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STRONG ELECTROMAGNETIC FIELDS

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Dynamics of ionisation and entanglement in the 'atom + quantum electromagnetic field' system

P.R. Sharapova, O.V. Tikhonova

Abstract. **The dynamics of a model Rydberg atom in a strong nonclassical electromagnetic field is investigated. The field-induced transitions to the continuum involving different numbers of photons (with intermediate states in the discrete spectrum) are taken into account and the specific features of ionisation in 'squeezed' field states are considered in comparison with the case of classical light. A significant decrease in the ionisation rate is found, which is caused by the interference stabilisation of the atomic system. The entanglement of the atomic and field subsystems, the temporal dynamics of the correlations found, and the possibility of measuring them are analysed.**

Keywords: *interference stabilisation, nonclassical electromagnetic fields, entangled states.*

1. Introduction

The recent development of experimental laser physics has made it possible to generate few-cycle laser pulses with an intensity comparable with intra-atomic or even higher. Radically new effects arise in such strong fields. One of them, which appears to be the most striking one, is stabilisation [1, 2]. It consists in suppression (saturation at a level below unity or even reduction) of the ionisation probability when the intensity of external laser field exceeds some critical value [1–4].

In the case of weak fields the interaction of laser radiation with atomic systems can be described in terms of the perturbation theory [1]. However, the perturbation-theory methods cannot be applied to strong fields. In this case, it is necessary to search for new approaches that would strictly take into account the interaction with strong fields. A new system is formed in a strong field: the so-called field-dressed atom, whose properties radically differ from those of unperturbed atom. A field-dressed atom is characterised by new energy states, which are referred to as dressed states. Stabilisation can be observed due to the formation of a dressed system in a strong light field. Two types of stabilisation can be distinguished: the interference stabilisation of initially excited Rydberg atoms, which was predicted by M.V. Fedorov [1, 5], and the adiabatic stabilisation of unexcited atoms according to the Kramers–Henneberger mechanism [3, 4].

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The physical nature of interference stabilisation is based on coherent repopulation of Rydberg levels during ionisation due to the Raman-type transitions. This repopulation may occur via virtual transitions through the continuum (Λ-type transitions) [1, 5, 6] or through a lower state (V-type transitions) [6–8]. These transitions become effective in sufficiently strong fields. In this case, the coherent repopulation of closely spaced Rydberg levels due to the Λ- and V-type transitions leads to phasing of the population amplitudes for these levels in such a way that the subsequent transitions from these levels to the continuum interfere to suppress partially each other and stabilise the atom, thus reducing its ionisation rate in strong fields. In weak fields Raman transitions between levels with different energies are forbidden by the law of conservation of energy. These transitions become possible when the Rydberg levels have a corresponding ionisation broadening, which overlaps the energy spacing between neighbouring levels. Due to this, interference stabilisation may occur, which arises in a strong classical field only at intensities exceeding some threshold value.

Although interference stabilisation is observed even in strong classical fields, the case of nonclassical fields (in particular, squeezed states), which have a number of exceptional properties, is of particular interest. For example, the essentially nonclassical properties of these fields are retained at any (even very large) mean numbers of quanta. Thus, the probability of the processes involving many photons is high, a circumstance making these processes promising [9, 10]. A striking representative of these fields is a field in the squeezed-vacuum state [11]. The squeezed state of a quantum field is interesting from both theoretical and experimental points of view. For example, generation of harmonics in atoms in the case of squeezed light is much more efficient in comparison with other field states. The squeezed state is also characterised by diminished noise and quantum errors, which is important for various methods of quantum measurements [11]. Thus, the aforementioned stabilisation effect is expected to be more pronounced for quantum fields than for classical light. To date, quantum fields have been obtained experimentally [12–15]. The main features of squeezed states were considered in [11], and the specificity of interaction of atomic systems with nonclassical fields was discussed in [9, 10, 16]. It was demonstrated in [9, 10] that squeezed states increase the efficiency of processes involving many photons; nevertheless, the question of possible stabilisation of atomic systems in nonclassical fields has not been discussed and remains open.

It should be noted that, in contrast to the semiclassical approach (where the given-field approximation is applied), in the case of quantum fields one can analyse the field evolution, i.e., the changes in the field properties in time. As a consequence, correlations may arise between the atomic and field subsystems during their interaction. This problem is also of great interest and has not been studied in detail. For example, having measured the properties of one subsystem, one can obtain information about the parameters of the other subsystem in a given instant, which is rather convenient if the properties of some subsystem are difficult to measure.

Our purpose was to investigate the dynamics of a model atomic system in a strong nonclassical electromagnetic field; analyse the possibility and conditions for the occurrence of interference stabilisation, entanglement, or correlations between atomic and field subsystems; and study the temporal dynamics of the correlations found and the possibility of measuring them.

2. Model of the atom + field system

The interaction between an atom and one mode of a quantum field is described by the time-dependent Schrödinger equation

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = (\hat{H}_{\rm at} + \hat{H}_{\rm field} + \hat{V}_{\rm int})\psi. \qquad (1)$$

Here, $\hat{H}_{\rm at}$ is the Hamiltonian of unperturbed atom; $\hat{H}_{\rm field}$ is the Hamiltonian of one field mode; $\hat{V}_{\rm int} = -\boldsymbol{D\varepsilon} = -\boldsymbol{Dq}\varepsilon_0$ is an operator describing the interaction of the atomic system with an external electromagnetic field in the dipole approximation; $\boldsymbol{q}$ is the dimensionless electric field; and$\varepsilon_0 = \sqrt{4\pi\hbar\omega/L^3}$ is the normalisation constant, which is formally determined by the characteristic cavity volume L^3 and is actually controlled by the efficiency of interaction between the atomic system and field. In this study the atom is considered within the model of two or three discrete levels and a continuum [5–7], which allows one to investigate the evolution of excited Rydberg states in a quantum electromagnetic field and analyse the possibility of interference stabilisation. The so-called schemes of Λ- and V-types are considered separately. The Λ-type scheme considers a combination of two discrete levels (1 and 2) and the continuum; this scheme implies coherent repopulation of these levels due the Λ-transitions through the continuum. The V-type scheme includes two discrete levels (1 and 2), a lower resonant state 0, and the continuum. In this case, along with the Λ-type transitions, one must also take into account the V-type transitions through the lower resonant state. It is assumed that the two upper atomic levels are populated at the initial instant; the complex probability amplitudes of the states have a phase difference ϕ, and the external field is characterised by some initial distribution over Fock states with the probability amplitudes α_k:

$$\psi\big|_{t=0} = C_1\varphi_1\sum_k \alpha_k \Phi_k + C_2\exp(\mathrm{i}\phi)\varphi_2\sum_k \alpha_k \Phi_k, \qquad (2)$$

where φ_1, φ_2, and Φ_k are, respectively, the wave functions of the stationary state of the atomic system and the field oscillator, with their interaction disregarded. The normalisation condition for the field and atomic coefficients is given by the expressions

$$\sum_k |\alpha_k|^2 = 1, \quad C_1^2 + C_2^2 = 1.$$

We took the initial state of the field in the form of coherent states with Poissonian distribution of photons (which corresponds to classical light at large mean numbers of quanta $\langle N\rangle$), as well as squeezed-vacuum states, and performed comparison for the interaction of this field with the atomic system at a fixed $\langle N\rangle$. The field in the squeezed-vacuum state is an essentially nonclassical electromagnetic field, which is characterised by a wide distribution over the number of photons:

$$|\alpha_{2k}|^2 = \frac{|\alpha_{2k-2}|^2\langle N\rangle}{\langle N\rangle + 1}\frac{2k-1}{2k}, \quad |\alpha_0|^2 = \frac{1}{\sqrt{\langle N\rangle + 1}}.$$

Here, the populations for different Fock states are expressed recurrently. It can easily be seen that the distribution slowly decreases with an increase in k, which leads to a nonzero probability of populating the Fock states with large k and a necessity for taking into account the processes involving many photons. Concerning the zero Fock state, it has the maximum population amplitude (proceeding from the distribution properties) and does not cause ionisation of the atomic system when the latter interacts with the field.

Note also that only the Fock states with even numbers are populated in the squeezed-vacuum state. Specifically this feature characterises its nonclassical properties, because the mean electric field for this distribution is zero, and the characteristic strength is determined by the dispersion, i.e., quantum fluctuations.

We will search for a solution to (1) in the form of an expansion in the state basis of the unperturbed system:

$$\psi(t) = \sum_{k=0}^{\infty}\Big[\sum_n a_{nk}(t)\varphi_n(\boldsymbol{r})\,\Phi_k + \int \mathrm{d}E a_{Ek}(t)\varphi_E(\boldsymbol{r})\,\Phi_k\Big] \times\exp[-\mathrm{i}\omega(k+1/2)t], \qquad (3)$$

where $n = 1, 2$ for the Λ scheme and 0, 1, 2 for the V scheme; the integration is over the continuum region.

Having substituted the wave function (3) into the time-dependent Schrödinger equation and neglected the direct transitions between closely spaced Rydberg levels and the transitions between different continuum states, in the case of the Λ scheme, we obtain a system of differential equations for the amplitudes α_{nk} of the probability of detecting the system in a discrete state with respect to the atomic degree of freedom and in a Fock state $|k\rangle$ in the field degree of freedom and for the amplitude α_{Ek}, corresponding to the existence of the atomic subsystem in the continuum:

$$\mathrm{i}\hbar\frac{\mathrm{d}a_{nk}}{\mathrm{d}t} = E_n a_{nk} - \sum_{p=0}^{\infty}\int a_{Ep}\exp[\mathrm{i}\omega(k-p)t] \times\langle\varphi_n|\hat{D}|\varphi_E\rangle\langle\Phi_k|\varepsilon_0 q|\Phi_p\rangle\mathrm{d}E,$$
$$\mathrm{i}\hbar\frac{\mathrm{d}a_{Ek}}{\mathrm{d}t} = Ea_{Ek} - \sum_{n=1}^{2}\sum_{p=0}^{\infty}a_{np}\exp[\mathrm{i}\omega(k-p)t] \times\langle\varphi_E|\hat{D}|\varphi_n\rangle\langle\Phi_k|\varepsilon_0 q|\Phi_p\rangle, \qquad (4)$$

where $\hat{D}$ is the operator of the dipole-moment projection on the field direction and E_n is the energy of the atom in the state n.

Using the rotating-wave approximation, we will leave only the resonant term in (4). Then, having adiabatically excluded the continuum [17], we obtain the following system of differential equations for the probability amplitudes:

$$i\hbar\frac{da_{1k}}{dt} = E_1 a_{1k} - i\frac{\Gamma_k}{2}\sum_{n=1}^{2} a_{nk}(t),$$
$$i\hbar\frac{da_{2k}}{dt} = E_2 a_{2k} - i\frac{\Gamma_k}{2}\sum_{n=1}^{2} a_{nk}(t), \quad (5)$$

where $\Gamma_k = 2\pi k\varepsilon_0^2|d_{nE}|^2/2$ is the ionisation width, which is due to the interaction between the atom and field in the Fock state $|k\rangle$ (it is assumed to be the same for all atomic states), and d_{nE} is the matrix element of the dipole-moment operator.

A similar approach was used for the model atomic system, with allowance for its resonant coupling with the lower atomic level. In this case, the differential system of equations for the probability amplitudes of discrete atomic states has the form

$$i\hbar\frac{da_{0k}}{dt} = E_0 a_{0k} + \sum_{n'=1}^{2}\Omega_k a_{n'k-1}\exp(i\omega t),$$
$$i\hbar\frac{da_{nk-1}}{dt} = \Omega_k a_{0k}\exp(-i\omega t) + E_n a_{nk-1} - i\frac{\Gamma_{k-1}}{2}\sum_{n'=1}^{2} a_{n'k-1}. \quad (6)$$

Here, $\Omega_k/\hbar = \varepsilon_0(k/2)^{1/2}d_{0n}/(2\hbar)$ is an analogue of the Rabi frequency for the resonant transition. Thus, in contrast to classical light, in the quantum case Ω_k and Γ_k depend on k; i.e., the field-induced transitions for each Fock state have their own probabilities. In addition, three important parameters arise in this case: Ω_k, Γ_k, and $d = E_2 - E_1$. The relationship between them determines different regimes of dynamics of the atomic system placed in a quantum field.

Note that an analogue of classical field in the quantum case is the coherent state of a field with Poissonian distribution over the number of photons at $\langle N\rangle \gg 1$, and the classical intensity I is determined by the mean number of field quanta $\langle N\rangle$ [18]: $I = c\varepsilon_0^2\langle N\rangle/(4\pi)$.

For a quantum field (by analogy with classical), there must be some critical number of quanta, which will serve as a criterion for separating the problem into the domains of strong and weak fields. This separation can be performed for a field with Poissonian distribution of coefficients, which is related to the fairly narrow localisation of populated field states near the mean number of field quanta. Another situation is observed for a field in the squeezed-vacuum state: here, at any $\langle N\rangle$ value, there are always regions of both weak and strong fields, and one must perform summation over two regions simultaneously, due to the wide distribution over the number of photons of this quantum field. The boundary between these two regions is determined by solving the problem and corresponds to some critical number of photons, which will be found below. The number of populated Fock states that are taken into account depends on the specificity of distribution: for example, for a field in the squeezed-vacuum state, states with $k \sim \langle N\rangle^2$ can make a contribution.

Thus, the Λ scheme contains three important parameters: d (the spacing between discrete levels), $\Gamma_k = \Gamma k$ (the ionisation width), and $\langle N\rangle$ (the mean number of photons for a given state of quantum field). The V scheme is characterised by an additional parameter: an analogue of the Rabi frequency $\Omega_k/\hbar = \Omega(k/2)^{1/2}/\hbar$. The effects observed are rather sensitive to the choice of initial conditions: stabilisation is practically absent in their certain range. We used the following values of the parameters: $d = 0.1$ eV, $\Omega = 0.1$ eV, $\Gamma = 2\times10^{-4}$ eV, $E_2 = -0.4$ eV, $E_1 = -0.5$ eV, $E_0 = -1.5$ eV, and $\hbar\omega = 1$ eV. Note that the parameters chosen correspond to a fairly strong coupling with the resonant state in the V scheme, in comparison with the continuum; this condition is necessary for implementing stabilisation, by analogy with [6].

3. Results and discussion

Consideration of this problem within the Λ scheme yielded an exact analytical solution to system (5). The time dependences of the population amplitudes for discrete atomic states were found provided that the field is in different Fock states $|k\rangle$. The solutions obtained differ significantly for the weak- and strong-field regions. The strong-field regime corresponds to overlap of atomic levels due to the ionisation width $\Gamma_k > d$. For the problem parameters chosen, this situation yields a critical number of photons $k_{cr} = 500$, which corresponds to the boundary between these two regions.

It is more convenient to consider this problem in terms of quasi-energies and quasi-energy wave functions, i.e., the energies and wave functions corresponding to the dressed system. In this case, the expression for the amplitude of the probability of finding the atom in the state n and the field in the state $|k\rangle$ takes the form

$$a_{nk}(t) = A_{nk}\exp(-i\gamma_+^{(k)}t/\hbar) + B_{nk}\exp(-i\gamma_-^{(k)}t/\hbar), \quad (7)$$

where the coefficients A_{nk} and B_{nk} are time-independent; they are found from the initial conditions. System (5) is similar to that obtained for an atom in a classical field [6]; however, in our case Γ_k depends on the number k. The quasi-energies arising, $\gamma_\pm^{(k)}$, are different for different photon states:

$$\gamma_\pm^{(k)} = \frac{E_1+E_2}{2} - \frac{i\Gamma_k}{2} \pm \sqrt{\frac{d^2}{4} - \frac{\Gamma_k^2}{4}}, \quad d > \Gamma_k, \quad (8a)$$

$$\gamma_\pm^{(k)} = \frac{E_1+E_2}{2} - \frac{i\Gamma_k}{2} \pm i\sqrt{\frac{\Gamma_k^2}{4} - \frac{d^2}{4}}, \quad d < \Gamma_k. \quad (8b)$$

The ionisation probability W_i can be expressed in these terms as follows:

$$W_i = 1 - \sum_{k=0}^{\infty}\Big\{|A_{1k}|^2 + |B_{1k}|^2 + |A_{2k}|^2 + |B_{2k}|^2$$
$$+\Big\{A_{1k}B_{1k}^*\exp\Big[\frac{i(\gamma_+^{(k)}-\gamma_-^{(k)})t}{\hbar}\Big] + B_{1k}A_{1k}^*\exp\Big[\frac{i(\gamma_+^{(k)}-\gamma_-^{(k)})t}{\hbar}\Big]\Big\}$$
$$+\Big\{A_{2k}B_{2k}^*\exp\Big[-\frac{i(\gamma_+^{(k)}-\gamma_-^{(k)})t}{\hbar}\Big] + B_{2k}A_{2k}^*\exp\Big[\frac{i(\gamma_+^{(k)}-\gamma_-^{(k)})t}{\hbar}\Big]\Big\}\Big\}. \quad (9)$$

Thus, in a weak field ($d > \Gamma_k$), the time dependence of the ionisation probability is described by the expression

$$W_i = 1 - \sum_{k=0}^{\infty}\frac{|\alpha_k|^2\exp(-\Gamma_k t/\hbar)}{4\beta_k^2}\{(C_1^2 + C_2^2)$$
$$\times[d^2 - \Gamma_k^2\cos(2\beta_k t/\hbar)] - 2\Gamma_k C_1 C_2[2\beta_k\cos\phi\sin(2\beta_k t/\hbar)$$
$$+ d\sin\phi[\cos(2\beta_k t/\hbar) - 1]]\}, \quad (10)$$

where

$$\beta_k = \sqrt{\frac{d^2}{4} - \frac{\Gamma_k^2}{4}}.$$

In a strong field ($d < \Gamma_k$), we have

$$W_{\rm i} = 1 - \sum_{k=0}^{\infty} \frac{|\alpha_k|^2 \exp(-\Gamma_k t/\hbar)}{4\beta_k^2}\{(C_1^2 + C_2^2)$$

$$\times[\Gamma_k^2 \cosh(2\beta_k t/\hbar) - d^2] - 4\Gamma_k C_1 C_2 \sinh(\beta_k t/\hbar)$$

$$\times[\beta_k \cos\phi \cosh(\beta_k t/\hbar) + d \sin\phi \sinh(\beta_k t/\hbar)]\}, \qquad (11)$$

where

$$\beta_k = \sqrt{\frac{\Gamma_k^2}{4} - \frac{d^2}{4}}.$$

Expressions (10) and (11) take into account that the chosen field state corresponds to the regime of a weak (or strong) field for all Fock states present in the initial distribution. This situation can be implemented for a Poissonian initial distribution of the field if $\langle N \rangle$ differs significantly from $k_{\rm cr}$. However, even a Poissonian distribution with $\langle N \rangle \sim k_{\rm cr}$ (and especially for case of squeezed vacuum) contains different photon states, some of which interact with the atom in the weak-field regime, while the others interact in the strong-field regime. Therefore, it is necessary to perform summation in the expression for the ionisation probability separately over each region, taking into account the difference between the solutions in the aforementioned regimes. Thus, the dynamics of the atomic system in the case of squeezed vacuum is expected to differ from that in the case of classical field, which was considered in [6]; this difference is caused by different contributions of a large number of Fock states to ionisation.

3.1. Dynamics of the system and the interference-stabilisation regime

As follows from solution (10), in a weak field ($\Gamma_k < d$) the difference in quasi-energies leads to characteristic oscillations of atomic-level populations with a frequency on the order of d/h against the background of exponential decay with time. For a field with a Poissonian distribution of the coefficients the characteristic decay constant is close to the width Γ_k, calculated for $k = \langle N \rangle$. Thus, the ionisation probability is saturated at a level of unity. At long times atomic levels decay, and the system becomes completely ionised (Fig. 1, dashed curve).

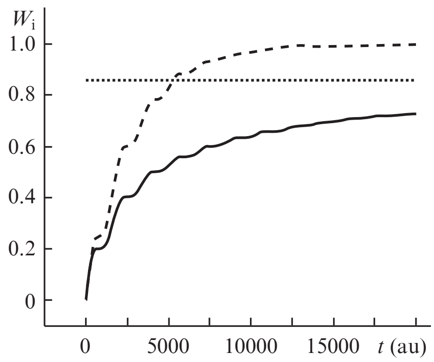

Figure 1. Time dependences of the ionisation probability $W_{\rm i}$ for the Λ scheme at $\langle N \rangle = 50$ and the initial phase difference $\phi = 0$ for the field in coherent state (dashed line) and in the squeezed-vacuum state (solid line). The horizontal straight line shows the level $1 - W_0$, where W_0 is the vacuum state population (1 au $\approx 2.42 \times 10^{-17}$ s).

In a strong field ($\Gamma_k > d$) the character of quasi-energies (8) radically changes: one of them (γ_-) corresponds to the quasi-energy state that rapidly decays with time, while the other (γ_+) corresponds to the state decaying more slowly. Due to the occurrence of a slowly decaying quasi-energy level in the strong field, one can observe the stabilisation effect. In the case of classical field [6], stabilisation manifests itself as follows: an increase in the laser intensity leads to an increase in the part of atomic population that is captured into a more stable (with respect to ionisation) quasi-energy state. Therefore, a higher laser intensity at a fixed instant yields a higher probability of detecting the atom in the bound state. Thus, in the general case, the ionisation dynamics of the system is characterised by biexponential decay.

The situation with a quantum field characterised by a Poissonian initial distribution over the number of photons at $\langle N \rangle \gg k_{\rm cr}$ is similar. Indeed, in this case, the ionisation dynamics of the system exhibits a sharp increase in the population of the continuum in the initial stage, after which the population tends much more slowly to unity (Fig. 2, dashed curve).

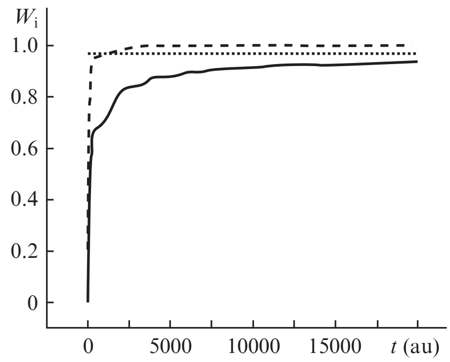

Figure 2. The same as in Fig. 1 but for $\langle N \rangle = 1000$.

In addition, stabilisation manifests itself in the phase sensitivity of the ionisation probability to the initial phase of the atomic-level population amplitude. In the limit $\Gamma_k \gg d$, the quasi-energies (8) characterise the rapidly decaying and practically stable states with the wave functions $\psi_k^{\mp} = [(\varphi_1 \pm \varphi_2)/\sqrt{2}]\Phi_{\rm k}$; therefore, the equiprobable initial population of the atomic states with a difference in the initial phases of the population amplitudes equal to π leads to a very slow decay of this state. However, at long times, the more stable energy level also decays, and the system becomes completely ionised; this situation corresponds to that for the classical field. Thus, the results for a field with a Poissonian initial statistics completely correspond to the data obtained in [6] within the semiclassical approach, although the $\langle N \rangle$ value is not very large and, strictly speaking, the coherent state under consideration is not classical light.

For a field in the squeezed-vacuum state, as was indicated above, the problem cannot be separated into strong- and weak-field regions: summation is performed simultaneously over two regions, as a result of which the features of both weak and strong fields manifest themselves in the dynamics of the system. For example, the time dependence of the ionisation probability exhibits characteristic oscillations with a frequency on the order of d/h, as for weak fields. Simultaneously, one can observe the stabilisation (strong-field) effect. As was expected, the stabilisation is most pronounced in this quantum field. Two factors contribute to the stabilisation: (i) the population of the zero Fock state (which is maximum for this distribution) and (ii) nonzero populations of the Fock states with large numbers k and their contribution to the dynamics of ionisation-stable quasi-energy states. Since the population of the zero Fock (vacuum) state corresponds to the populations of the atomic states that do not decay with time, the corresponding stabilisation occurs at any instant. The smaller the mean number of photons $\langle N\rangle$ for a given field, the higher the population of this state and the higher the degree of the related stabilisation. However, a decrease in $\langle N\rangle$ leads to a decrease in the number of states with large numbers k that were taken into account (i.e., the states with $k \sim \langle N\rangle^2$), which are involved in the formation of ionisation-stable quasi-energy states. Thus, for the problem parameters chosen, there is an optimal $\langle N\rangle$ value at which the stabilisation effect for a field in the squeezed-vacuum state is maximum.

The ionisation dynamics of the atomic system under field in the squeezed-vacuum state is shown in Figs 1 and 2 for $\langle N\rangle = 50$ and 1000, respectively. The horizontal straight line is the level corresponding to the $1 - W_0$ value, where W_0 is the vacuum-state population. This is the upper limit of the ionisation probability. It can be seen that for both $\langle N\rangle$ values the ionisation probability in squeezed vacuum is below this level and tends to saturate at a level below the aforementioned one. At $\langle N\rangle = 50 \ll k_{cr}$r the characteristic saturation time of the dependence $W_i(t)$ for a field in the squeezed-vacuum state is much longer than in the case of Poissonian distribution with the same $\langle N\rangle$. Therefore, even at small $\langle N\rangle$, the ionisation in squeezed vacuum slows down, and the asymptotic (at $t \to \infty$) value of its probability is much smaller than not only unity but also the upper boundary, which is due to the stabilising role of the Fock states with a large number of the photons ($k \gg k_{cr}$) entering this field state.

A similar situation is observed for $\langle N\rangle = 1000$. However, in this case the population of bound states for both the classical field and the field in the squeezed-vacuum state are characterised by rapid decay in the initial stage, which is followed by a much slower decay. Thus, when an atom is ionised by a squeezed nonclassical field, interference stabilisation is observed for both rather small and large $\langle N\rangle$ values. This stabilisation is much more pronounced than in the case of classical fields; it manifests itself in the formation of a wave packet of bound atomic states that are more stable to ionisation and in the long-term conservation of population in the states with a large number of photons.

Note also that a quantum field is characterised by even stronger (in comparison with classical fields) ionisation sensitivity to the initial phase difference for the atomic-level population amplitudes. In the case of antiphase population of atomic levels at the initial instant, the system exhibits a higher stability in comparison with the in-phase population. Thus, it is shown that the interaction of an atom with a quantum field in the squeezed-vacuum state leads to the formation of a rather stable system at a certain instant, which hardly decays in the course of time.

The situation with the V system is more complicated. Here, the analytical solution is rather cumbersome, and the pattern observed is fairly sensitive to the initial parameters of the problem. An interaction of the atom with an external field gives rise to induced transitions between discrete levels of both the Λ type (through the continuum) and the V type (through the lower resonant state); therefore, the interference stabilisation is much more pronounced than for the Λ scheme. Here, the relation between the frequency of a light photon and the Rabi frequency is of great importance: if the parameters are chosen poorly, the probability of resonant transitions is low, and they can hardly affect the dynamics of the system.

The consideration of the problem in terms of quasi-energies and quasi-energy wave functions gives rise to three quasi-energies, two of which correspond to the states decaying fairly rapidly, while the third describes the state that hardly decays and thus leads to stabilisation. The quasi-energies can be obtained in the explicit form by equating the determinant of the matrix

$$\begin{pmatrix} E_0 - \gamma & \Omega_k & \Omega_k \\ \Omega_k & E_1 - \mathrm{i}\dfrac{\Gamma_{k-1}}{2} - \hbar\omega - \gamma & -\mathrm{i}\dfrac{\Gamma_{k-1}}{2} \\ \Omega_k & -\mathrm{i}\dfrac{\Gamma_{k-1}}{2} & E_1 - \mathrm{i}\dfrac{\Gamma_{k-1}}{2} - \hbar\omega - \gamma \end{pmatrix}$$

to zero. Since the analytical expressions in the V scheme are fairly cumbersome, the problem is solved numerically; it is assumed that the resonant level is depopulated at the initial instant but then becomes populated. In contrast to classical field, squeezed vacuum within the V scheme, even at long times, contains oscillations of populations of both the resonant state (Fig. 3a) and the other two atomic levels (Fig. 3b), which is related to the influence of a large number of Fock

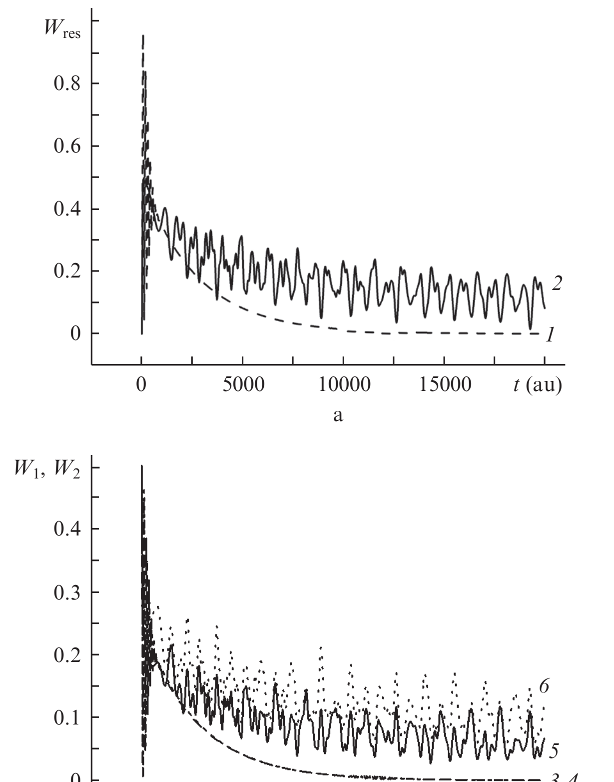

Figure 3. Time dependences of the populations of the (a) resonant state, W_{res}, and (b) the first (W_1; *3*, *5*) and second (W_2; *4*, *6*) atomic levels for the V scheme at $\langle N\rangle = 50$ and the initial phase difference $\phi = 0$ for fields in the (*1*, *3*, *4*) coherent and (*2*, *5*, *6*) squeezed-vacuum states.

states with large numbers k, populated for the nonclassical field. Therefore, the ionisation probability in a nonclassical field is always below unity; i.e., the stabilisation effect is directly observed at any initial phase difference, which does not hold true for the classical field (Figs 3, 4). In the case of initially populated atomic levels with a phase difference of π, a rather stable wave packet is formed almost immediately and does not decay in the course of time, which is characteristic of both classical and quantum fields (Fig. 5). Under these conditions, the stabilisation in the classical field is even stronger due to the higher degree of phase coherence of states (with respect to the validity of the destructive-interference conditions for the transitions into continuum).

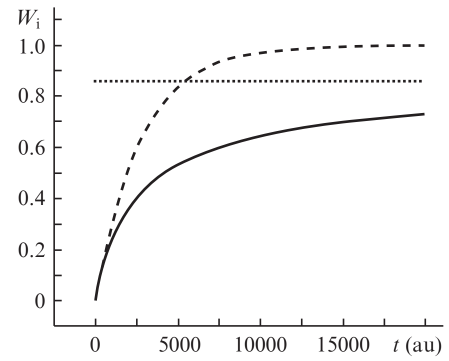

Figure 4. Time dependences of the ionisation probability W_i for the V scheme at $\langle N \rangle = 50$ and the initial phase difference $\phi = 0$ for fields in the coherent (dashed line) and (solid line) squeezed-vacuum states. The horizontal straight line shows the $1 - W_0$ level.

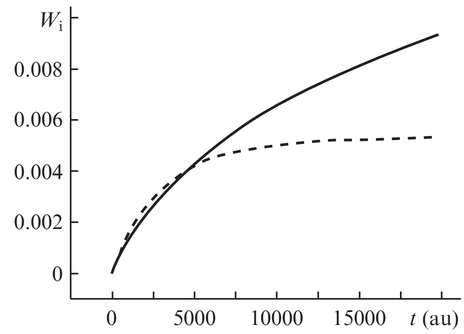

Figure 5. The same as in Fig. 4 but for the initial phase difference of the atomic states $\phi = \pi$.

3.2. Entanglement of the atomic and field subsystems

An important property of a combined quantum-mechanical system is its entanglement or impossibility of being factorised into combined parts, in other words, the wave function of the system cannot be presented as the product of the wave functions of the atomic and field subsystems: $\psi(r,q) \neq \varphi(r)\Phi(q)$. There are different measures of entanglement [19–22]. Here, the entanglement of the system was analysed using the Schmidt parameter, which is directly related to the reduced matrix density ρ_r of the system [19, 21]:

$$K = \mathrm{Tr}(\rho_r^2)^{-1}. \qquad (12)$$

For the states depending on a continuously changing variable, the expression for the Schmidt parameter can be written in another form [21]:

$$K = \frac{\int dx_1 dx_2 |\psi(x_1,x_2)|^2}{\int dx_1 dx_2 dx_1' dx_2' \, \psi(x_1,x_2)\psi^*(x_1' x_2)\psi^*(x_1 x_2')\psi(x_1' x_2')}.$$

It is easy to calculate the Schmidt parameter by considering only the states of the system that remained bound during ionisation. Having substituted the time-dependent wave function of the system in the form of expansion in the eigenfunctions of the bound states and taken into account the orthogonality of the basis states, we obtain the expression for K in terms of the time-dependent probability amplitudes:

$$K = \frac{\left(\sum_{ik} |a_{ik}|^2\right)^2}{\sum_{nmkj} a_{nk} a_{mk}^* a_{nj}^* a_{mj}}, \qquad (13)$$

where n and m take values of 1 or 2 in the Λ scheme and 0, 1, or 2 in the V scheme; double summation is performed over the field states.

The same expression can be derived proceeding from the matrix concepts. The elements of the density matrix of the coupled atom + field system have the form

$$\rho_{ij}^{(kl)} = a_{ik} a_{jl}^*.$$

Averaging the matrix over the field degrees of freedom leads to the so-called reduced density matrix, which characterises the state of the atomic subsystem, independent of the field state:

$$\rho_r = \mathrm{Sp}(\rho) = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} = \begin{pmatrix} \sum_k |a_{1k}|^2 & \sum_k a_{1k} a_{2k}^* \\ \sum_k a_{2k} a_{1k}^* & \sum_k |a_{2k}|^2 \end{pmatrix}. \qquad (14)$$

Using the definition of the Schmidt parameter (12), one can easily make sure that the matrix approach also yields expression (13). Thus, the physical meaning of the Schmidt parameter becomes clear: in the case of pure atomic state, the trace of the square of reduced density matrix is unity, i.e., $K = 1$; if the state is mixed, K > 1.

Having performed some transformations, one can obtain simpler expressions for the Schmidt parameter in terms of the single sum over the field states:

$$K = \frac{W_{\mathrm{bound}}^2}{\left(\sum_k |a_{1k}|^2\right)^2 + \left(\sum_k |a_{2k}|^2\right)^2 + 2\left|\sum_k a_{1k} a_{2k}^*\right|^2} \qquad (15)$$

for the Λ scheme and

$$K = W_{\mathrm{bound}}^2 \Big[\left(\sum_k |a_{0k}|^2\right)^2 + \left(\sum_k |a_{1k}|^2\right)^2 + \left(\sum_k |a_{2k}|^2\right)^2$$

$$+ 2\left|\sum_k a_{0k} a_{1k}^*\right|^2 + 2\left|\sum_k a_{0k} a_{2k}^*\right|^2 + 2\left|\sum_k a_{1k} a_{2k}^*\right|^2\Big]^{-1} \qquad (16)$$

for the V scheme, where

$$W_{\mathrm{bound}} = \sum_{nk} |a_{nk}|^2.$$

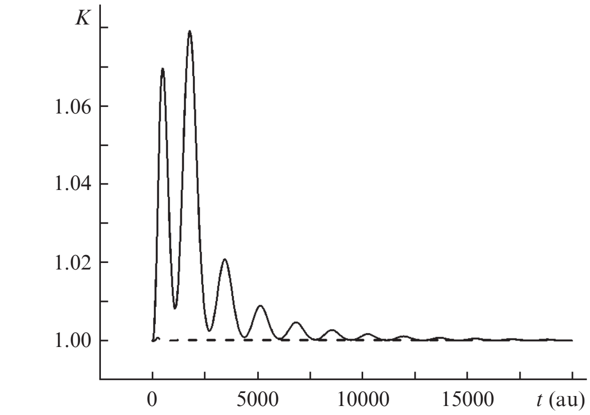

Figure 6. Time dependences of the Schmidt parameter K for the Λ scheme at $\langle N\rangle = 1000$ and the initial phase difference $\phi = 0$ for fields in the coherent (dashed line) and squeezed-vacuum (solid line) states.

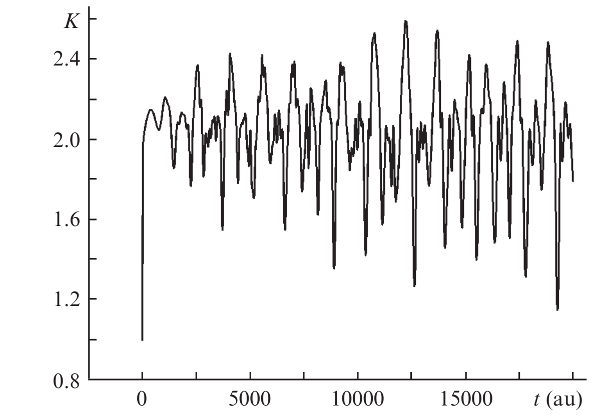

Figure 7. Time dependence of the Schmidt parameter K for the V scheme at $\langle N\rangle = 50$ and the initial phase difference $\phi = 0$ for a field in the squeezed-vacuum state.

As was suggested, the Schmidt parameter for the Λ scheme in the case of classical field at $\langle N\rangle = 1000$ is practically constant and equal to unity; i.e., no correlations are observed between the atomic and field subsystems (Fig. 6, dashed curve).

The situation is different in the case of the quantum field: here, the Schmidt parameter exceeds unity on some time interval (Fig. 6, solid curve). During ionisation the system is entangled, and factorisation can be performed only at complete decay of one of the states, corresponding to one of the quasi-energies. The initial population of atomic states in antiphase is more preferred, because the characteristic time of ionisation (and, therefore, entanglement) significantly increases. Note that an increase in the mean number of quanta increases the degree of entanglement of the system; i.e., the highest correlation between the atomic and field subsystems occurs in a stronger field. Thus, entanglement is inherent in only quantum fields, and the degree of entanglement depends on the field intensity.

Even higher degree of entanglement can be obtained in the presence of resonant coupling with the lower state. In the case of the V scheme, an effect of coherent field even with a small $\langle N\rangle$ value may cause a weak entanglement of system at short times, while the resonant state remains populated. As the system becomes ionised and the resonant state becomes depleted (Fig. 3a), the Schmidt parameter tends to unity and equals unity at long times. A significant entanglement arises in a quantum field even at a small mean number of photons: $\langle N\rangle = 50$ (Fig. 7). Here, one can also observe a correlation between the residual population of the resonant state and the K value. Thus, while the resonant state is populated, the Schmidt parameter $K > 1$, and the system is entangled; if the population of the lower state becomes zero, the V system degenerates into the Λ system, and the Schmidt parameter tends to unity. In a classical field, the population of the resonant state becomes zero at a certain instant, and the V scheme degenerates into the Λ scheme. A quantum field, even at long times, contains some residual population of the resonant state, degeneration does not occur, and the system is constantly entangled (Figs 3, 7). Note that within the V scheme the dependence of the degree of entanglement on the initial phase of atomic-level population is opposite to that for the Λ scheme: the residual population of the resonant state for the in-phase population exceeds that for the antiphase population; therefore, the degree of entanglement in the case of in-phase population is also higher. In addition, since the population of the resonant state is retained, the Schmidt parameter for the V scheme exceeds that for the Λ scheme. Therefore, the V scheme is more favorable for observing the entanglement effect.

However, entanglement can also be observed in a coherent field with a Poissonian distribution of photons. This situation occurs, for example, in the Λ scheme, if the mean number of photons is close to $k_{\rm cr}$. As a result, an intermediate regime arises, where the ionisation rate Γ_k and the spacing d between discrete levels become values of the same order of magnitude. In this case, a degree of entanglement close to maximum ($K = 2$) is periodically observed in the system. Thus, along with the regions of strong and weak fields, one can select the third region, where the behaviour of the Schmidt parameter is radically different; this is the region near $k_{\rm cr}$. Figure 8 shows the behaviour of the Schmidt parameter in the weak-field regime (Fig. 8a) and in the case where $\langle N\rangle \approx k_{\rm cr}$ and the K values are close to the maximum value $K_{\max} = 2$ (Fig. 8b). Note that one would expect the coherent state of the field with $\langle N\rangle \sim k_{\rm cr}$ to be classical, which is indirectly confirmed by the correspondence between the dynamics of the atomic system in this field and the solution of the semiclassical problem. However, the observed high degree of correlation between the atomic and field subsystems reveals nonclassical features of this field state. Therefore, the degree of entanglement that arises upon interaction of coherent light with a Rydberg atom, for which the mean ionisation width is of the same order of magnitude as the energy spacing to the neighboring levels, can actually be a measure of acting-field nonclassicity.

Note that, with allowance for the renormalisation of the elements of the reduced density matrix $\tilde{\rho}_{\rm r} = \rho_{\rm r}/W_{\rm bound}$, expression (15) yields the following relation for the Schmidt coefficient within the Λ scheme:

$$P = 2K^{-1} - 1 = (\tilde{\rho}_{22} - \tilde{\rho}_{11})^2 + 4\tilde{\rho}_{12}\tilde{\rho}_{21}. \qquad (17)$$

This quantity characterises the degree of purity of the atomic state upon interaction between the atom and quan-

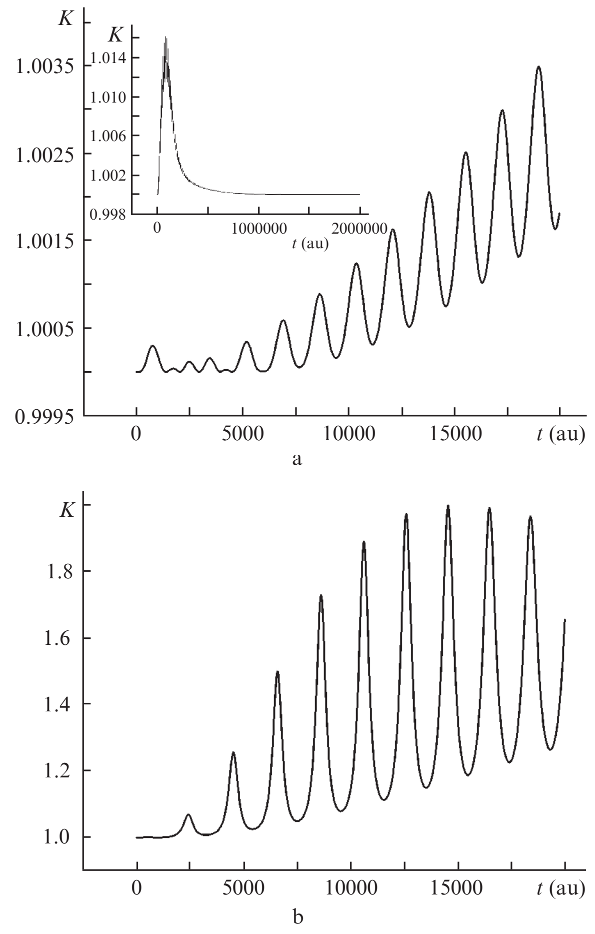

Figure 8. Time dependences of the Schmidt parameter K for the Λ scheme in the case of weak field at $\langle N \rangle = 50$ (a) and 300 ($\langle N \rangle$ is close to $k_{cr} = 500$) (b) and the initial phase difference $\phi = 0$ for the field in coherent state. The inset shows the behaviour of the dependence at long times.

tum field. Here, $K = 1$ yields $P = 1$ and corresponds to the factorisation of the total wave function and, therefore, to the possibility of characterising the atomic subsystem by a pure state.

As for the experimental measurement of the degree of entanglement, it is sufficient to determine either the Schmidt parameter K or the degree of atomic-state purity. To this end, it is necessary to measure the inverse population of the atomic state $(\tilde{\rho}_{22} - \tilde{\rho}_{11})$, which can easily be implemented experimentally, and the last term in (17), which contains information about the phases; the latter is a fairly difficult problem. One of the ways to measure this quantity is based on the subsequent interaction between the atom in the state $\tilde{\rho}_r$ and the classical field with the same frequency as the initial quantum field. In this case, the dynamics of the atomic system depends directly on the values of the off-diagonal elements of the density matrix and contains information about their amplitudes and phases.

Note that, for a real atom, which is characterised by a large number of neighboring Rydberg states, the model under consideration gives only a qualitative concept of the dynamics of the system, whereas the degree of entanglement is much more difficult to determine. However, when there is a resonance with one of the lower states, all the found features of the dynamics of the V scheme considered above are present, and the population of the initially depleted resonant state can serve as a measure of entanglement of the system.

4. Conclusions

We investigated the dynamics of a model atomic system in a strong nonclassical electromagnetic field and performed comparison with the case of interaction of an atomic system with classical light. It was shown that an atom in a quantum electromagnetic field in the squeezed-vacuum state is ionised more slowly than in the coherent state of the field, which is due to the much wider distribution over the number of photons for squeezed nonclassical light. At the same time, the dynamics of an atom in the squeezed-vacuum field demonstrates features that are characteristic of the dynamics of an atom in a classical field in both strong- and weak-field regimes. It was established that in the case of resonant coupling with the lower state a fairly stable wave packet can be formed, which barely decays even over long times. The found features of the dynamics of atomic systems in nonclassical fields qualitatively manifest themselves in real atoms, which are characterised by a large number of closely spaced Rydberg levels.

We also investigated the correlations arising in the atom + field system and demonstrated the occurrence of entanglement of the field and atomic subsystems, which exists for a fairly long time. The presence of entanglement in the system makes it possible to obtain information about one subsystem by performing measurements with the other one. In particular, the detection of the field system in one of the specific Fock states allows one to find the atomic wave function in the form of a superposition of atomic states (wave packet) with exact (including the phase difference) determination of the amplitudes of entering-state probabilities. The Schmidt parameter, which is a quantitative measure of entanglement, was calculated for the system under consideration, its temporal dynamics was investigated, and the regimes in which the degree of entanglement reaches a maximum were found. The possibility of measuring experimentally the degree of entanglement in the system under study was analysed and it was demonstrated that the population of the lower, resonantly populated state can be a measure of entanglement of the system even for real atoms, characterised by a large number of closely spaced Rydberg states.

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