a wide range of practical applications of such structures, in particular in the problems of information transmission and processing [2].

The known procedure for coherent writing, storing and reading of optical information using the states of the atomic system [3] is based on the effect of electromagnetically induced transparency, corresponding to the linear operation regime of the lambda-type atom–optical interaction scheme under near-resonance conditions. Significant delays of the probe pulse by the medium in this case are due to the efficient excitation of atomic (dark) polaritons under near-resonance conditions. In this case, the polarition propagation velocity is determined by the atomic excitation transfer rate and can tend to zero in the limiting case of switching-off of the pump field. The resulting phase of storage of (quantum) optical information on the states of an atomic ensemble is limited by the characteristic time of development of decoherence effects, in particular the processes of diffusion in gaseous media. At the same time, the lifetime of a quantum state in such a system can be increased by orders of magnitude using atomic coherent ensembles in the form of a Bose–Einstein condensate.

The scheme under consideration acquires new features when stable topological structures are used as information carriers, the structures being formed in the transverse profile of the probe optical beam, in particular, dissipative spatial solitons [2]. Of considerable interest is a special class of optical topological structures – optical vortices [4], the central dip in the intensity distribution of which is securely recorded in the experiment, even in the case of a strong diffraction spreading of the optical beam [5]. At present, optical vortices are obtained experimentally using different laser schemes that can be conventionally divided into two groups. The first one involves direct generation of topological light structures in laser resonators [6, 7]; the second group is based on the modulation of the laser beam as it passes through spatially inhomogeneous media: specially synthesised holographic plates [8] and optical masks of variable thickness [9].

However, stable behaviour of optical vortices (dissipative vortex solitons) in the laser scheme during their generation can be extremely rarely observed [10]. The explanation for this is the fact that designing and developing modern laser devices, which combine stable generation and subsequent control of optical beams with complex topology require correct identification of the regions of their stability, specified in the parameters that describe a specific laser experiment. When using extended media it can be done by analysing the propagation equation obtained by direct derivation, for example in solving a self-consistent problem on the atoms and the field [2]. Appearance of nonlinear terms (required for the formation of stable solitons) in the final form of the propagation equation may be associated with the use of the Raman regime of lambda-scheme operation [11], implemented at considerable lengths of the medium and with increasing concentration of active atoms [12].

A promising medium for the formation and control of soliton regimes can be a recently fabricated hollow-core photonic-crystal fibre loaded with cold atoms [13]. The coherent state in such an atomic medium was confined for a long time, sufficient for the development of competitive dispersion–diffraction, nonlinear and dissipative processes that are necessary to stabilise dissipative solitons.

However, the possibility of achieving high concentrations of optically active particles in such a system necessarily requires consideration of the local field effects [14, 15]. Corrections made in this case are caused by the presence of near dipole–dipole interactions in the medium that are manifested when the Rabi frequency for the transition in an atom is comparable with the magnitude of the corresponding correction to these interactions. Estimates for the ensemble of resonant particles show that the need to take into account close dipole–dipole interactions begins at a concentration of 10^{15} cm^{-3}.

In this paper we consider the formation and subsequent optical control of the dynamics of a vortex optical soliton [4] in the Raman operation regime of the lambda-scheme and

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taking into account (in our case) the effects of the local response of the medium. The problem is similar to obtaining self-induced transparency solitons with account for the effect of the local field in resonant media [16]. The aim of the research is to find the estimated values of the medium and field parameters for which the model of a segment of a hollow-core photonic-crystal fibre filled with a gas of cold atoms ^{87}Rb can be used in the practical task of efficient generation of stable optical topological structures for the needs of telecommunications and optical manipulation of microscopic objects in light beams [17]. Accounting for the local field should help identify the nature of the influence of close dipole–dipole interactions on the stabilisation of vortex solitons in the optical scheme under study.

6. Basic relations for the lambda-type interaction scheme in an optically dense medium

In the problem under consideration we assume that the probe light pulse of given shape with centre radiation frequency ω_p and intensity $\boldsymbol{E}_p$ propagates along the z axis of a hollow-core optical fibre (waveguide channel filled with a gas of ^{87}Rb atoms) in a direction, opposite to that of propagation of a cw pump pulse with intensity $\boldsymbol{E}_c$ (Fig. 1a). In the Raman limit for the lambda-type interaction scheme (Fig. 1b), when the field frequency detuning from the resonance is $|\Delta_c| > d_0\Gamma_{ac}$ [12], and assuming that all the atoms are initially located at the level $|b\rangle$, a self-consistent problem of atoms + probe field is described by the equations

$$\begin{aligned}
\dot{\sigma}_{ba} &= -\Gamma_1\sigma_{ba} - ig\varepsilon - i\Omega\sigma_{bc} - i\chi_{ba}\sigma_{ba},\\
\dot{\sigma}_{ca} &= -\Gamma_2\sigma_{ca} - ig\varepsilon\sigma_{cb} - i\chi_{ba}\sigma_{ba}\sigma_{cb},\\
\dot{\sigma}_{bc} &= i\Delta_3\sigma_{bc} + ig\varepsilon\sigma_{ac} - i\Omega^*\sigma_{ba},\\
\dot{\sigma}_{bb} &= ig\varepsilon\sigma_{ab} - ig\varepsilon^*\sigma_{ba},
\end{aligned} \tag{1a}$$

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z} - ic\frac{D_0}{2}\nabla_\perp^2\right)\varepsilon = -igN\sigma_{ba}, \tag{1b}$$

where σ_{mn} is an element of the density matrix; $\Gamma_{mn} = |\mu_{mn}|^2\times\omega_{mn}^3(3\pi\hbar c^3\varepsilon_0)^{-1}$ is the relaxation rate; $\chi_{mn} = \rho|\mu_{mn}|^2/(3\hbar\varepsilon_0)$ is the value of the local response of the medium; μ_{mn} is the dipole moment of transition between atomic levels m and n; ρ is the atomic concentration; Ω and $g\varepsilon$ are the Rabi frequencies for the pump and probe fields; $g = \mu_{ba}[\omega/(2\hbar\varepsilon_0 V)]^{1/2}$ is the atomic coupling constant; $\varepsilon = A_p[\hbar\omega/(2\varepsilon_0 V)]^{-1/2}$; A_p is a slowly varying amplitude of the probe field; V is the quantization volume; $N = \rho V$ is the number of atoms in the interaction region; $\Gamma_1 = -[i\Delta_b - (\Gamma_{ab} + \Gamma_{ac})/2]$, $\Gamma_2 = -[i\Delta_c - (\Gamma_{ab} + \Gamma_{ac})/2]$; $\Delta_3 = \Delta_b - \Delta_c$; $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$; $D_0 = \lambda_p/\pi$ is a parameter that specifies diffraction in the direction, transverse to the z axis; c is the speed of light in vacuum; $\hbar$ is Planck's constant; ε_0 is the dielectric constant. As applied to the problem in question, the optical thickness of the medium d_0 can be determined by the characteristic linear dimension a_0 formed in the xy plane of topological structures: $d_0 = g^2Na_0/(c\Gamma_{ac})$ (cf. [12]).

In deriving (1) it was assumed that the contribution of the local response of the medium is comparable to the Rabi frequency for the probe transition, i.e., $g\varepsilon \approx \sigma_{ba}\chi_{ba}$; the local-field effects can be ignored in the pumped transition, provided the $\Omega > \sigma_{ca}\chi_{ca}$.

For a probe light pulse of duration T_0, problem (1) can be reduced to the Ginzburg–Landau equation describing its propagation in a medium (cf. [18], see Appendix):

$$\left(\frac{1}{v_g}\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right)\varepsilon + \frac{i}{2}\beta_2\frac{\partial^2\varepsilon}{\partial t^2} - i\frac{D_0}{2}\left(\frac{\partial^2\varepsilon}{\partial x^2} + \frac{\partial^2\varepsilon}{\partial y^2}\right)$$
$$- i\gamma_2|\varepsilon|^2\varepsilon + i\gamma_4|\varepsilon|^4\varepsilon = -\alpha_1\varepsilon - \alpha_2|\varepsilon|^2\varepsilon - \alpha_4|\varepsilon|^4\varepsilon, \tag{2}$$

where the coefficients are given in the Appendix.

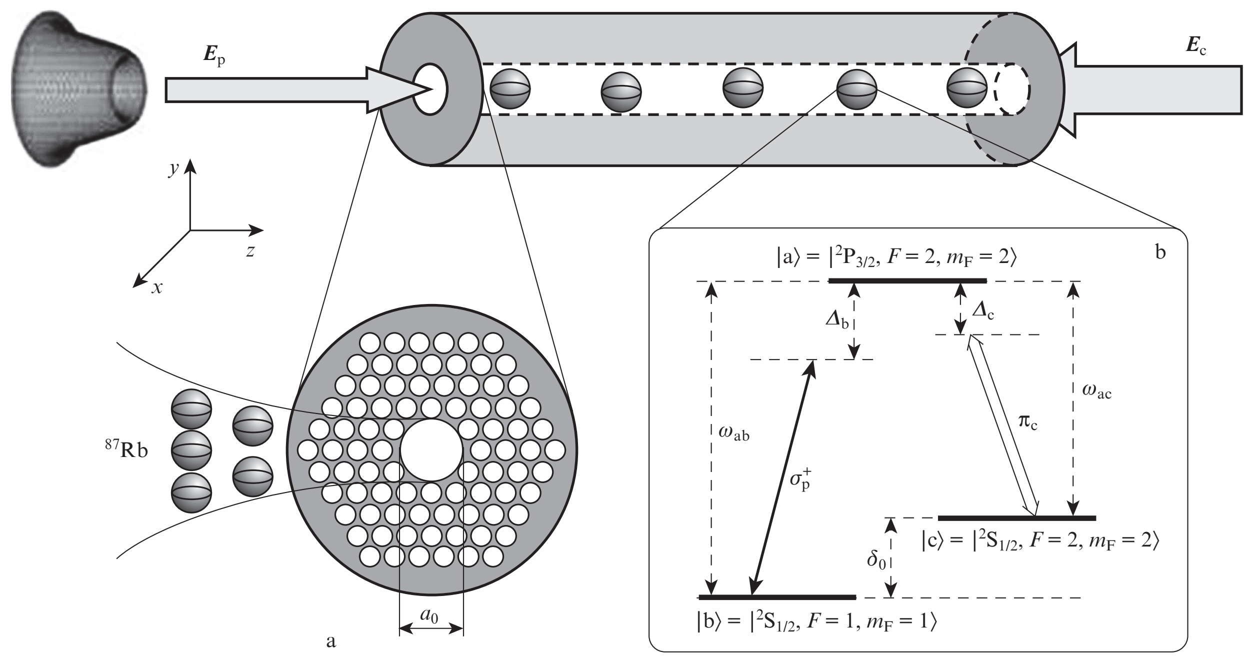

Figure 1. Model of a hollow-core optical gas-filled fibre (a) and lambda-scheme of atomic–optical interaction for ^{87}Rb atoms (b). Frequency detuning δ_0 of levels $|c\rangle$ and $|b\rangle$ is 6.834 GHz, the dipole matrix element μ_{ab} of transition $|a\rangle \to |b\rangle$ at a wavelength $\lambda = 780.241$ nm is 3.58×10^{-29} C m.

Equation (2) describes complex nonlinear probe field – pump field interaction which arises solely due to the presence of polarisation on the pump-induced transition ($\sigma_{ac} \neq 0$) in the Raman limit of the lambda-scheme [11]. Accounting for the local field in the medium although does not introduce new terms into equation (2), but clarifies the picture of the nonlinear interactions with the corresponding corrections (the terms with a co-factor χ determining the local field are given in the Appendix) in the case of an optically dense medium.

Note that in the other limit of the lambda-scheme under the near-resonance conditions, $\Delta_c \ll \Gamma_{ac}$, the polarisation on the transition $|a\rangle \to |c\rangle$ disappears – either one-photon probe-field process underlying the linear effect of electromagnetically induced transparency [3] or its nonlinear analogue in the case of sufficiently intense pump radiation [19] can be implemented. Accounting for the local field effects in these problems leads to appearance of one more term in the expression for the frequency detuning Δ_b and to additional phase modulation of the probe pulse [20].

For the analysis of equation (2) we pass to the travelling coordinate system with $T = t - z/v_g$, and after replacing the variables $u = \varepsilon/\sqrt{|\varepsilon_{in}|^2}$, $\xi = z/L_{dif}$, $X = x/a_0$, $Y = y/a_0$, we introduce the following characteristic parameters with the dimensionality of length: $L_{\beta_2} = T_0^2/|\beta_2|$ for the second order dispersion; $L_{\gamma_2} = 1/(\gamma_2|\varepsilon_{in}|^2)$ and $L_{\gamma_4} = 1/(\gamma_4|\varepsilon_{in}|^4)$ for third- and fifth-order nonlinearities, respectively; $L_{\alpha_1} = 1/\alpha_1$ for linear loss; $L_{\alpha_2} = 1/(\alpha_2|\varepsilon_{in}|^2)$ and $L_{\alpha_4} = 1/(\alpha_4|\varepsilon_{in}|^4)$ for the third- and fifth-order nonlinear losses, respectively; and $L_{dif} = a_0{}^2/D_0$ for diffraction, where a_0 and ε_{in} are the initial width and reduced amplitude of the optical vortex. The signs of L_{β_2}, L_{γ_2}, L_{γ_4}, L_{α_1}, L_{α_2} and L_{α_4} depend on the sign of the corresponding coefficients.

After multiplying both sides of equation (2) by L_{dif} and the transition to cylindrical coordinates, we finally obtain

$$\mathrm{i}\frac{\partial U}{\partial \xi} + \frac{1}{2}\left(\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} + \frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2}\right) + |U|^2 U - \nu|U|^4 U = Q, \quad (3)$$

where $Q = \mathrm{i}[-\delta U - \phi|U|^2 U - \mu|U|^4 U]$ is the dissipative term; $r = (X^2 + Y^2)^{1/2}$; θ is the angle in spherical coordinates; $U = uM$; $M^2 = L_{dif}/L_{\gamma_2}$. In (3) we introduced the basic parameters of the problem: $\delta = L_{dif}/L_{\alpha_1}$, $\phi = L_{\gamma_2}/L_{\alpha_2}$, $\mu = L_{\gamma_2}^2/(L_{\alpha_4}L_{dif})$, $\nu = L_{\gamma_2}^2/(L_{\gamma_4}L_{dif})$.

Since the inequality $L_{dif} \ll L_{\beta_2}$ is valid for the interaction parameters used below, the dispersion term in equation (3) is excluded and we are talking below only about the spatial effects in the case of propagating radiation.

7. Formation of vortex solitons in gas-filled optical fibres

Following the concept of dissipative solitons [14], to maintain the energy of bright solitons at the same level during their distribution in the medium, it is needed to alternate the effects of absorption and amplification for various segments on the probe light pulse envelope. This can occur if the inequalities $\delta > 0$, $\phi < 0$, $\mu > 0$ are fulfilled, their signs being determined by a combination of signs of the corresponding coefficients.

Figure 2 shows the dependences of typical dissipative parameters on the frequency of pump field detuning from resonance Δ_c, when a single-mode optical fibre filled with a gas of ^{87}Rb atoms is used as a model medium under the following conditions: atomic concentration, $\rho = 7.3\times10^{21}$ m^{-3}; frequency of probe field detuning, $\Delta_b = 0$; relaxation rate, $\Gamma_{ab} = \Gamma_{ac} = 2.5\times10^8$ Hz; magnitude of the local response, $\chi_{ab} \equiv \chi = 3.36\times10^9$. The intensities of the fields used are chosen as follows: $I_c = 58$ W cm^{-2} for the pump field and $I_p = 58$ mW cm^{-2} for the probe field. The corresponding Rabi frequencies can be calculated by the formulas $\Omega = \mu_{ac}E_c/\hbar$, and $g\varepsilon = \mu_{ab}E_p/\hbar$ through the field strengths $E_{c,p} = [2I_{c,p}/(c\varepsilon_0)]^{1/2}$ and will be equal to $\Omega = 7.13\times10^9$ s^{-1} and $g\varepsilon = 2.25\times10^8$ s^{-1}. With $a_0 = 11$ μm and the calculated parameters $g = 1.3\times10^6$ s^{-1} and $N = 19.7\times10^8$ taken into account, the correspondence of the problem to the Raman interaction limit is due to the implementation of the ratio $\Delta_c/(d_0\Gamma_{ac}) \approx 6.5$.

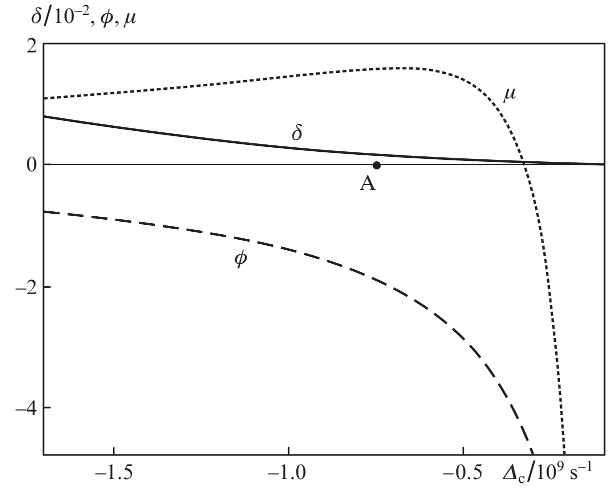

Figure 2. Frequency dependences of the coefficients of the dissipative part of the dimensionless Ginzburg–Landau equation (3) for the initial probe pulse duration $T_0 = 10$ ns.

The region, in which the inequalities required for the soliton stabilisation can be satisfied in the model under study, is located on the left side of Fig. 2. For further simulations, near the lasing threshold we fix point A to which the detuning frequency $\Delta_c^A = -7.5\times10^8$ s^{-1} corresponds. An additional condition of conservation of shape of a dissipative soliton $L_{dif} \simeq L_{\gamma_2}$ is fulfilled at this point, taking into account the fact that $D > 0$ and $\gamma_2 > 0$ [21].

Using variational methods, we find with the help of equation (3) the regions of the parameters at which stable solitons can appear in our problem. In this case, we will focus on an important class of dissipative vortex solitons described by the expression [18]

$$U = A_0 A\left(\frac{r}{R_0 R}\right)^S \exp\left[-\frac{r^2}{2(R_0 R)^2} + \mathrm{i}\left(C\frac{r^2}{R_0^2} + S\theta + \Psi\right)\right], \quad (4)$$

where A, R, C, Ψ are the amplitude, spatial width, wavefront curvature and phase of the pulse, respectively. The parameter S defines the topological charge of the vortex soliton and is assumed to be unity in the present paper.

The coefficients A_0 and R_0 can be obtained from the normalisation condition of the total laser power

$$P = \int_0^{2\pi}\int_0^{\infty}|U(r,\theta)|^2 r\,\mathrm{d}r\,\mathrm{d}\theta = \pi S! A_0^2 R_0^2 A^2 R^2;$$

in the simplest case, $P = A^2R^2$, we have $A_0 = 1/(R_0\sqrt{\pi})$ and assume below $R_0 = 1$.

In practice, the effects of diffusion spreading in atomic ensembles lead, as a rule, to the light filling of the central dip in the intensity distribution of the vortex soliton (4) even in the presence of long-term preservation of the external profile in the form of a bright soliton during its propagation in such media [5]. To have the form of vortex solitons (4) self-sustained during their propagation, apart from the balance of nonlinear dispersive and dissipative effects [21] the implementation of additional conditions is also require, such as those associated with the presence of optical diffusion [2, 22] or refractive index modulation [23] and/or absorption through the functions of complex form [18]. In the latter case, the parameter of the linear absorption can be replaced by a new space-dependent (coordinate r) effective parameter $\delta_{\rm eff} = \delta - Vr^2$, which corresponds to the introduction of an additional saturable absorber into a gas-filled fibre; we consider this case below, assuming $V = -0.03$ everywhere.

A detailed analysis of the stability of vortex solitons during linearization of the master equation of form that is more general than in (3) (which, apparently, can be obtained by solving a self-consistent problem through a series of successive approximations and without expansion of field-nonlinear functions for the density matrix in a power series; see the Appendix) and with the optical diffusion taken into account is given in [24].

Using the Euler–Lagrange equation

$$\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\frac{\partial \bar{L}_{\rm c}}{\partial \eta'}\right) - \frac{\partial \bar{L}_{\rm c}}{\partial \eta} = 2\,\mathrm{Re}\left(\int_0^\infty\int_0^{2\pi} Qr\frac{\partial U^*}{\partial \eta}\,\mathrm{d}r\,\mathrm{d}\theta\right),$$

where

$$\bar{L}_{\rm c} = \int_0^\infty\int_0^{2\pi} L_{\rm c}\,\mathrm{d}r\,\mathrm{d}\theta$$

is the averaged conservative Lagrangian of equation (3), and $\eta = \{A(\xi), R(\xi), C(\xi), \Psi(\xi)\}$ is a set of variable parameters of the functions of the spatial coordinates ξ, and taking into account (4) we obtain

$$\frac{\mathrm{d}A}{\mathrm{d}\xi} = -\frac{5\phi A^3}{16\pi} - \frac{8\mu A^5}{81\pi^2} + A(-\delta - C + VR^2),$$

$$\frac{\mathrm{d}R}{\mathrm{d}\xi} = \frac{\phi A^2 R}{16\pi} + \frac{2\mu A^4 R}{81\pi^2} + CR + VR^3, \qquad (5)$$

$$\frac{\mathrm{d}C}{\mathrm{d}\xi} = -C^2 + \frac{1}{8R^4} - \frac{A^2}{16\pi R^2} + \frac{2\nu A^4}{81\pi^2 R^2},$$

$$\frac{\mathrm{d}\Psi}{\mathrm{d}\xi} = \frac{3A^2}{8\pi} - \frac{10\nu A^4}{81\pi^2} - \frac{1}{2R^2}.$$

In the approximation of low-frequency modulation ($C^2 \simeq 0$; cf. [18]) the following system of equations that simplifies the search for stationary points of the solutions of system (5) is valid:

$$C = \frac{A^2(-81\pi\phi - 32A^2\mu)}{1296\pi^2} - \frac{162\pi^2 V}{A^2(81\pi - 32A^2\nu)},$$

$$R^2 = \frac{162\pi^2}{A^2(81\pi - 32A^2\nu)}, \qquad (6)$$

$$-\frac{5\phi A^3}{16\pi} - \frac{8\mu A^5}{81\pi^2} + A(-\delta - C + VR^2) = 0.$$

The system of equations (6) has 16 solutions, only two of which correspond to the physical constraints imposed on the energy and width of the vortex soliton ($A > 0$, $R > 0$ and $A, R \in$ R), while only one of them with a larger value of A and negative frequency modulation ($C < 0$) will be stable [18].

Figure 3 shows the parametric plane formed by the following parameters: the density of resonant atoms in the ρ system and the frequency detuning of the pump field from resonance, $\Delta_{\rm c}$. The gray highlighted region shows the stability of a vortex soliton for the selected physical solution of equations (6) [18]. This region of stability was determined from the analysis of the eigenvalues of λ_j of the Jacobian matrix of the system of equations (5), i.e., by the condition $\mathrm{Re}(\lambda_j) < 0$, where $j = 1, 2, 3$ [25], and corresponds to the point of a stable focus.

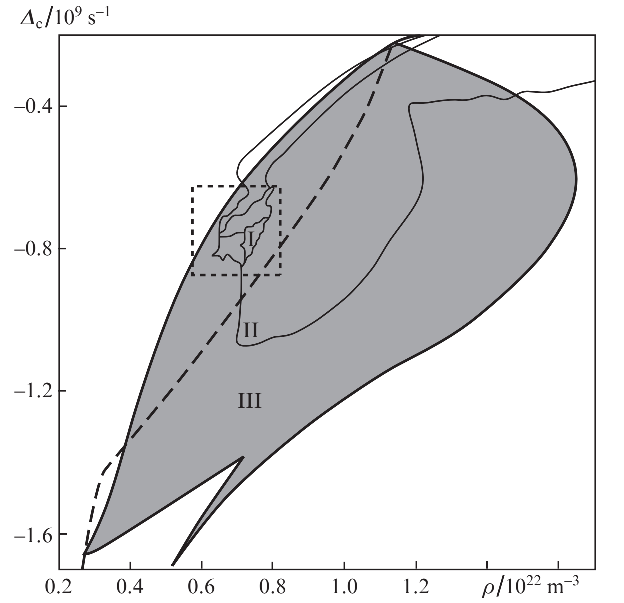

Figure 3. Parametric plane in the coordinates $\Delta_{\rm c}$ (the pump field frequency detuning from resonance) and ρ (the density of resonant atoms loaded in the fibre). Gray area is the region of existence of stationary vortex solitons, obtained by the variational method. The numbers denote the regions [found by direct numerical simulations of equation (3)] of true stability of axisymmetric vortex solitons (I), the transition to the vortex solitons with $S = 0$ (II) and loss of stability (III). Dashed line is the separatrix $C = 0$. Interaction parameters correspond to Fig. 2 (see description of Fig. 2 in the text).

Direct numerical simulation of equation (3) with exhaustive search of parameters η for the function of form (4) and with the initial perturbations R and C taken into account [24, 26] shows that the set of obtained stable solutions to the simplified system (6) is very approximate, but the true stability region (denoted by I in Figs 3 and 4) has a much smaller size. Inside the stability region obtained by the variational method there appears a 'fine' structure in the form of separate regions of stability for solitons in modified forms, as well as region III, where the optical vortices decay.

In particular, for region II [the parameters $\nu = 0.5052$, $\delta = 0.0023$, $\phi = -1.6255$ and $\mu = 1.5319$ of equation (3) correspond to point B in Fig. 4 from this region with $\rho = 7.3\times10^{21}$ m^{-3} and $\Delta_{\rm c} = -8.7\times10^8$ s^{-1}] the phase portrait of the system under the condition $\lambda_3 < \mathrm{Re}(\lambda_{1,2})$ is transformed into two narrow beams of the phase trajectories, significantly narrowing near the singular point. Thus, even strong initial perturbations of the substitution parameters (4) are rapidly quenched when the phase trajectories from the far region of the phase space tend to such a singular point. However, small fluctuations of the vortex soliton in the singular point transfer the system to an unstable trajectory. As a result, the vortex soliton spontaneously loses the topological charge and passes into a new stable state with $S = 0$ (see insets in Fig. 4). This is a new type of evolution of the vortex solitons, not described in [18].

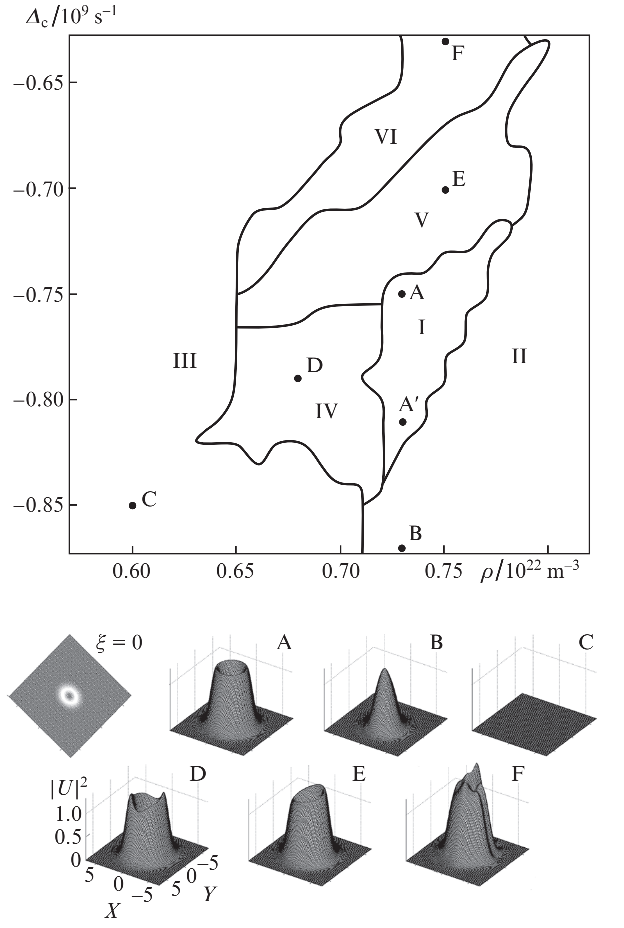

Figure 4. Scaled area around the region of true stability I in Fig. 3. Below are shown spatial profiles [obtained by direct numerical simulations of equation (3)] (in the plane XY) of optical beams after passing the distance $\xi = 100000$, corresponding to $\sim$50 m of a gas-filled fibre. Letters above each distribution correspond to points on the parametric plane whose coordinates are used to calculate the parameters of equation (3); the distribution for $\xi = 0$ corresponds to the shape of the optical vortex at the input to the medium (top view) in the presence of azimuthal perturbations.

Regions IV and V are transitional: in region IV [the parameters $\nu = 0.3240$, $\delta = 0.0018$, $\phi = -1.7989$ and $\mu = 1.5183$ of equation (3) correspond to point D from this region at $\rho = 6.8\times10^{21}$ m^{-3} and $\Delta_c = -7.9\times10^8$ s^{-1}] there occurs a spontaneous transition of an axisymmetric vortex in double-humped solitons, and in region V [the parameters $\nu = 0.3448$, $\delta = 0.0015$, $\phi = -2.0409$ and $\mu = 1.6160$ of equation (3) correspond to point E from this region at $\rho = 7.5\times10^{21}$ m^{-3} and $\Delta_c = -7\times10^8$ s^{-1}] – in single-humped asymmetric stable vortex solitons. Bifurcations of this type are described in [24].

In region VI [the parameters $\nu = 0.2372$, $\delta = 0.0013$, $\phi = -2.2755$ and $\mu = 1.6131$ of equation (3) correspond to point F from this region at $\rho = 7.5\times10^{21}$ m^{-3} and $\Delta_c = -6.3\times10^8$ s^{-1}] account for angular effects in (3) leads to the destruction of the vortex solitons with the emergence of some time-dependent multi-humped localised structures in their place, which, however, do not decay but experience constant evolution [27]. In region III [the parameters $\nu = 0.2261$, $\delta = 0.0018$, $\phi = -1.6652$ and $\mu = 1.4236$ of equation (3) correspond to point C from this region at $\rho = 6\times10^{21}$ m^{-3} and $\Delta_c = -8.5\times10^8$ s^{-1}] there occurs a loss of stability and decay of vortex solitons. When selecting the initial values of A, R and C that are different from the values corresponding to equations (6), in region III the optical beam is split into separate filaments, which either decay or continue to evolve over time in the same way as described in [26].

Note that point A (see also Fig. 2) actually falls in the stability region, calculated on the basis of both – variational and numerical – methods, which corresponds to the qualitative analysis of equation (3) given above. At the same time to form a vortex soliton in the medium under study, the initial spatial width r_R and linear frequency modulation C_R of a light pulse of form (4) will be 12 μm and -7.7×10^8 m^{-2}, respectively (dimensionless parameters $A = 3.496$, $R = 1.096$, $C = -0.093$); the calculated parameters of equation (3) for point A in Fig. 4 are as follows: $\nu = 0.3723$, $\delta = 0.0017$, $\phi = -1.8995$ and $\mu = 1.5817$.

The fundamental point in this problem is consideration of the local field effects. Indeed, in the case $\chi = 0$, i.e., without taking into account these effects, the stability region not only is transformed, but also is completely beyond the parametric plane shown in Figs 3 and 4. In this case, all the solutions obtained for the vortex solitons become unstable. Thus, a strategy of a possible experiment is changing dramatically due to the local response of the medium.

Further analysis and obtaining (different from those found) the stability regions of a vortex soliton require solutions to the complete nonlinear system (5), as well as an advanced multi-dimensional numerical experiment in the parameter space of the problem.

8. Optical control of dynamics of vortex solitons without affecting the conditions of their stability

Consider the possibility of controlling the dynamics of vortex solitons obtained while following the conditions of the problem, which do not violate their stability.

Figure 5a presents a time-dependent piecewise linear function, when the frequency detuning of the pump field (parameter ζ) changes externally, and the corresponding changes in the group velocity Δv_g (the expression for v_g is given in the Appendix) during the propagation of a vortex soliton in a gas-filled fibre.

Choosing the parameters for point A in Fig. 4 as initial conditions (at a constant detuning Δ_c^A at the initial time interval $t \in [0; 10$ ns$]$), we obtain the stabilisation of a corresponding vortex soliton during a characteristic time $\sim$5 ns (Fig. 5b).

Changing the frequency detuning to a value of $\Delta_c^{A'}$ for point A′ in the same region of stability (Fig. 4) leads to a decrease in the group velocity of the vortex soliton $\Delta v_g = 1.8\times10^4$ m s^{-1} while maintaining a steady regime for the axisymmetric soliton. Intensity oscillations of the vortex soliton emerging in this case decay rapidly for time $\tau_d = 3$ ns (Fig. 5b).

When the frequency detuning acquires the previous value of Δ_c^A, a similar scenario of stabilisation of the symmetric vortex soliton is implemented at a level preceding the onset of changes in Δ_c.

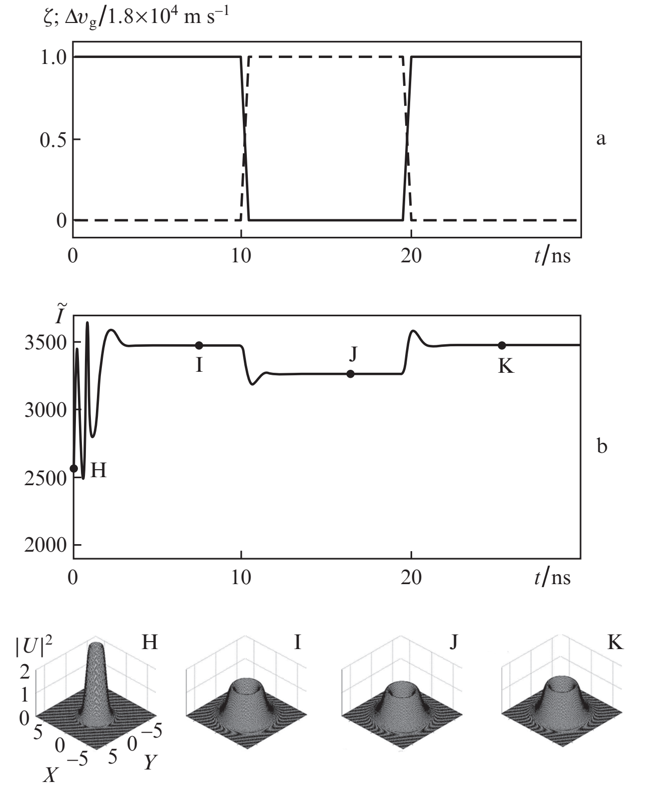

Figure 5. Optical control of dynamics of a vortex soliton: time dependences of the relative increment of the frequency detuning $\zeta = (\Delta_c - \Delta_c^{A'})/(\Delta_c^{A} - \Delta_c^{A'})$ (solid line) and change in the group velocity $\Delta v_g = v_g^A - v_g$ (dashed line) (a) and time dependence [obtained by numerical simulation of (3)] of the normalised intensity $\tilde{I}$ of the vortex soliton (4), corresponding to almost instant modification from point A to point A′ in Fig. 4 (b). Below is shown the shape of the vortex soliton at different times, corresponding to Fig. 5b.

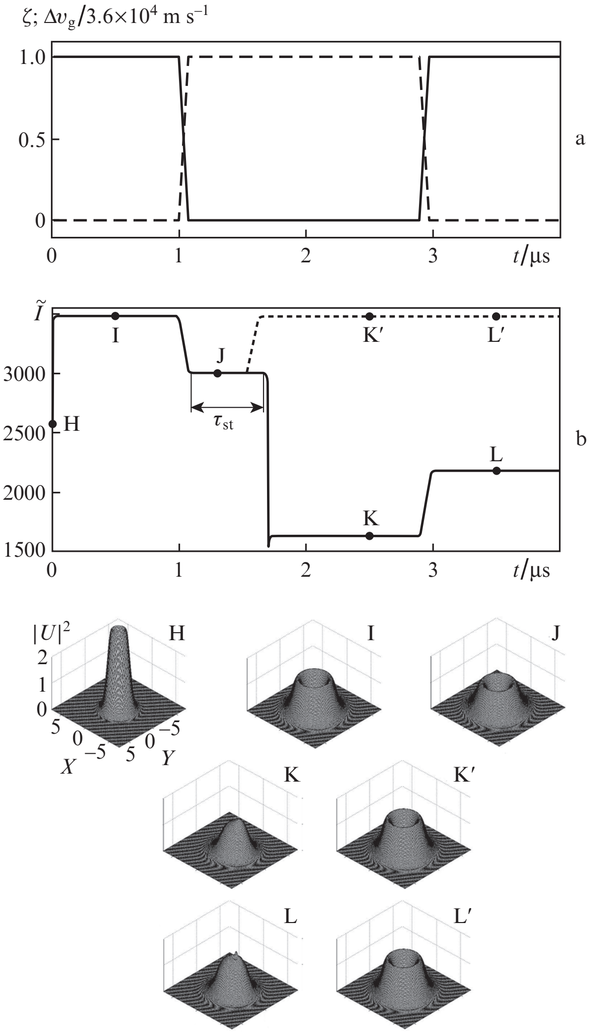

Figure 6. Same as in Fig. 5, but with the modification of the parameter Δ_c to the conditions corresponding to point B in Fig. 4. The dashed line in Fig. 6b corresponds to the reverse modification from point B to point A in Fig. 4 for the settling time τ_{st} with preservation of the shape of the vortex soliton.

The lower part of Fig. 5 presents the process of deformation of the vortex soliton as it propagates in a gas-filled fibre. These results were obtained by numerical simulation of equation (3) with the initial function (4) upon modulation of $\Delta_c(t)$, corresponding to Fig. 5a. In this case, the total length of the gas-filled fibre, corresponding to the entire time interval $t \in (0; t_K)$, is 14 cm.

Note that changing the switching time affects only the type of the transition to the new conditions of the vortex soliton stability with the settling time τ_{st} when points A and A′ are located in region I (Fig. 4).

However, the offset of the frequency detuning $\Delta_c(t)$ beyond the stability region – in going from point A to point B in Fig. 4 – leads to the pump-field-induced loss of the topological charge by the vortex soliton and to the transition to the new conditions of stability, but for a vortex-free soliton (see Fig. 6a and the shape of the optical beam for point K in Fig. 6b). An attempt to implement a scenario of return of the system to initial conditions, similar to that given in Fig. 5, fails: changing the frequency detuning Δ_c in the reverse transition to point A also leads to stabilisation of the soliton, but its vortex structure for $S = 1$ is not maintained (see the shape of an optical beam for point L in Fig. 6b).

A feature of the system for the conditions corresponding to Fig. 6 is the presence of the establishment process (with characteristic settling time τ_{st}) appearing after achieving the conditions (upon modulation of Δ_c) corresponding to point B in Fig. 4 and preceding almost instantaneous switching from a vortex soliton to the vortex-free regime. In Fig. 6b the settling time is 0.5 μs. The inverse modulation of Δ_c (performed during this time) to the conditions corresponding to point A does not allow the decay of the vortex structure of the soliton (see profiles of the optical beam for points K′ and L′ in Fig. 6b). This feature causes the additional possibility of using the proposed scheme of optical control of dissipative vortex solitons for optical information processing.

Note that the implementation of optical modulation beyond the region of stability of dissipative solitons, obtained by numerical methods (Fig. 3), leads to a rapid decrease in the intensity of the vortex soliton to full decay without the possibility of further recovery of its shape.

9. Conclusions

We have considered the problem of forming and controlling the dynamics of optical vortex solitons of the probe field, emerging in the Raman regime of the lambda-type interaction scheme, taking into account the local field effects. Using variational methods and direct numerical simulation we have predicted the regions of the parameters of the medium and fields at which optical vortex solitons can be efficiently formed in a model of a hollow-core fibre filled with a gas of ^{87}Rb atoms. Accounting for the dipole–dipole interactions in such a problem has a stabilising effect on the formation of dissipative solitons. We have found that the modulation of the detuning frequency of the pump field from the resonance does not lead to a dramatic destruction of a vortex soliton in the case when such changes do not exceed the limits of the stability region obtained. This results in additional topological noise immunity of the proposed scheme for the generation of optical vortex solitons with much more stable values of the parameters. Other advantages of the scheme include the possibility of controlling the parameters of the optical vortex solitons, a relatively simple interface with fibre-optic communication lines and compactness of the scheme.

Experimental observation of the considered effects is a challenge and possible, for example, when using hollow-core photonic-crystal fibres filled with a gas of an atomic Bose–Einstein condensate; the lifetime of the coherent state of the atomic system in this case reaches 400 μs [13].

A significant increase in phase density of an ensemble of atoms, as they are loaded into the fibre from a magneto-optical trap containing a Bose–Einstein condensate, is possible when use is made of the geometry of a tapered fibre [28] with a horizontal end-face of diameter of about several micrometres.

An additional factor increasing the lifetime of the coherent state is the effect of channelling of atoms in the waveguide when a surface light wave is produced along its hollow core; atoms interacting with it can lose energy. The estimates given in [28] show that at an achievable concentration of the ensemble of atoms 10^{15} cm^{-3}, the temperature of the atomic system in such a waveguide can be evaluated as 1.5×10^{-5} K.

In the absence of significant broadening of spectral lines and with the diffusion effects minimal for this gaseous medium, it is possible to observe confidently the effects analysed in a sample of a thin, gas-filled fibre.

Note also that of interest is the problem of an almost complete stop and acceleration of formation of solitons without affecting their stability in the optical system. This approach avoids the procedures of writing the profile of the wave packet on the states of atomic excitations of the medium in which the storage time of quantum information is limited by the coherence time of the atomic medium. This is especially urgent for the implementation of the original quantum information processing algorithms based on the coding and control of time delays of probe pulses in multi-beam optical schemes of atomic-optical interactions [29], including the use of quantum solitons. To solve this problem, it is necessary to search for a wider region of stability of vortex solitons based on the analysis of phase portraits of the complete system of equations (5), similar to that described in [30] for the case of conventional bright solitons.

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Appendix. Derivation of the coefficients of the Ginzburg–Landau equations for the Raman limit of lambda-type atomic–optical interaction scheme

In deriving equation (2) we use the equation of propagation of the probe field, which generally takes the form (1b). This requires finding an explicit expression for the density matrix element $\sigma_{\rm ba}$ (polarisation) on a probe transition, which depends only on the material parameters of the medium and the characteristics of the optical fields. We solve the system of equations (1a) in two stages.

In the first stage we define the polarisation of the system at lower levels $\sigma_{\rm cb}$, using the approximation of constant population levels in the stationary regime, i.e., when $\dot\sigma_{\rm aa} = \dot\sigma_{\rm bb} = \dot\sigma_{\rm cc} = 0$. In addition, we assume that the probe pulse duration is large enough compared to the inverse Rabi frequency for the pump field $[T_0 > (\Omega d_0)^{-1}]$ to ensure 'smooth input' [12]; this allows one to ignore the fast oscillations $\sigma_{\rm ba}$ [31] and, therefore, deep modulation of the probe pulse envelope is absent. Thus, the conditions $\dot\sigma_{\rm ba} = \dot\sigma_{\rm ca} = 0, \dot\sigma_{\rm bc} = 0$ must be valid for the leading edge of the probe pulse.

Taking into account this approximation from (1a) we can obtain an algebraic equation for the polarisation $\sigma_{\rm bc}$:

$$\Omega\chi g\varepsilon^*\sigma_{\rm bc}^2 + {\rm i}(\Gamma_1 g^2|\varepsilon|^2 + \Gamma_2^*\Theta)\sigma_{\rm bc} + {\rm i}g\varepsilon\Omega^*\Gamma_2^* = 0, \quad \text{(A1)}$$

whose roots have the form:

$$\sigma_{\rm bc} = \frac{-{\rm i}(\Gamma_1 g^2|\varepsilon|^2 + \Gamma_2^*\Theta) \pm \sqrt{D}}{2\Omega\chi g\varepsilon^*}, \quad \text{(A2)}$$

where $D = -(\Gamma_1 g^2|\varepsilon|^2 + \Gamma_2^*\Theta)^2 - 4{\rm i}\chi g^2|\Omega|^2\Gamma_2^*|\varepsilon|^2$; $\Theta = |\Omega|^2 - {\rm i}\Delta_3(\Gamma_1 + {\rm i}\chi)$; $\chi \equiv \chi_{\rm ba}$.

Solutions (A2) define actually two branches of spin excitations that occur at the transition between the levels $|{\rm b}\rangle$ and $|{\rm c}\rangle$ (Fig. 1). The solution containing the minus sign in (A2) leads to a problem with a saturable nonlinearity of type $\sigma_{\rm bc} \approx 1/\varepsilon$ and in the present work is not considered.

Expansion of the other, containing a plus sign, solution (A2) in series over the probe field ε leads to the relation:

$$\sigma_{\rm bc} \approx -\frac{g\Omega^*}{\Theta}\varepsilon + \frac{g^3\Omega^*}{\Gamma_2^*\Theta^2}\left(\Gamma_1 + {\rm i}\frac{|\Omega|^2\chi}{\Theta}\right)|\varepsilon|^2\varepsilon$$
$$-\frac{g^5\Omega^*}{(\Gamma_2^*)^2\Theta^3}\left(\Gamma_1^2 + \frac{3{\rm i}|\Omega|^2\chi\Gamma_1}{\Theta} - \frac{2\chi^2|\Omega|^4}{\Theta^2}\right)|\varepsilon|^4\varepsilon. \quad \text{(A3)}$$

In the second stage we express from the equations for $\dot\sigma_{\rm ca}$ and $\dot\sigma_{\rm bc}$ of system (1a) the polarisation of the atomic system at the probe transition in regime of propagation of the probe pulse at $\dot\sigma_{\rm bc} \neq 0$. Then we obtain

$$\sigma_{\rm ba} = \left[{\rm i}\left(\dot\sigma_{\rm bc} - {\rm i}\frac{g\varepsilon\chi}{\Omega\Gamma_2^*}\sigma_{\rm bc}\dot\sigma_{\rm cb}\right) + \left(\Delta_3 + {\rm i}\frac{g^2|\varepsilon|^2}{\Gamma_2^*}\right)\sigma_{\rm bc}\right.$$
$$\left. + {\rm i}\frac{g\varepsilon\chi}{\Omega\Gamma_2^*}|\sigma_{\rm bc}|^2\left(\Delta_3 - {\rm i}\frac{g^2|\varepsilon|^2}{\Gamma_2}\right)\right]\left(\Omega^* - \frac{g^2|\varepsilon|^2\chi^2}{\Omega|\Gamma_2|^2}|\sigma_{\rm bc}|^2\right)^{-1}. \quad \text{(A4)}$$

Substituting expansion (A3) into (A4) and performing an additional expansion, we have

$$\sigma_{\rm ba} \approx -{\rm i}\frac{g}{\Theta}\dot{\varepsilon} - \frac{g\Delta_3}{\Theta}\varepsilon - {\rm i}\frac{g^3}{\Gamma_2^*\Theta}\Bigg[1 + {\rm i}\Delta_3\Bigg(\frac{\Gamma_1}{\Theta} + {\rm i}\frac{\chi}{\Theta^*} + {\rm i}\frac{\chi|\Omega|^2}{\Theta^2}\Bigg)\Bigg]|\varepsilon|^2\varepsilon + g^5\Bigg\{\frac{1}{(\Theta\Gamma_2^*)^2}\Bigg[{\rm i}\Bigg(1 - \frac{\chi\Delta_3}{\Theta^*}\Bigg)$$
$$\times\Bigg(\Gamma_1 + {\rm i}\frac{\chi|\Omega|^2}{\Theta}\Bigg) - \frac{\Delta_3}{\Theta}\Bigg(\Gamma_1^2 - 2\frac{\chi^2|\Omega|^4}{\Theta^2} + 3{\rm i}\frac{\chi\Gamma_1|\Omega|^2}{\Theta}\Bigg)\Bigg]$$
$$-\frac{{\rm i}\chi}{|\Theta|^2|\Gamma_2|^2}\Bigg[{\rm i} + \frac{\Delta_3}{\Theta^*}\Bigg(\Gamma_1^* - {\rm i}\frac{\chi|\Omega|^2}{\Theta^*}\Bigg) - {\rm i}\frac{\chi\Delta_3}{\Theta}\Bigg]\Bigg\}|\varepsilon|^4\varepsilon. \quad \text{(A5)}$$

In order to determine the contributions of the effects that must be taken into account in the system, we substitute the obtained expression (A5) into (1b) and obtain the equation

$$\left(\frac{1}{v_{\rm g}}\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right)\varepsilon + \frac{\rm i}{2}\beta_2\frac{\partial^2\varepsilon}{\partial t^2} - {\rm i}\frac{D_0}{2}\left(\frac{\partial^2\varepsilon}{\partial x^2} + \frac{\partial^2\varepsilon}{\partial y^2}\right)$$
$$- {\rm i}\gamma_2|\varepsilon|^2\varepsilon + {\rm i}\gamma_4|\varepsilon|^4\varepsilon = -\alpha_1\varepsilon - \alpha_2|\varepsilon|^2\varepsilon - \alpha_4|\varepsilon|^4\varepsilon, \quad \text{(A6)}$$

where the corresponding coefficients are given by

$$v_{\rm g} = \frac{c}{1 + {\rm Re}(g^2N/\Theta)}$$

is the group velocity;

$$\beta_2 = \frac{{\rm d}\beta_1}{{\rm d}\omega_{\rm p}} = \frac{1}{v_{\rm g}^2}\frac{{\rm d}v_{\rm g}}{{\rm d}\Delta_{\rm b}}$$

is the second-order group-velocity dispersion;

$$\gamma_2 = {\rm Im}\left\{-\frac{g^4N}{\Theta\Gamma_2^*c}\left[1 + {\rm i}\Delta_3\left(\frac{\Gamma_1}{\Theta} + {\rm i}\frac{\chi}{\Theta^*} + {\rm i}\frac{\chi|\Omega|^2}{\Theta^2}\right)\right]\right\}$$

is the coefficient of cubic nonlinearity;

$$\gamma_4 = {\rm Im}\left\{\frac{{\rm i}g^6N}{c}\left(\frac{1}{(\Theta\Gamma_2^*)^2}\left[{\rm i}\left(1 - \frac{\chi\Delta_3}{\Theta^*}\right)\left(\Gamma_1 + {\rm i}\frac{\chi|\Omega|^2}{\Theta}\right)\right.\right.\right.$$
$$\left. - \frac{\Delta_3}{\Theta}\left(\Gamma_1^2 - 2\frac{\chi^2|\Omega|^4}{\Theta^2} + 3{\rm i}\frac{\chi\Gamma_1|\Omega|^2}{\Theta}\right)\right]$$
$$\left.\left. - \frac{{\rm i}\chi}{|\Theta|^2|\Gamma_2|^2}\left[{\rm i} + \frac{\Delta_3}{\Theta^*}\left(\Gamma_1^* - {\rm i}\frac{\chi|\Omega|^2}{\Theta^*}\right) - {\rm i}\frac{\chi\Delta_3}{\Theta}\right]\right)\right\}$$

is the fifth-order nonlinearity coefficient;

$$\alpha_1 = {\rm Im}\left(\frac{g^2N\Delta_3}{\Theta c}\right)$$

is the linear loss coefficient;

$$\alpha_2 = {\rm Re}\left\{\frac{g^4N}{\Theta\Gamma_2^*c}\left[1 + {\rm i}\Delta_3\left(\frac{\Gamma_1}{\Theta} + {\rm i}\frac{\chi}{\Theta^*} + {\rm i}\frac{\chi|\Omega|^2}{\Theta^2}\right)\right]\right\}$$

is the cubic nonlinear loss coefficient;

$$\alpha_4 = {\rm Re}\left\{\frac{{\rm i}g^6N}{c}\left(\frac{1}{(\Theta\Gamma_2^*)^2}\left[{\rm i}\left(1 - \frac{\chi\Delta_3}{\Theta^*}\right)\left(\Gamma_1 + {\rm i}\frac{\chi|\Omega|^2}{\Theta}\right)\right.\right.\right.$$
$$\left. - \frac{\Delta_3}{\Theta}\left(\Gamma_1^2 - 2\frac{\chi^2|\Omega|^4}{\Theta^2} + 3{\rm i}\frac{\chi\Gamma_1|\Omega|^2}{\Theta}\right)\right]$$
$$\left.\left. - \frac{{\rm i}\chi}{|\Theta|^2|\Gamma_2|^2}\left[{\rm i} + \frac{\Delta_3}{\Theta^*}\left(\Gamma_1^* - {\rm i}\frac{\chi|\Omega|^2}{\Theta^*}\right) - {\rm i}\frac{\chi\Delta_3}{\Theta}\right]\right)\right\}$$

is the fifth-order nonlinear loss coefficient.

Equation (A6) corresponds to (2) and with the coefficients given above determines the basic relation for the problem under study.

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