 

*CONTROL OF THE LASER RADIATION PARAMETERS*

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# Control of the repetition rate and reduction of the jitter of the pulses of passively *Q*-switched solid-state miniature lasers using periodic diode-pump modulation

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*Abstract.* **Based on an extended point laser model, the lasing of passively *Q*-switched Nd : YAG minilasers has been numerically analysed. We have shown that periodically modulated diode pump may be used to control the pulse repetition rate and substantially reduce the timing pulse jitter. When the pump is modulated with rectangular pulses, switching from a high pump level to a low one upon generation of a successive laser pulse improves the stability of the pulse repetition rate relative to variations of the average pump level and broadens the range of repetition frequencies whereby they may be locked to the pulse modulation frequency. The feasibility of lowering the timing jitter and locking the pulse repetition frequency to the pump modulation frequency has been confirmed experimentally. For repetition rates ~1 kHz, the relative standard deviation of the pulse-repetition period is reduced from several percent for a continuous pumping to 0.06% by applying 'truncation' of modulation pulses. The experimental results agree well with simulations.**

***Keywords:*** *solid-state lasers, diode pumping, passive Q-switching, pulse jitter.*

## 1. Introduction

Many applications necessitate lasers with a high repetition rate of short pulses. Microlasers with *Q*-switching (QS) permit obtaining subnanosecond pulses [1]. However, the energy of these pulses (usually below 100 μJ) is quite often insufficiently high for their efficient application. That is why increasingly strong emphasis is now placed on miniature solid-state diode-pumped lasers with QS [2], which are capable of generating pulses with a substantially higher energy though with a longer duration. In this connection, the problem of efficient compression of short pulses with the use of nonlinear optical techniques is topical as before [3]. Especially simple are solid-state lasers with passive QS, which find diverse applications at the present time [1–3]. However, a significant disadvantage of lasers with continuous diode pumping is the instability of their pulse repetition frequency. Uncontrollable fluctuations of the pump power are the main cause of the timing jitter (TJ) of the generated-pulse repetition period $T_g$, the standard period deviation $\Delta T_g$ normally being equal to several percent of $T_g$ [1, 2].

As a rule, lasers with an active resonator QS, especially with a negative feedback [4], exhibit a smaller TJ in comparison with lasers in which use is made of passive switching techniques. And so to obtain single-mode lasing with a low TJ advantage is quite often taken of combined active-passive resonator QS [5–8]. In this case, an auxiliary part is assigned to the active elements (AEs) of the *Q*-switch, and the requirements on their technical characteristics are therefore not very high. Illuminating a passive laser switch (PLS) with an additional stabilised optical source permits not only lowering the TJ [9, 10], but also realising, for instance, synchronisation of the pulses of a passively *Q*-switched microlaser to the femtosecond pulses of external source [11].

However, lasers with external illumination, combined active-passive QS, or controllable feedback are substantially more complex devices than conventional lasers with passive QS. In this connection, the search for new simpler ways of lowering the TJ of the pulses generated by lasers with passive QS is a topical problem. In particular, Huang et al. [12] state that the use of certain ring resonator configurations may lower the relative TJ $\delta_g = \Delta T_g / T_g$ from 8% to 3%. It is pertinent to note that the thus improved pulse TJ is still too high and is hardly different from the ordinarily observed TJ in lasers with a passive QS. That is why considerable interest was generated by Refs [13, 14], which reported a significant reduction of the TJ due to additional modulation of the pumping laser-diode current by rectangular pulses. This method of reducing the TJ is particularly attractive for its simplicity of implementation. In the former of the cited publications, Ref. [13], it was established that the periodic modulation of the pump current by rectangular pulses with a filling factor of 0.5 resulted, in a certain range of modulation periods $T_p^m$ for a given average pump level, in the synchronisation of pump modulation periods to pulsed lasing periods ($T_g \approx T_p^m$). It was emphasised that this technique enabled the pulse repetition frequency to be experimentally stabilised at $\sim 10^{-6}$ and that this was in agreement with numerical simulations reliant on the balance equations of a point laser model (PLM). The reached degree of stabilisation was called in question, the more so as the details of the simulations and the parameters of the PLM equations were not given. The authors of Ref. [14] made only a qualitative consideration of the variation of the moment the threshold inversion was reached under pumping by single rectangular pulses and composite pulses, when a short high-intensity pulse is added on termination of a rectangular pulse. In this case, the scatter of the moments of the onset of pulse lasing [which will be referred to as local timing jitter (LTJ)] lowered from 6 to 0.5 μs, i.e. by about an order of magnitude.

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Reference [15] and its following Refs [16,17] contained only a qualitative analysis of the system of two equations (for the photon density in the resonator and the population inversion in the AE) of a conventional PLM under some assumptions, whose validity is discussed below. We only mention that experimental measurements were made of the scatter of the onset of pulsed lasing relative to the leading edge of a pump pulse, which was taken as the sought-for jitter of the laser pulse-repetition period. This quantity supposedly corresponds to the LTJ defined above. The minimal absolute LTJ value recorded for a laser pulse repetition rate $\nu_g$ = 192 Hz was rather large (3 μs), while the formally calculated relative jitter appeared to be rather low: $\Delta T_g/T_g \approx 0.06\%$. It was not indicated how the jitter was measured for a constant pump current, but it was quite high (8.6%) for a repetition rate $\nu_g$ = 3.9 kHz.

That is why, in our opinion, of interest are direct numerical simulations of the pulse TJ for diode-pumped minilasers with passive QS with the use of an extended PLM which takes into account the splitting of working laser levels to Stark sublevels, the finiteness of the lifetime of the lower working level, the depopulation of the ground level via diode pumping to higher lying excited levels and via direct pumping to the upper laser level, absorption from the excited level of a PLS, fluctuations of the number of seed photons in the mode being generated, the slow drift and modulation of diode pump radiation power, as well as other factors which affect the dynamics of lasing. The data obtained by simulations testify to the possibility of additional stabilisation of the pulse-repetition period by interrupting rectangular pump pulses upon generation of a successive laser pulse and were borne out by direct experiments.

## 2. Model

In dimensionless quantities the system of equations of the extended PLM model, which includes the AE and PLS properties indicated in the foregoing and some other ones, is of the form

$$\chi \frac{d\phi}{dt} = \left[ n_u - n_l - \sum_i (\tilde{\sigma}_0 n_{a1}^{(i)} + \tilde{\sigma}_e n_{a2}^{(i)}) f_i(\theta) \right] \phi - \bar{\alpha}_\Sigma(t)\phi + \tilde{\varepsilon}(t)\frac{n_3}{\tau_{32}^*}, \quad (1)$$

$$\frac{dn_3}{dt} = R_p^*(t)\left(1 - \frac{n_2}{n_{Nd}} - \frac{n_3^*}{n_{Nd}}\right) - \phi(n_u - n_l) - \frac{n_3}{\tau_{32}^*}, \quad (2)$$

$$\frac{dn_2}{dt} = \phi(n_u - n_l) - \frac{n_2}{\tau_{21}} - \frac{n_3}{\tau_{32}^*}, \quad (3)$$

$$\frac{dn_{a1}^{(i)}}{dt} = -\beta_0 \bar{\sigma}_0 n_{a1}^{(i)} f_i(\theta)(\phi + \phi_a) + \frac{n_{Cr}^{(i)} - n_{a1}^{(i)}}{\tau_{a21}}. \quad (4)$$

The notation in Eqns (1)–(4) largely corresponds to that accepted in Refs [5,6]. The passage from the travelling wave model to the PLM is detailed also in these papers. That is why here we provide only a brief explanation of the physical meaning of the parameters used in the PLM. So, $\chi = \{l/v + l_a/v_a + l_{eff}/v_{eff} + [L - (l + l_a + l_{eff})]/c\}/(l/v)$ is the coefficient which characterises the degree of resonator filling by the AE; $L$ is the resonator length; $l$ and $l_a$ are the AE and PLS lengths; $v$ and $v_a$ are the group velocities of light in the AE and the PLS; $l_{eff}$ and $v_{eff}$ are the effective length and the effective light velocity in the remaining optical elements placed in the resonator; $\phi = \langle I_+ \rangle / I_0$ is the averaged intensity of the total radiation in the resonator normalised to $I_0 = \hbar\omega_0/(\sigma t_0)$; $\hbar$ is the Planck constant; $\omega_0$ is the cyclic frequency of the main transition in Nd : YAG with a wavelength $\lambda$ = 1064 nm; $\sigma = 8.8\times10^{-19}$ cm$^2$ is the effective spectroscopic cross section of stimulated emission [18]; $t_0 = l/v$ is the duration which the current time and the relaxation times are normalised to; $n_u - n_l = f_u n_3 - f_l n_2$ is the effective population difference with consideration of the fact that the overall lasing profile is formed by the overlap of two transitions [18]; $f_u$ and $f_l$ are the generalised Boltzmann occupation numbers [5,6]; $n_j$ ($j$ = 1, 2, 3) and $n_{ai}^{(m)}$ ($i, m$ = 1, 2) are the level population densities in the AE and the PLS normalised to $N_0 = 1/(\sigma l)$; $\tilde{\sigma}_{0,e} = \sigma_{0,e} l_a/(\sigma l)$ and $\bar{\sigma}_{0,e} = \sigma_{0,e}/\sigma$ are the effective and relative cross sections of absorption from the ground and excited levels; $f_1(\theta) = \cos^2\theta$ и $f_2(\theta) = \sin^2\theta$ are the transmission coefficients for the PLS cut along the $Cr^{4+}$ : YAG crystallographic axes, which depend on the angle of rotation $\theta$ relative to the optic axis; $\bar{\alpha}_\Sigma(t) = \bar{\alpha} + \bar{\alpha}_a(t) + \bar{\alpha}_Q(t) + \bar{\alpha}_R(t)$ are the dimensionless nonresonance net losses, which comprise distributed losses in the AE ($\bar{\alpha}$) and the PLS ($\bar{\alpha}_a$) as well as useful losses for the radiation extractable through the mirrors, which, in our opinion, should be specified in the form $\alpha_R = (1 - R_1)/(1 + R_1) + (1 - R_2)/(1 + R_2)$ [5,6]. We note that it is possible to describe the resonator dumping regime using time-dependent mirror reflectivities $R_1$ and $R_2$. The coefficient $\tilde{\varepsilon}(t)$ is responsible for accounting spontaneous emission; its magnitude depends on the specific experimental situation and may be selected by way of comparison of simulated and experimental data (its specific realisations are discussed in greater detail below); $\tau_{32}^*$ is the radiation relaxation time of the upper laser level with the inclusion of the possible action of amplified spontaneous emission (ASE) on it [15]. The same designations are used for the dimensional and dimensionless (normalised to $t_0$) current time and relaxation times.

As will be clear from the data outlined below, under continuous pumping the generated-pulse TJ in real conditions is defined not by the fluctuations of spontaneous emission in the AE, but primarily by the technical temporal instability of the diode pump: by its slow drift in time. A modulated pump, including its temporal drift and modulation type, is modelled by function $R_p^*(t)$, which is proportional to the pump power, the density of active ions, and the absorption cross sections at a given frequency. These cross sections are different for the pumping via the upper excited levels of the AE ($n_3^* = n_3$) and the direct pumping to the upper working level, when its effective population is defined by the expression $n_3^* = n_3(1 + f_3^{(13)}/f_1^{(13)})$, which depends on the Boltzmann occupation numbers of the lower ($f_1^{(13)}$) and upper ($f_3^{(13)}$) Stark sublevels. It is assumed in the model that the relaxation between the sublevels and from the upper excited level is very rapid, with the result that the excited level population $n_4$ is assumed to be zero and, consequently, the conservation law $n_1 + n_2 + n_3 = n_{Nd}$ holds true. It is commonly assumed that $n_2, n_3 \ll n_{Nd}$ and the saturation of pump radiation absorption is neglected. However, the authors of Ref. [19] believe that is precisely the inclusion of ground state population depletion alone may lead to dynamic chaos in the balance laser model.

The remaining terms in Eqns (2), (3) are generally accepted and describe the stimulated emission between the given sublevels, the contribution of ASE to changes in population densities and the lowering of the lower level population arising from radiationless relaxation to the ground level. We note that until the present time different authors adhere to different opinions about the value of the relaxation time $\tau_{21}$, although our long-standing measurements [20] showed that it is in the subnanosecond range and that it should therefore be explicitly taken into account for minilasers generating short (below 1 ns) pulses in the QS regime. The last-written equation describes population variations of the ground level of $Cr^{4+}$ YAG-crystal impurity centre with a dipole moment directed along the $X$ ($m = 1$) or $Y$ ($m = 2$) crystallographic axes [21]. The excitation relaxation from the intermediate and upper excited levels is assumed to be very fast. It would therefore suffice to consider the population variation of only the ground PLS level, because the population of the upper metastable level with a rather long relaxation time $\tau_{a21}$ is found from the conservation law $n_{a1}^{(i)} + n_{a2}^{(i)} = n_{Cr}^{(i)}$. Coefficient $\beta$ is equal to the ratio between the beam cross section areas in the AE and the PLS. The PLS illumination by an additional external source may be taken into account by using an expression similar to the first term in Eqn (2) or simply by introducing a given time-dependent function $\phi_a$.

To numerically solve the system of equations (1)–(4) subject to the $Nd^{3+}$ and $Cr^{4+}$ ion number conservation law, advantage was taken of the Dormand–Prince method with a variable (adaptive) time step [5, 6]. This method enabled us to directly calculate the generation of a large number of successive pulses, which is an intricate task when use is made of the Runge–Kutta technique [13, 22]. To statistically process the simulation data, we employed an original program, which made it possible to investigate the variation and the statistics of generated pulse parameters like the peak intensity, the energy, the duration, and the pulse-repetition period as functions of AE and PLS parameters as well as of resonator and pump parameters. To decrease the size of the files processed, the data were retained only for those points in time when the normalised intensity $\phi$ exceeded some small preassigned value. In the simulation results outlined below it was usually assumed that the initial populations of the upper levels in the AE and the PLS were equal to zero. We emphasise that even a very high intracavity intensity (the initial $\phi_0 = \phi(t = 0)$ or the remaining one after the generation of a successive pulse) is insufficient for giant pulse generation, because it decreases drastically owing to losses in the resonator and may be so low by the next instant the threshold population is reached that the development of a giant pulse is impossible. That is why it is fundamentally necessary that the spontaneous emission coefficient $\tilde{\varepsilon}$ in Eqn (1) should be nonzero. The findings of Refs [6, 21] suggest that the rotation angles $\theta = 0$ and 90° are optimal for the generation of pulses of higher energy and shorter duration., whereby the liner pulse polarisation coincides with the direction of PLS crystallographic axes. In this connection, in simulations we used $\theta = 0$.

## 3. Simulation results and their discussion

As discussed above, generating giant pulses requires the term $\tilde{\varepsilon}(t)$ in Eqn (1) to describe spontaneous emission. This term was present in Ref. [13] in the equation for intracavity intensity, but its fluctuations in time were neglected. In Refs [14–17] this term was missing from a similar equation, because they actually calculated the time taken to reach the threshold of lasing in the absence of not only spontaneous, but also of stimulated emission. The fluctuations of $\tilde{\varepsilon}(t)$ are responsible for the lowest attainable value of the TJ for strictly constant values of the pump and all other laser parameters. In the numerical simulation it was assumed that $\tilde{\varepsilon}$ comprised a constant part $\varepsilon$ and a fluctuating part $\Delta\tilde{\varepsilon}(t)$, which varied randomly in time at every time step: $\tilde{\varepsilon}(t_j) = \varepsilon + \Delta\tilde{\varepsilon}(t_j)$. It was also assumed in the simulations that the $\tilde{\varepsilon}(t)$ probability density obeyed the normal Gaussian distribution

$$p(\tilde{\varepsilon}) = \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{(\tilde{\varepsilon}-\varepsilon)^2}{2\sigma_{\varepsilon}^2}\right] \quad (5)$$

with variance $\sigma_{\varepsilon}$. Use was also made of a uniform probability density distribution $\tilde{\varepsilon}(t)$ in some value interval. No appreciable difference in results was observed. It is quite evident that the constant part $\varepsilon$ and the ratio $\sigma_{\varepsilon}/\varepsilon$ for a constant pump may practically affect only the pulse-repetition period $T_g^{cw}$ and the relative TJ $\delta_g^{cw} = \Delta T_g^{cw}/T_g^{cw}$.

Applying the variable-step method made the simulations substantially faster, but a standard version of lasing with the number of laser pulses $N \geqslant 100$ for a repetition rate $\nu_g \sim 1$ kHz nevertheless took more than one hour to calculate on a personal computer. To this repetition rate there corresponds a pump with a normalised constant power $R_p^{cw} \sim 10^{-6}$. By and large the parameters employed in our simulations corresponded to those of a passively $Q$-switched pulsed-pumped laser generating pulses shorter than 1 ns [3]. Owing to an induced thermal lens, in reality such a laser could operate at a maximum repetition rate of $\sim$200 Hz. However, conventional balance PLMs, including the model employed in our paper, neglect thermal effects. That is why to shorten the computation time we raised the pump power $R_p^{cw}(t)$ to a value $2\times10^{-6}$, whereby the pulse-repetition period $T_g^{cw} = 0.442$ ms and the repetition frequency $\nu_g = 2.262$ kHz for an AE of length $l = 1$ cm and $t_0 = l/v \approx 60.67$ ps. For the specified $R_p^{cw}$ this average repetition frequency was obtained for $\varepsilon = 10^{-6}$ and $\sigma_{\varepsilon}/\varepsilon = 0.1$. We note that the TJ arising from spontaneous quantum noise, which is defined as the average period deviation $\Delta T_g^{qn}$, did not exceed the pulse duration (less than 1 ns) even when $\varepsilon$ was increased to $10^{-5}$ provided that $\sigma_{\varepsilon}/\varepsilon \leqslant 0.1$. With increasing the $\sigma_{\varepsilon}/\varepsilon$ ratio to 0.3, the relative TJ, defined as $\delta_g^{qn} = \Delta T_g^{qn}/T_g^{qn}$, increased linearly from zero to $10^{-3}\%$ for $\varepsilon = 10^{-5}$ and lowered to $3\times10^{-4}\%$ for $\varepsilon = 10^{-6}$, while the pulse-repetition period lengthened by about 1%. These values are substantially smaller than those recorded experimentally. There is no way to determine the values of $\varepsilon$ and $\sigma_{\varepsilon}/\varepsilon$ by comparing the calculated TJ with experimental data, because in reality the main contribution to the TJ is made by technical noise of different nature. In this connection, in our calculations we used $\varepsilon = 10^{-6}$ and $\sigma_{\varepsilon}/\varepsilon = 0.1$, whereby the relative jitter $\delta_g^{qn} \approx 10^{-4}\%$ for $R_p^{cw}(t) = 2\times10^{-6}$. This value was later taken as the initial one in the investigation of the effect of its temporal modulation on the pulse repetition frequency and the TJ.

We note that lower values of $\varepsilon$ and $\sigma_{\varepsilon}/\varepsilon$ entail lower values of the TJ. The value of the TJ resulting from simulations in this case is therefore limited by the digital noise of the simulations. So, when the parameter values in use are not within the narrow intervals whereby dynamic chaos can occur in a laser system [19], with neglect of technical instabilities the TJ may be very low also for a continuous and strictly constant pump. Periodic modulation of some parameters, for instance the modulation of the pump power by rectangular pulses [13], in this case may only impose on the laser system an externally given pulse-repetition period (in some range of its values) for the same average pump level. The periodic temporal modulation of the pump by rectangular pulses may be specified as

$$R_{\rm p}^{\rm m}(t) = \begin{cases} R_{\rm p0}(1-\delta_{\rm m}), & t_i \leqslant t < t_i + T_{\rm p}^{\rm m}/2, \\ R_{\rm p0}(1+\delta_{\rm m}), & t_i + T_{\rm p}^{\rm m}/2 \leqslant t < t_i + T_{\rm p}^{\rm m}, \end{cases} \quad (6)$$
$$t_i = (i-1)T_{\rm p}^{\rm m}, \ i = 1,2,\dots,N,\dots,$$

where $R_{\rm p0}$ is the average pump level; $\delta_{\rm m}$ and $T_{\rm p}^{\rm m}$ are the depth ($0 \leqslant \delta_{\rm m} \leqslant 1$) and the modulation period, respectively. We note that the initial ($t = 0$) pump level may be lower [13] or higher than the average one. Periodic composite pump may be described in the following way:

$$R_{\rm p}^{\rm c}(t) = \begin{cases} R_{\rm p1}, & t_i \leqslant t < t_i + T_{\rm p1}^{\rm c}, \\ R_{\rm p2}, & t_i + T_{\rm p1}^{\rm c} \leqslant t < t_i + T_{\rm p}^{\rm c}, \end{cases} \quad (7)$$
$$t_i = (i-1)T_{\rm p}^{\rm c}, \ i = 1,2,\dots,N,\dots,$$

where $T_{\rm p}^{\rm c} = T_{\rm p1}^{\rm c} + T_{\rm p2}^{\rm c}$. We are reminded that the authors of Ref. [14] only qualitatively considered the variation of the upper level population during one composite pulse subject to conditions $R_{\rm p1} \ll R_{\rm p2}$ and $T_{\rm p2}^{\rm c} \ll T_{\rm p1}^{\rm c}$, following which the pump terminated. Expression (7) may be used to describe conventional pulsed pump by putting, for instance, $R_{\rm p2} = 0$.

Although in Ref. [13] it is stated that there are many technical reasons for the instability of the pulse-repetition period, including the instability of the power and the spectrum of diode pump, actually they were not included in their numerical simulations. In recent work [23], which was concerned with the investigation of the effect of classical noise on the development time of the first peak of relaxation oscillations in a continuously pumped Nd:YAG laser, a random number generator with a Gaussian probability distribution was employed to select the initial pump level and the magnitude of distributed internal loss. In this case, the random pump remained invariable throughout the simulation, while the loss was changed at every step, like in our case in the consideration of spontaneous noise. In our opinion, to describe a slow variation of the pump, i.e. the drift of its parameters arising from technical instability, it would suffice to employ a periodic sinusoidal modulation with a long period $T_{\rm d}$. That is why in our work the pump was defined in the form

$$\tilde{R}_{\rm p}(t) = R_{\rm p}(t)\left[1 + \delta_{\rm d}\sin\left(\frac{t}{T_{\rm d}} + \varphi_{\rm d}\right)\right], \quad (8)$$

where $R_{\rm p}(t)$ is the theoretical time dependence of the pump power and the technical noise is defined by parameters $\delta_{\rm d} \ll 1$ and $T_{\rm d} \gg T_{\rm p}^{\rm m,c}$. Evidently the effect of phase $\varphi_{\rm d}$ is small for simulation times much longer than the modulation period. It is well to bear in mind that this description of technical pump instability has the effect that the distribution of the time spacing between pulses is not Gaussian and has a characteristic shape with two peaks corresponding to the periodicity of the occurrence of the maxima and minima of the pump. To quantitatively characterise the TJ, use was made of the calculated statistical period deviation, although in this case it does not lend itself to a simple interpretation as with Gaussian statistics.

For a constant-pump instability $\delta_{\rm d} = 0.01$ ($R_{\rm p}^{\rm cw}(t) = 2\times10^{-6}$) and a drift period $T_{\rm d} = 7.28\times10^8 t_0 = 44.2$ ms, the calculated standard period deviation $\Delta T_{\rm g}^{\rm cw} = 10.63$ μs and the relative TJ $\delta_{\rm g}^{\rm cw} = \Delta T_{\rm g}^{\rm cw}/T_{\rm g}^{\rm cw} \approx 2.4\%$. Therefore, the seemingly acceptable 1% instability of constant pump leads to a rise in the TJ by more than four orders of magnitude in comparison with the TJ caused by quantum noise for a strictly constant pump. The magnitude of TJ which follows from our simulations amounts to several percent and agrees nicely with the known data.

To impose an externally prescribed pulse-repetition period on a laser system for the same average pump level, advantage may be taken of periodic rectangular modulation of certain parameters of the laser system, for instance of the pump power, but not necessarily by periodic pulses [13]. Our simulations suggest that this may be achieved by employing sinusoidal modulation

$$R_{\rm p}^{\rm s}(t) = R_{\rm p0}\left[1 + \delta_{\rm s}\sin\left(\frac{2\pi}{T_{\rm s}}t + \varphi_{\rm s}\right)\right] \quad (9)$$

with a sufficient modulation depth $\delta_{\rm s}$. Specifically, for a modulation depth $\delta_{\rm s} = 0.2$ and a modulation period $T_{\rm s}$ coinciding with the average period $T_{\rm g}^{\rm cw}$ for a constant pump, the relative TJ $\delta_{\rm g}^{\rm s} = \Delta T_{\rm g}^{\rm s}/T_{\rm g}^{\rm s}$ lowers to ~0.12%, which is practically similar to the result obtained under rectangular modulation with the same period. Realised in experiment was the rectangular pulse modulation, including that with pump truncation after generation of a successive pulse, and therefore we restrict ourselves to the presentation of primarily the rectangular-pulse modulation data. However, before doing this we mention that the dependences of the pulse-repetition period and the relative TJ on the constant pump power (Fig. 1), which were obtained in our numerical simulations, are in qualitative agreement with similar experimental dependences observed previously in Refs [13, 24]. In particular, increasing the pump power entails not only a rapid shortening of the pulse-repetition period, but also a rapid lowering of the relative TJ.

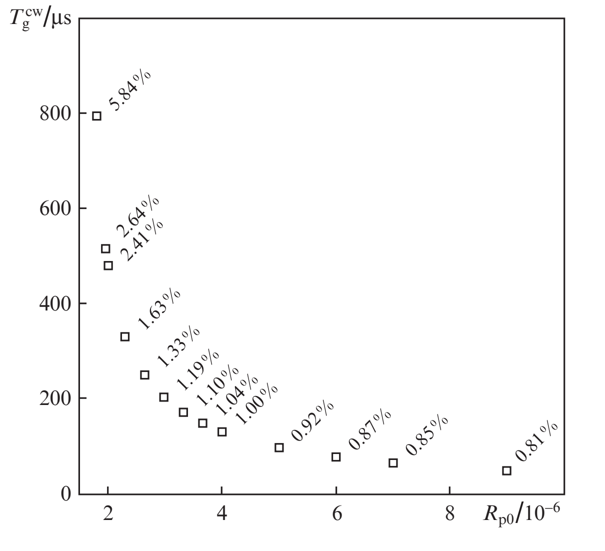


**Figure 1.** Pulse-repetition period and relative TJ as functions of continuous pump power.

Pump modulation by rectangular pulses (with the same average pump level) first of all stabilises the pulse-repetition period and lowers the TJ from several percent to 0.12% for $\delta_{\rm m} = 0.2$, i.e. by approximately 20 times. With a composite pump, by selecting $R_{\rm p1,p2}$ and $T_{\rm p1,p2}^{\rm c}$ it is possible to further lower the relative TJ by nearly an order of magnitude with retention of the same average power. However, the rectangular modulation permits not only reducing the jitter for a fixed power

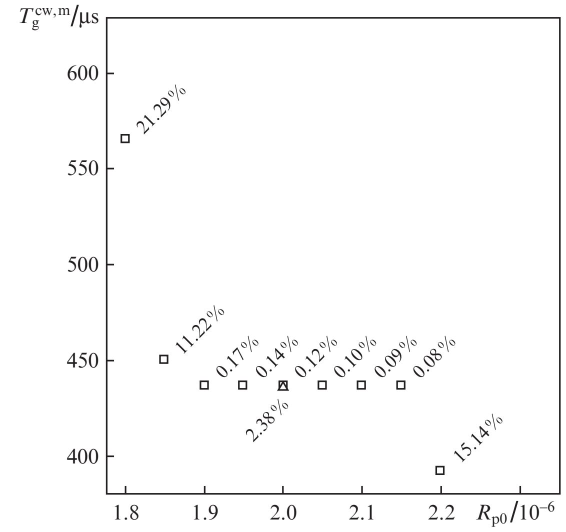


**Figure 2.** Pulse-repetition period and relative TJ as functions of average pump power for rectangular-pulse modulation with a depth of 20% (□) and for continuous pumping (△).

level, but also retaining the repetition frequency as well as the low pulse TJ for a substantial change (by no more than 10%) of the average pump level (Fig. 2). The same change in the level of continuous pumping would result in a significant change of the pulse-repetition frequency (Fig. 1). Figure 3 provides a more compelling explanation of the mechanism of frequency stabilisation under rectangular modulation. With an increase in modulation depth and retention of the average pump level, one would expect first of all a reduction of the relative TJ, which does take place but is hard to notice in Fig. 3 because of the long time scale. At the same time, as is clearly seen, for a small modulation depth ($\delta_m = 0.2$) the upper level population rises steadily in the intervals between pulses, while with an increase in modulation depth it builds up rapidly during stronger pumping (not attaining the threshold of lasing, though), but then lowers due to spontaneous emission during the weaker-pump half-cycle to become even lower than the population for the lower modulation depth. But prior to the onset of lasing the population growth rate is higher, and the TJ therefore becomes lower. It is evident that the energy losses due to spontaneous emission, as well as due to ASE, are quite high. This naturally brings up the idea of lowering them by terminating the pump and resetting it to a lower value immediately after the production of a successive pulse.

This pump pulse 'truncation' after the generation of laser pulses has the effect that the population of the upper laser level remains lower than the corresponding population under non-truncated modulation pump throughout the period up to the attainment of the threshold population (Fig. 4). With reference to Fig. 4, a lower energy is inputted into the AE when the pump pulses are truncated, i.e. the loss by spontaneous emission becomes lower. A pumping with interruption resembles composite pumping. For this reason alone one might expect a lowering of the TJ under variation of the average pump level and its corresponding variation of the modulation period. In the simulations we used the previously obtained (Fig. 1) average pulse-repetition periods for a given average pump power. The results of simulations are depicted in Fig. 5. One can see that truncating the rectangular pump pulses makes the relative TJ several tens of times lower and stabilises it at a level below 0.1%.

In the regime of pump pulse truncation, there occurs a broadening of the interval of average pump levels whereby

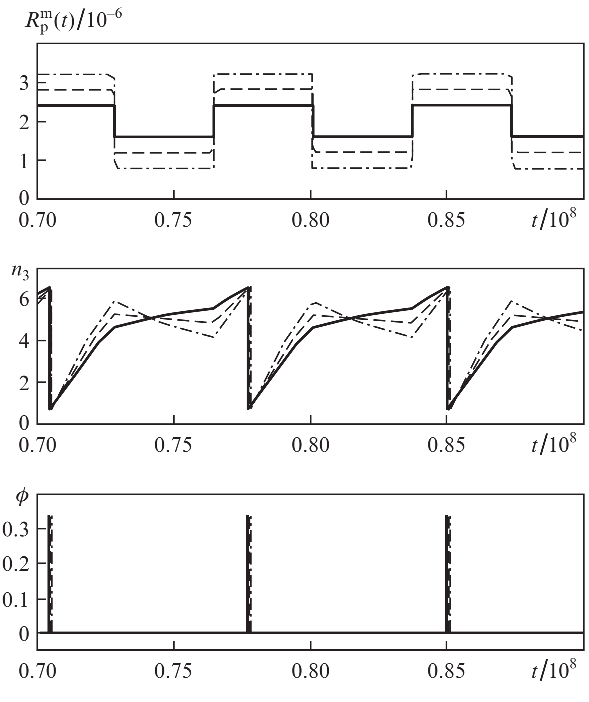


**Figure 3.** Time dependences of the modulated pump power $R_p^m$, the upper level population $n_3$, and the intensity $\phi$ of laser pulses for pump modulation depths $\delta_m = 0.2$ (solid curves), 0.4 (dashed curves), and 0.6 (dash-dotted curves) in a stationary lasing regime.

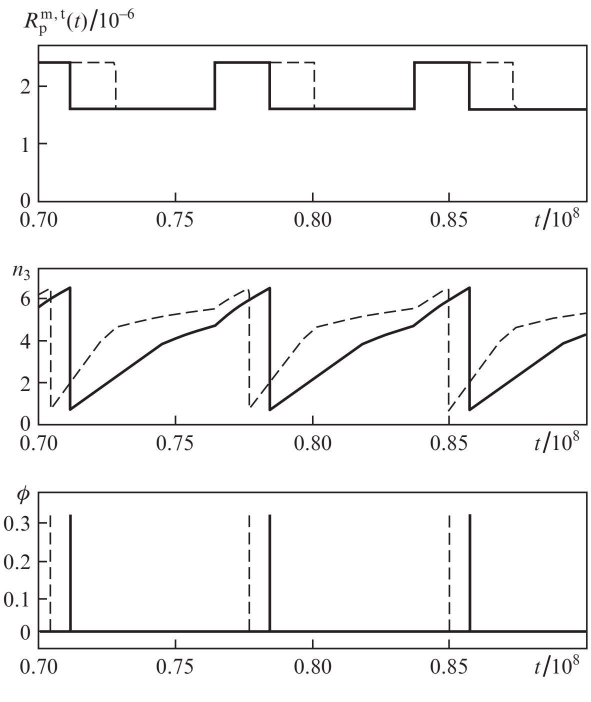


**Figure 4.** Time dependences of the modulated pump power $R_p^{m,t}$, the upper level population $n_3$, the intensity $\phi$ of laser pulses for a pump modulation depth of 20% in a stationary lasing regime with (solid lines) and without (dashed lines) truncation of the pump pulses.

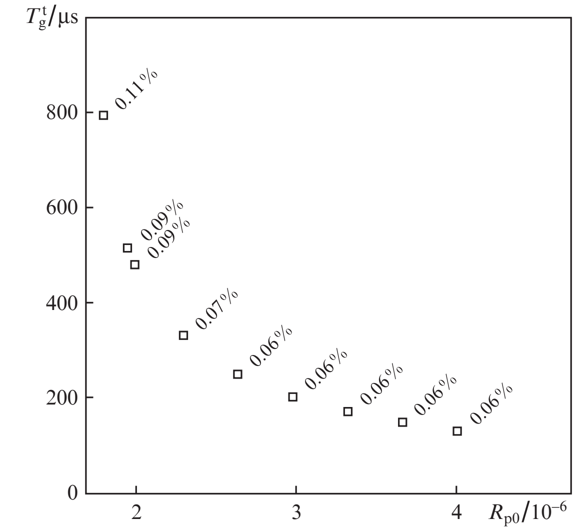


**Figure 5.** Pulse-repetition period and relative TJ as functions of average power of the pump modulated by rectangular pulses with a depth of 20% in the regime with truncation of the pump pulses.

the relative TJ remains sufficiently low for a given modulation pulse repetition frequency. Figure 6 shows the dependences of the relative TJ on the average pump level, the depth and type of pump modulation for a standard repetition frequency obtained under continuous pumping with $R_{p0} = 2\times10^{-6}$. In each case we performed simulations whose results are similar to those represented in Fig. 2. The relative TJ increased sharply beyond the intervals of the average pump level under consideration, signifying that outside of these intervals there is no way of imposing the pulse repetition rate prescribed by the modulation period. The truncation of pump pulses leads to a substantial broadening of the interval in which the prescribed repetition frequency is stable with respect to variations in average pump level, especially towards the higher pump level. Indeed, for a modulation-imposed pulse repetition rate which is

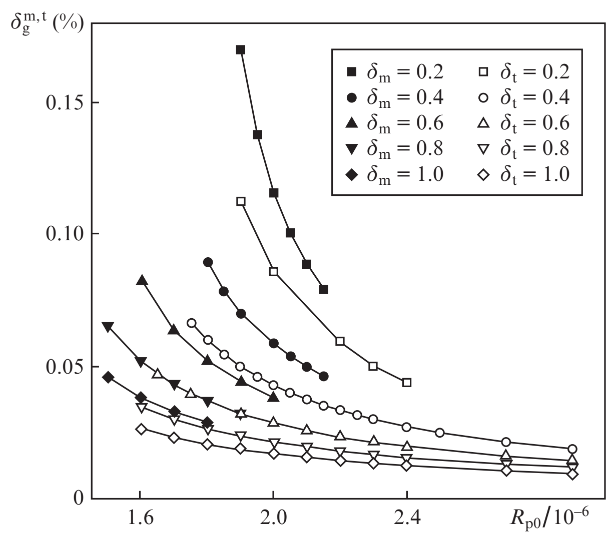


**Figure 6.** Dependences of the relative TJ on the average pump power modulated by rectangular pulses with different modulation depths in the regime without (full points) and with (empty points) truncation of the pump pulses.

higher than that realised for a constant pump equal to the lower level of composite pump, the pump energy is automatically stabilised due to the interruption of pulses at the upper level. Furthermore, increasing the modulation depth and truncating the pump pulses lowers the relative TJ. Evidently a modulation depth of 100% corresponds to pulsed pumping with a filling factor of 0.5. For a sufficiently high pump power this may result in the generation of a pulse train during one pump pulse. Truncating the pump pulses, evidently, should entail the generation of only one pulse per period.

In the regime of pump pulse truncation for a given average pump level and a given modulation depth there also broadens the range of externally prescribed pulse-repetition periods with a low TJ (Fig. 7). One can see that for a 20% depth of pump power modulation by rectangular pulses it is possible to impose a pulse-repetition period which is 1.3 times (regime without pulse truncation) and 2.6 times (regime with truncation) longer than the pulse-repetition period for the constant pump with the same average level. It is possible to implement a master-slave synchronisation of the repetition frequency to the pump modulation frequency, whereby there occurs a lowering of the pulse-repetition period by about 20%. The resultant ranges of possible variation of the pulse-repetition period are much broader than the synchronisation ranges specified in Ref. [13]. For a sinusoidal pump modulation the control range of pulse-repetition period is considerably shorter than for rectangular modulation with the same depth. Furthermore, in the case of sinusoidal modulation the truncation regime makes no sense at all. In addition, rectangular pulse modulation is simpler to realise in experiment. However, the fundamental importance of this modulation is that by way of a purely harmonic action it is possible to impose on the system the requisite pulse-repetition frequency.

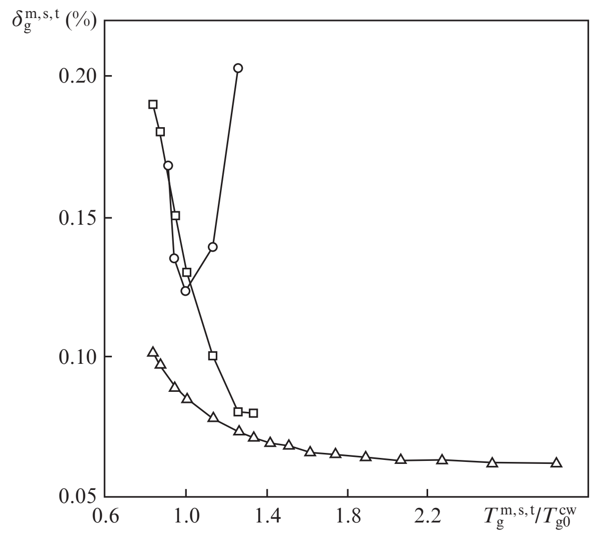


**Figure 7.** Dependences of the relative TJ on the periods of pump power modulation (of depth 20%) in the case of rectangular pulse modulation in the regime without (□) and with (△) truncation of the pump pulses, as well as in the case of harmonic (sinusoidal) modulation (○) for the same average pump power.

Concluding the exposition of our simulation data, we make several remarks about the history of investigations involving the use of modulated pumping. To the best of our knowledge, the possibility of lowering the TJ of semiconductor laser pulses under the periodic modulation of injection current by rectangular pulses was investigated back in Refs [25, 26]. Slightly later this idea, as applied to fibre laser pumping by the radiation of a diode laser modulated by rectangular pulses, was elaborated in Ref. [27], though for the generation of long (tens of microseconds) pulses in the absence of a PLS. The authors of Ref. [27] emphasised the significance of the memory effect associated with the slow relaxation of population inversion and the presence of subthreshold pre-pumping for the control of the onset of pulsed lasing. The idea of subthreshold pre-pumping was accurately formulated more recently in Ref. [28] and elaborated in Refs [13–17, 29] primarily for lowering the pulse TJ, although such pre-pumping is intended and employed also for stabilising the thermal laser regime when the pulse repetition frequency is varied [30].

A comparison of our simulation data with those obtained earlier in Ref. [13] shows that they are mostly in agreement with each other. For a pump power drift of 1%, the relative TJ is stabilised at above $10^{-4}$ for a deep modulation and pump pulse truncation (Fig. 6). To attain stabilisation at a level of $10^{-6}$–$10^{-5}$ requires extremely stable pump sources, for which the effect of technical noise is practically the same as the effect of quantum radiation noise.

We next compare our results with those of Ref. [29]. It makes use of a standard simplified balance PLM which disregards the splitting into sublevels in the energy level diagram of the AE, the relaxation of excitation from the lower working level is described by lowering the population inversion, etc. What is in doubt is the correctness of the coefficient $[1 - (n_3 - n_2)/n_{\rm Nd}]$ introduced by $R_{\rm p}^*(t)$ and supposedly required to take into account the depletion of ground level population. The contribution of spontaneous emission is not taken into account in explicit form. By and large it is stated that the presence of a nonzero constant pump may substantially improve the TJ stability in comparison with the case of pulsed pumping. This assertion is substantiated as follows: for a constant pump the initial population inversion will always be the same, while for pulsed pumping for some reason will not. However, one can see from the drawings presented that a 2.5 times higher energy is inputted into the AE for a constant pump than for a pulsed one; in both cases, the population inversion lowers by only 20% during the generation of a laser pulse, i.e. the lasing process is organised extremely inefficiently. It is quite evident that the presence of a constant pump can lower the jitter only when it is highly stable. And indeed, as indicated in Ref. [29], in simulations the Gaussian instability was assumed to be equal to 0.6% for the long-duration (~1 ms) constant pump and to 1.4% for the short (30 μs) pulsed pump. The same instability was supposedly assumed for the pulsed pump. In our simulations, the same slow power drift equal to 1% was used for any kind of pump modulation. In this connection, increasing the modulation depth and going over to pulsed pumping result only in a lowering of the TJ. It is evident that different versions of technical pump power fluctuations may occur in practice, which calls for a careful analysis of experimental data in every specific case.

In Ref. [15], like in Ref. [29], it is also stated that the jitter has a local minimum, when the constant part of the pump power amounts to a certain fraction of the threshold power. However, it is rather strange that this power is independent of the duration of the pulsed component and of the pulse-repetition period. We also note that the scatter of the delay of lasing onset (defined as the point in time at which the time derivative of the photon density is equal to zero) relative to the leading edge of a pump pulse is taken as the sought-for jitter of the pulse-repetition period. In our view, this quantity corresponds to the LTJ for a pulsed pumping. The authors of Ref. [15] did not explain how the jitter was determined for a constant pump power. We shall discuss the experimental results of the aforementioned and some other works after describing our experimental data.

## 4. Experimental results

Our experimental investigations of the TJ were carried out using several miniature *Q*-switched Nd:YAG laser with a longitudinal diode-laser pumping. The objective of the experiments conducted was to bear out the possibilities of lowering the TJ determined by way of our simulations. That is why here we restrict ourselves to only a summary of the TJ measurement data without a detailed description of the lasers employed. At first we outline the results for lasers with a pump modulated by rectangular pulses and the possibility of their truncation, and next for lasers with pulsed pumping for a passive QS and negative-feedback active QS.

In lasers with a passive QS, use was made of different pump systems as well as of AEs and PLSs with strongly different initial transmittances. This enabled obtaining pulses with different duration (from tens of nanoseconds to subnanoseconds), energy, and repetition frequency. When employing continuous pumping with rectangular pulse modulation, a randomly selected sequence of pulses (usually numbering several hundred) was stored in oscilloscope memory and was statistically processed for determining the average period, the standard deviation, etc. Therefore, in this case we measured the absolute TJ and, of course, it was possible to calculate the relative TJ for different types of modulation and repetition frequencies.

In particular, for a laser with an initial Cr:YAG PLS transmittance of 45%, the pulse duration was equal to 5 ns (Fig. 8a) and their repetition frequency could be varied from one to over ten kilohertz. When use was made of a modulated pump with a filling factor of 0.5, laser pulses were generated for a higher pump power (Fig. 8b). The constant and pulsed pump components could be varied independently. By putting the pulsed or constant component equal to zero it was possible to effect transition to continuous or pulsed pumping. On engaging the truncation regime the pulsed pump component was disengaged after the generation of a successive laser pulse. We note that with the use of a sufficiently high pump power a train of pulses could be generated during a higher power pulse [31]; this train was cut off on engaging the truncation regime, following which only one pulse would persist throughout the modulation period. On exiting the stable generation regime due to a change of the pump components the time intervals between the pulses became irregular and measurements of the TJ made no sense. That is why on changing the pump level the generated pulse sequence was visually monitored (Fig. 8c) and only after that the sequence was stored in the memory of a Tektronix DPO72004C oscilloscope and processed for obtaining the values of the TJ. Since special measures to stabilise the level of constant pumping were not taken, the relative TJ amounted to 5% for a pulse repetition frequency ranged from one to several kilohertz. With pump modulation under the conditions corresponding to Fig. 8b, for a pulse-repetition period $T_{\rm g}^{\rm m} = 1.0002$ ms the jitter $\Delta T_{\rm g}^{\rm m} = 2.405$ μs, i.e. $\delta_{\rm g}^{\rm m} = 0.24\%$. By a small increase in modulation depth a value $\delta_{\rm g}^{\rm m} = 0.09\%$ was readily achieved,

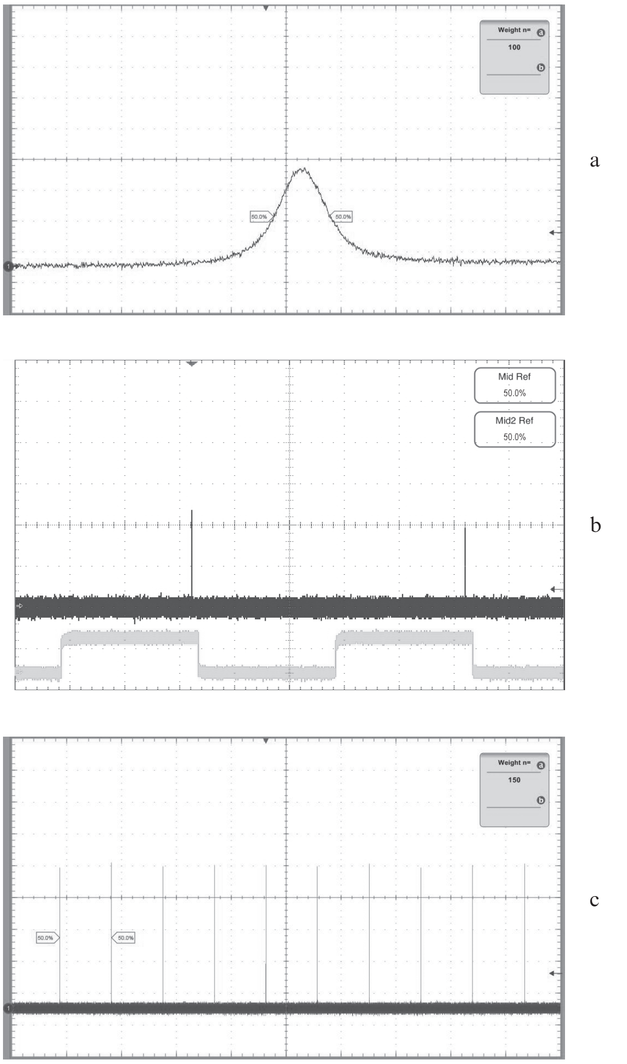


**Figure 8.** Oscilloscope traces of a laser pulse for a time scale of 4 ns div.$^{-1}$ (a), laser (the upper trace) and pump (the lower trace) pulses for a time scale of 200 μs div.$^{-1}$ (b), and a sequence of laser pulses for a time scale of 400 μs div.$^{-1}$ (c).

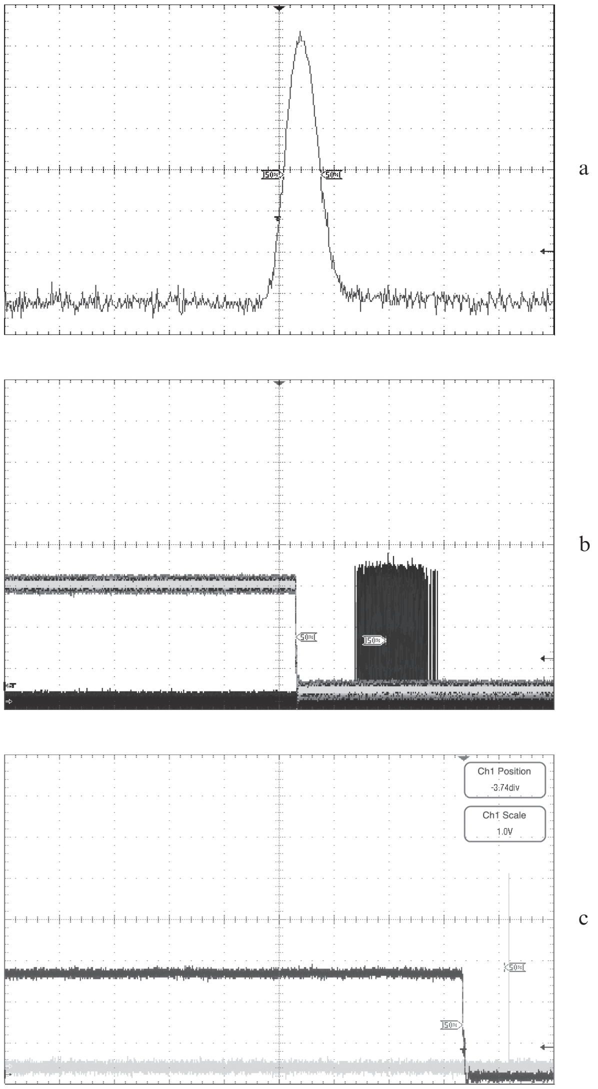


**Figure 9.** Oscilloscope traces of a laser pulse for time scale of 1.25 ns div.$^{-1}$ (a), laser (at the right) and sinchro (at the left) pulses for a time scale of 1 μs div.$^{-1}$ (b), as well as of sinchro pulses (at the left) and a sequence of superposed laser pulses (at the right) for a time scale of 400 ns div.$^{-1}$ (c).

and engaging the regime of pump pulse truncation resulted in $\delta_g^t = 0.06\%$.Therefore, the resultant experimental data agree nicely with simulation data.

Next we present the LTJ measurement data for lasers which generate short high-power pulses and therefore operate at relatively low repetition frequencies (below 200 Hz) under pumping by ~200-μs-long pulses. Lasers with a Cr:YAG crystal PLS with an initial transmittance of 12%, a highly reflecting mirror, and an exit mirror (with a reflectivity of 20%) generated short (under 1 ns) pulses (Fig. 9a), which were compressed in the stimulated Mandel'shtam–Brillouin scattering [3]. In this case we recorded (Fig. 9b) the delay of pulsed lasing relative to the synchronising pulse, which controlled the instant of the onset of diode lasing and was fed to a Tektronix TDS6124C oscilloscope. In this case, the standard deviation of the time of lasing development was less than 200 ns and was independent of the pulse repetition frequency, because the memory of the parameters of the system was not retained after the generation of pulses in the intervals between them. That is why there is no point in dividing this value by the pulse separation, for instance for a repetition rate of 10 Hz, to obtain a relative LTJ of $\sim 10^{-6}$, because the absolute jitter of the synchronising pulses themselves was not taken into account in these measurements. The absolute TJ of the compressed pulses [3] for their repetition frequency of 10 Hz was earlier measured at ~0.25 ms. Therefore, in this case the TJ is almost entirely due to the TJ of synchronising pulses themselves, because the jitter of the time delay of the compressed pulses relative to the pump pulses did not exceed 40 ps [3].

However, lowering the LTJ of pulses is of major significance, because in many cases the phenomena investigated with the help of a laser and the laser firing are triggered by the same synchronizing pulse or by time-locked synchronising pulses. Lasers with an active QS and negative feedback are convenient to use for obtaining low absolute values of the LTJ [4]. In this case, during a pump pulse there initially sets in a cw lasing and only then a special synchronising pulse is applied, the onset of a laser pulse being recorded relative to this synchronising pulse (Fig. 9c). Measurements performed in the case of short (under 1 ns) pulse lasing yield an LTJ value of 50 ps. In our opinion, this precision is quite often sufficient for timing to external events.

As a result of our literature search, which we carried out after we had conceived the idea of lowering the TJ with the use of pump pulse truncation and after its confirmation by direct numerical simulations and experiments, we managed to find only two earlier publications [32, 33] in which it was mentioned. First, this is a patent of 1996 [32], in which practically all of the above possibilities of controlling the instants of the onset of pulsed lasing and stabilising the repetition frequency were formulated in general terms. The regime of pump pulse truncation upon laser pulse generation, which was investigated in our work, was not considered directly in Ref. [32] and the specified possibility of pulsed pump disengagement in this case was associated only with the capability of controlling the intervals between the generated pulses. The experimental values of the TJ equal to $\sim$60 ns in the regime of continuous pumping for a pulse repetition frequency of 13 kHz and to 33 ps for a modulated pump and a repetition frequency of 11.7 kHz appear to be too small even if the LTJ was involved. Indeed, this signifies that a relative jitter $\delta_{\rm g}^{\rm cw} = 0.078\%$ was obtained in the conventional regime of continuous pumping, which is nearly two orders of magnitude lower than the values usually realised in experiments of other authors. Also noteworthy is the fact that the aforementioned patent has not been cited in any of the papers known to us, including the more recently published papers of the patent authors themselves (see, for instance, Ref. [7] and their citation in Refs [5, 6]). The possibility of disengaging the pump pulse after the generation of a laser pulse is mentioned also in Ref. [33], but it contains no evidence of the effect of this disengagement on the TJ, either.

## 5. Conclusions

We have performed a numerical analysis of the lasing of Nd : YAG minilasers with passive QS and longitudinal laser diode pumping. This analysis is based on an extended PLM which takes into account the splitting of AE energy levels to Stark sublevels, the time of excitation relaxation from the lower working laser level to the ground state, the possibility of direct pumping to the upper level, the dependence of PLS transmittance on the angle of rotation relative to the plane of linear radiation polarisation, etc. For the purposes of the present work, account was specially taken of the possibility of population depletion of the ground level of the AE, of PLS bleaching by an additional radiation source, as well as the possibility of pump radiation modulation in different ways. The spontaneous radiation fluctuation powers employed in our simulations limit the period stability at a level $\delta_{\rm g}^{\rm gn} = \Delta T_{\rm g}^{\rm gn}/T_{\rm g}^{\rm gn} \sim 10^{-6}$ for a pulse repetition frequency of ~1 kHz, which defines the minimal attainable TJ of pulse; this limit may only be approached by taking advantage of different technical methods of lowering the TJ, including periodic modulation of the pump power.

By way of direct numerical simulations we showed the feasibility of controlling the repetition frequency and lowering substantially the TJ of pulses with the use of periodic modulation of the diode pump by rectangular pulses. It was noted that synchronising the pulse-repetition period to the modulation period is also possible for a harmonic (sinusoidal) pump modulation. However, the ranges of repetition frequency and average pump power whereby this synchronisation may be achieved are broader when the modulation is effected by rectangular pulses. When use is made of pump modulation by rectangular pulses, the higher-to-lower pump power switching upon generation of a successive laser pulse improves the immunity of the repetition rate to variations of the average pump power and broadens the range of repetition frequencies wherein they may be locked to the pump modulation frequency. The feasibility of lowering the TJ and locking the pulse repetition frequency to the pump modulation frequency was borne out by our experiments. For repetition rates of $\sim$1 kHz, the ratio between the standard period deviation to the repetition period was shown to lower from several percent for continuous pumping to 0.06% in the regime of modulation pulse truncation. The resultant experimental data are in good agreement with our simulation data.

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