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Refractive index gradient measurement across the thickness of a dielectric film by the prism coupling method

V.I. Sokolov, V.Ya. Panchenko, V.N. Seminogov

Abstract. **A method is proposed for measuring the refractive index gradient $n(z)$ in nonuniformly thick dielectric films. The method is based on the excitation of waveguide modes in a film using the prism coupling technique and on the calculation of $n(z)$ and film thickness H_f with the help of the angular positions of the TE or TM modes. The method can be used for an arbitrary shape of the index modulation over the film thickness in the limit of a small gradient $[\Delta n(z)/n(z) \ll 1]$.**

Keywords: *dielectric film, waveguide modes, prism coupling method.*

1. Introduction

The prism coupling method has been successfully used to determine the thickness H_f and refractive index n_f of uniformly thick dielectric films [1–5]. It allows one to measure n_f of films 0.3–15 μm in thickness with an accuracy of $\pm 1\times10^{-4}$, and their thickness – with an accuracy of ±0.5% [5]. This method was also used to measure the refractive index $n_f(z)$ of nonuniform films [6–9] with the following restrictions: $n_f(z)$ is a function continuously and monotonically decreasing for $z > 0$ whose maximum is located at the boundary of the film, $z = 0$ (Fig. 1). This paper presents a method for measuring the refractive index $n_f(z) = n_f + \Delta n_f(z)$ across the film thickness for an arbitrary shape of $n_f(z)$ modulation in the limit of a small gradient, i.e., at $\Delta n_f(z)/n_f \ll 1$. Here n_f is the average value of the refractive index of the film, and $\Delta n_f(z)$ is the contribution to $n_f(z)$, caused by its modulation.

2. Prism coupling method

The principle of excitation of waveguide modes in a dielectric film by the prism coupling technique is illustrated in Fig. 1. A monochromatic collimated light beam is focused on the film of thickness H_f with a refractive index $n_f(z)$ by a prism having a high refractive index N_p, at an angle θ. Between the prism and the film there is a gap of thickness H_i, filled with an immersion liquid with the refractive index n_i or air. If the angle of incidence exceeds a critical angle, there appears total internal reflection (TIR) of a light beam from the boundary of the prism. However, if the thickness of the gap is small (typical thicknesses are $H_i < 100$–200 nm), then under certain angles of incidence θ_i, for which the synchronism condition

$$N_p \sin\theta_i = \beta_i, \quad i = 0, 1, 2, 3, \ldots \tag{1}$$

is fulfilled, where β_i is the effective refractive index of the waveguide for the mode with number i, the TIR condition is violated and the light can penetrate into the film, exciting its waveguide mode. Therefore, when condition (1) is met, in the angular dependence of the reflection coefficient $R(\theta)$ of the beam from the prism–film interface, sharp and narrow minima are observed.

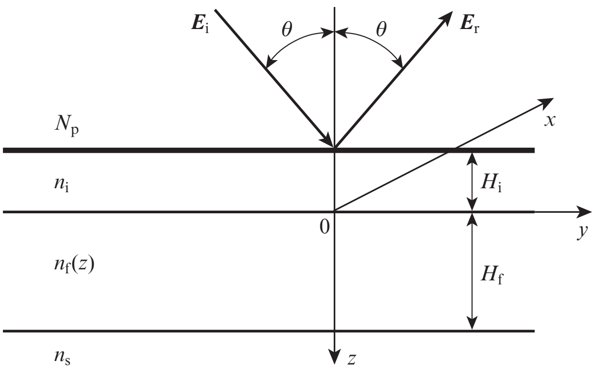

Figure 1. Scheme of excitation of waveguide modes in a dielectric film by the prism coupling method: N_p, n_i, n_f and n_s are the refractive indices of the prism, the immersion fluid (or air), the film and the substrate, respectively; H_i is thickness of the gap between the film and the prism; H_f is the film thickness; θ is the angle of incidence.

In the case of a uniformly thick film the values of β_i are uniquely determined by its thickness H_f and refractive index n_f (at specified n_i and n_s), so the two experimentally found values of θ_i can be used to calculate the two unknown parameters of the film, i.e., H_f and n_f [1–5]. If the film is not uniform in thickness, i.e., there is a refractive index gradient along the coordinate z (Fig. 1), then the problem becomes more complicated. Below we present a method for measuring H_f and $n_f(z)$, which can be used for an arbitrary shape of $n_f(z)$ modulation in the limit of a weak gradient $[\Delta n_f(z)/n_f \ll 1]$.

3. Dispersion equations for determining the effective refractive indices for TE and TM modes in a film with a refractive index gradient

In many practically important cases, the refractive index gradient $n_f(z) = n_f + \Delta n_f(z)$ in dielectric films is small. Therefore, below we will obtain the dispersion equations for TE

V.I. Sokolov, V.Ya. Panchenko, V.N. Seminogov Institute of Laser and Information Technologies, Russian Academy of Sciences,
ul. Svyatoozerskaya 1, 140700 Shatura, Moscow region, Russia;
e-mail: visokol@rambler.ru

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and TM modes by the perturbation method with respect to the small parameter $\Delta n_{\rm f}(z)/n_{\rm f} \ll 1$ for an arbitrary shape of $n_{\rm f}(z)$ modulation.

Consider a film with a thickness modulated (along the coordinate z, see Fig. 1) dielectric constant

$$\varepsilon_{\rm f}(z) = \varepsilon_{\rm f} + \Delta\varepsilon(z), \quad |\Delta\varepsilon(z)/\varepsilon_{\rm f}| \ll 1, \tag{2}$$

where $\varepsilon_{\rm f} = n_{\rm f}^2$ is an unperturbed dielectric constant; $\Delta\varepsilon(z)$ is a perturbation depending only on the coordinate z. We present expressions for the electromagnetic fields $\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}(\boldsymbol{r})\exp(-{\rm i}\omega t)$ + c.c. and $\boldsymbol{H}(\boldsymbol{r}, t) = \boldsymbol{H}(\boldsymbol{r})\exp(-{\rm i}\omega t)$ + c.c., where $\boldsymbol{r} = (x, y, z)$, in a three-layer dielectric nonabsorbing waveguide formed by a substrate, film and coating (immersion liquid or air), for TE modes. In the coating with a dielectric constant $\varepsilon_1 = n_{\rm i}^2$

$$\begin{aligned} &\boldsymbol{E}^{\rm I}(y, z) = \boldsymbol{e}_x A^{\rm I}\exp({\rm i}\beta k y + \gamma^{\rm I} z), \\ &\boldsymbol{H}^{\rm I}(y, z) = -\Big(\boldsymbol{e}_y\frac{{\rm i}\gamma^{\rm I}}{k} + \boldsymbol{e}_z\beta\Big)A^{\rm I}\exp({\rm i}\beta k y + \gamma^{\rm I} z), \end{aligned} \tag{3}$$

where $\boldsymbol{e}_x$, $\boldsymbol{e}_y$ and $\boldsymbol{e}_z$ are the unit vectors along the axes x, y and z; $A^{\rm I}$ is a complex amplitude of the wave in the coating; $k = 2\pi/\lambda$; λ is the wavelength of light in vacuum; $\gamma^{\rm I} = k(\beta^2 - \varepsilon_1)^{1/2} \geqslant 0$. In the substrate with a dielectric constant $\varepsilon_3 = n_{\rm s}^2$

$$\begin{aligned} &\boldsymbol{E}^{\rm III}(y, z) = \boldsymbol{e}_x A^{\rm III}\exp({\rm i}\beta k y - \gamma^{\rm III} z), \\ &\boldsymbol{H}^{\rm III}(y, z) = \Big(\boldsymbol{e}_y\frac{{\rm i}\gamma^{\rm III}}{k} - \boldsymbol{e}_z\beta\Big)A^{\rm III}\exp({\rm i}\beta k y - \gamma^{\rm III} z), \end{aligned} \tag{4}$$

where $A^{\rm III}$ is the amplitude of the wave in the substrate; $\gamma^{\rm III} = k(\beta^2 - \varepsilon_3)^{1/2} \geqslant 0$. In the film with a z-coordinate-dependent dielectric constant (2)

$$\begin{aligned} &\boldsymbol{E}^{\rm II}(y, z) = \boldsymbol{e}_x\exp({\rm i}\beta k y)[A_1 e_1(z) + A_2 e_2(z)], \\ &\boldsymbol{H}^{\rm II}(y, z) = -\Big\{\boldsymbol{e}_y\frac{\rm i}{k}\Big[A_1\frac{{\rm d}e_1(z)}{{\rm d}z} + A_2\frac{{\rm d}e_2(z)}{{\rm d}z}\Big] \\ &\qquad + \boldsymbol{e}_z\beta[A_1 e_1(z) + A_2 e_2(z)]\Big\}\exp({\rm i}\beta k y), \end{aligned} \tag{5}$$

where A_1 and A_2 are the amplitudes of waves $e_1(z)$ and $e_2(z)$ in the film, which are linearly independent solutions of the equation

$$\frac{{\rm d}^2 e_{1,2}(z)}{{\rm d}z^2} + k^2[\varepsilon_{\rm f}(z) - \beta^2]e_{1,2}(z) = 0. \tag{6}$$

Equation (6) can be solved analytically in the limit $\Delta\varepsilon(z)/\varepsilon_{\rm f} \to 0$ for an arbitrary shape of $\Delta\varepsilon(z)$ modulation (2). As a result, we obtain

$$\begin{aligned} e_{1,2}(z) = &\exp(\pm{\rm i}\gamma^{\rm II} z)\Big[1 \pm \frac{{\rm i}k^2}{2\gamma^{\rm II}}\int_{z_0}^{z}\Delta\varepsilon(\xi)\,{\rm d}\xi\Big] \\ &\mp \frac{{\rm i}k^2}{2\gamma^{\rm II}}\exp(\mp{\rm i}\gamma^{\rm II} z)\int_{z_0}^{z}\Delta\varepsilon(\xi)\exp(\pm{\rm i}2\gamma^{\rm II}\xi)\,{\rm d}\xi, \end{aligned} \tag{7}$$

where $\gamma^{\rm II} = k(\varepsilon_{\rm f} - \beta^2)^{1/2} \geqslant 0$.

Taking into account expressions (3)–(7) and sewing electromagnetic fields at the film–immersion liquid ($z = 0$) and film–substrate interface ($z = {\rm H_f}$) (Fig. 1), we obtain a dispersion equation for determining the effective refractive indices β_i for TE modes of the waveguide

$$\begin{aligned} &\{\gamma^{\rm II}(\gamma^{\rm I} + \gamma^{\rm III})\cos(\gamma^{\rm II} H_{\rm f}) + [\gamma^{\rm I}\gamma^{\rm III} - (\gamma^{\rm II})^2]\sin(\gamma^{\rm II} H_{\rm f})\} \\ &+ {\rm Im}\Big\{\frac{k^2(\gamma^{\rm II} + {\rm i}\gamma^{\rm I})}{2\gamma^{\rm II}}\Big[(\gamma^{\rm III} - {\rm i}\gamma^{\rm II})\exp(-{\rm i}\gamma^{\rm II} H_{\rm f}) \\ &\times\int_0^{H_{\rm f}}\Delta\varepsilon(z)\,{\rm d}z - (\gamma^{\rm III} + {\rm i}\gamma^{\rm II})\exp({\rm i}\gamma^{\rm II} H_{\rm f}) \\ &\times\int_0^{H_{\rm f}}\Delta\varepsilon(z)\exp(-{\rm i}2\gamma^{\rm II} z)\,{\rm d}z\Big]\Big\} = 0. \end{aligned} \tag{8}$$

The first term in curly brackets in equation (8) describes the contribution of the unperturbed waveguide with a constant refractive index, and the second term is associated with modulation of the refractive index $n_{\rm f}(z)$ across the thickness of the film. Equation (8) holds true for TE modes of a three-layer nonabsorbing waveguide with an arbitrary shape of $n_{\rm f}(z)$ modulation in the limit of small gradients $[|\Delta\varepsilon(z)/\varepsilon_{\rm f}| \ll 1]$.

Similarly, we write the expression for the electromagnetic fields for the TM modes of a three-layer waveguide. In the coating with a dielectric constant $\varepsilon_1 = n_{\rm i}^2$

$$\begin{aligned} &\boldsymbol{H}^{\rm I}(y, z) = \boldsymbol{e}_x B^{\rm I}\exp({\rm i}\beta k y + \gamma^{\rm I} z), \\ &\boldsymbol{E}^{\rm I}(y, z) = \Big(\boldsymbol{e}_y\frac{{\rm i}\gamma^{\rm I}}{k\varepsilon_1} + \boldsymbol{e}_z\frac{\beta}{\varepsilon_1}\Big)B^{\rm I}\exp({\rm i}\beta k y + \gamma^{\rm I} z); \end{aligned} \tag{9}$$

in the substrate with a dielectric constant $\varepsilon_3 = n_{\rm s}^2$

$$\begin{aligned} &\boldsymbol{H}^{\rm III}(y, z) = \boldsymbol{e}_x B^{\rm III}\exp({\rm i}\beta k y - \gamma^{\rm III} z), \\ &\boldsymbol{E}^{\rm III}(y, z) = \Big(-\boldsymbol{e}_y\frac{{\rm i}\gamma^{\rm III}}{k\varepsilon_3} + \boldsymbol{e}_z\frac{\beta}{\varepsilon_3}\Big)B^{\rm III}\exp({\rm i}\beta k y - \gamma^{\rm III} z); \end{aligned} \tag{10}$$

in the film with a z-coordinate-dependent dielectric constant (2)

$$\begin{aligned} &\boldsymbol{H}^{\rm II}(y, z) = \boldsymbol{e}_x\exp({\rm i}\beta k y)[B_1 h_1(z) + B_2 h_2(z)], \\ &\boldsymbol{E}^{\rm II}(y, z) = \Big\{\boldsymbol{e}_y\frac{\rm i}{k\varepsilon_{\rm f}(z)}\Big[B_1\frac{{\rm d}h_1(z)}{{\rm d}z} + B_2\frac{{\rm d}h_2(z)}{{\rm d}z}\Big] \\ &\qquad + \boldsymbol{e}_z\frac{\beta}{\varepsilon_{\rm f}(z)}[B_1 h_1(z) + B_2 h_2(z)]\Big\}\exp({\rm i}\beta k y), \end{aligned} \tag{11}$$

where $B^{\rm I}$ and $B^{\rm III}$ are the amplitudes of TM waves in the coating and substrate, respectively; and B_1 and B_2 are the amplitudes of the wave $h_1(z)$ and $h_2(z)$ in the film, which are linearly independent solutions of the equation

$$\begin{aligned} &\frac{{\rm d}^2 h_{1,2}(z)}{{\rm d}z^2} - \frac{1}{\varepsilon_{\rm f}(z)}\frac{{\rm d}\varepsilon_{\rm f}(z)}{{\rm d}z}\frac{{\rm d}h_{1,2}(z)}{{\rm d}z} \\ &\qquad + k^2[\varepsilon_{\rm f}(z) - \beta^2]h_{1,2}(z) = 0. \end{aligned} \tag{12}$$

The analytical solution of equation (12) in the limit $\Delta\varepsilon(z)/\varepsilon_{\rm f} \to 0$ for an arbitrary shape of $\Delta\varepsilon(z)$ modulation (2) can be written as

$$h_{1,2}(z) = \exp(\pm\mathrm{i}\gamma^{\mathrm{II}}z)\left\{1 \pm \frac{\mathrm{i}k}{2\gamma^{\mathrm{II}}}\int_{z_0}^{z}\left[k\Delta\varepsilon(\xi) \mp \frac{\mathrm{i}\gamma^{\mathrm{II}}}{k\varepsilon_{\mathrm{f}}}\frac{\mathrm{d}\Delta\varepsilon(\xi)}{\mathrm{d}\xi}\right]\mathrm{d}\xi\right\}$$

$$\mp \frac{\mathrm{i}k^2}{2\gamma^{\mathrm{II}}}\exp(\mp\mathrm{i}\gamma^{\mathrm{II}}z)\int_{z_0}^{z}\left[k\Delta\varepsilon(\xi) \mp \frac{\mathrm{i}\gamma^{\mathrm{II}}}{k\varepsilon_{\mathrm{f}}}\frac{\mathrm{d}\Delta\varepsilon(\xi)}{\mathrm{d}\xi}\right]\exp(\pm\mathrm{i}2\gamma^{\mathrm{II}}\xi)\,\mathrm{d}\xi. \quad (13)$$

After sewing the electromagnetic fields (9)–(13) on the film boundaries $z = 0$ and $z = H_{\mathrm{f}}$ (Fig. 1), we obtain the dispersion equation for TM modes:

$$\left\{\frac{\gamma^{\mathrm{II}}}{\varepsilon_{\mathrm{f}}}\left(\frac{\gamma^{\mathrm{I}}}{\varepsilon_1} + \frac{\gamma^{\mathrm{III}}}{\varepsilon_3}\right)\cos(\gamma^{\mathrm{II}}H_{\mathrm{f}}) + \left[\frac{\gamma^{\mathrm{I}}}{\varepsilon_1}\frac{\gamma^{\mathrm{III}}}{\varepsilon_3} - \left(\frac{\gamma^{\mathrm{II}}}{\varepsilon_{\mathrm{f}}}\right)^2\right]\sin(\gamma^{\mathrm{II}}H_{\mathrm{f}})\right\}$$

$$- \operatorname{Im}\frac{\mathrm{i}\gamma^{\mathrm{II}}}{\varepsilon_{\mathrm{f}}}\left[\left(\frac{\gamma^{\mathrm{III}}}{\varepsilon_3} - \frac{\mathrm{i}\gamma^{\mathrm{II}}}{\varepsilon_{\mathrm{f}}}\right)\frac{\Delta\varepsilon(z=0)}{\varepsilon_{\mathrm{f}}} + \left(\frac{\gamma^{\mathrm{I}}}{\varepsilon_1} - \frac{\mathrm{i}\gamma^{\mathrm{II}}}{\varepsilon_{\mathrm{f}}}\right)\frac{\Delta\varepsilon(z=H_{\mathrm{f}})}{\varepsilon_{\mathrm{f}}}\right]$$

$$\times \exp(-\mathrm{i}\gamma^{\mathrm{II}}H_{\mathrm{f}}) + \operatorname{Im}\left(\frac{\gamma^{\mathrm{I}}}{\varepsilon_1} - \frac{\mathrm{i}\gamma^{\mathrm{II}}}{\varepsilon_{\mathrm{f}}}\right)\frac{\mathrm{i}k}{2\gamma^{\mathrm{II}}}\left\{\left(\frac{\gamma^{\mathrm{III}}}{\varepsilon_3} - \frac{\mathrm{i}\gamma^{\mathrm{II}}}{\varepsilon_{\mathrm{f}}}\right)\right.$$

$$\times \exp(-\mathrm{i}\gamma^{\mathrm{II}}H_{\mathrm{f}})\int_0^{H_{\mathrm{f}}}\left[k\Delta\varepsilon(z) + \frac{\mathrm{i}\gamma^{\mathrm{II}}}{k\varepsilon_{\mathrm{f}}}\frac{\mathrm{d}\Delta\varepsilon(z)}{\mathrm{d}z}\right]\mathrm{d}z - \left(\frac{\gamma^{\mathrm{III}}}{\varepsilon_3} + \frac{\mathrm{i}\gamma^{\mathrm{II}}}{\varepsilon_{\mathrm{f}}}\right)$$

$$\left.\times \exp(\mathrm{i}\gamma^{\mathrm{II}}H_{\mathrm{f}})\int_0^{H_{\mathrm{f}}}\left[k\Delta\varepsilon(z) + \frac{\mathrm{i}\gamma^{\mathrm{II}}}{k\varepsilon_{\mathrm{f}}}\frac{\mathrm{d}\Delta\varepsilon(z)}{\mathrm{d}z}\right]\exp(-\mathrm{i}2\gamma^{\mathrm{II}}z)\,\mathrm{d}z\right\} = 0. \quad (14)$$

The first term in curly brackets in equation (14) describes the contribution of the unperturbed waveguide with a constant refractive index, and the remaining terms are associated with modulation of the refractive index $n_{\mathrm{f}}(z)$ across the thickness of the film. Equation (14) holds true for TM modes of a three-layer nonabsorbing waveguide with an arbitrary shape of $n_{\mathrm{f}}(z)$ modulation in the limit of small gradients [$|\Delta\varepsilon(z)/\varepsilon_{\mathrm{f}}| \ll 1$].

Using dispersion equations (8) and (14) we can calculate the effective refractive indices β_i^{th} for TE and TM waveguide modes for a given thickness H_{f} and refractive index $n_{\mathrm{f}}(z)$ of the dielectric film, if the refractive indices of the substrate and the coating are known.

4. Experimental excitation of TE and TM modes in nonuniformly thick SiO films with the refractive index $n_{\mathrm{f}}(z)$

The dielectric film of silicon monoxide, SiO, of thickness $H_{\mathrm{f}} \approx 1.1$ μm was fabricated by thermal evaporation in vacuum with subsequent deposition on the SiO_2 substrate. Figure 2 shows the experimentally measured dependences of the reflection coefficient $R(\beta)$ of this film for TE and TM polarised light. Measurements were performed using a Metricon-2010 prism coupler [5] by focusing a collimated beam from a 632.8-nm helium–neon laser through a measuring prism of cubic zirconia (ZrO_2) with $N_{\mathrm{p}}(\lambda = 632.8\mathrm{nm}) = 2.13825$ on the film. When measuring the effective refractive indices β_i^{exp} for different modes, use is made of the minimum force that presses the film to the prism. At this force the corresponding minimum in the dependence $R(\beta)$ was still clearly visible. This was done to ensure that the conditions of weak coupling, under which the presence of the measuring prism with a high refractive index in the close proximity with the film does not alter significantly the angular positions of the modes. The parameter β_i^{exp} was measured several times for each waveguide mode and the results were averaged to improve the measurement accuracy. According to our estimates, the experimental accuracy of β_i^{exp} in repeated measurements using the Metricon-2010 prism coupler was $\pm 2\times10^{-5}$.

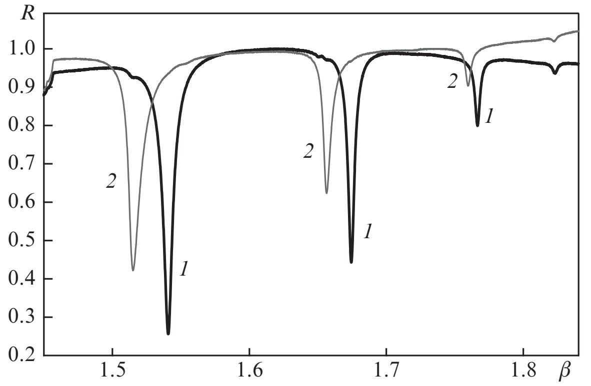

Figure 2. Experimentally measured dependences of the reflection coefficient R of the SiO film on the parameter β for a wavelength of 632.8 nm in the case of TE (*1*) and TM (*2*) polarisations.

One can see from Fig. 2 that four TE and four TM modes are observed in the SiO film. The experimentally measured effective refractive indices for these modes are presented in Table 1. Taking into account the values of $\pm 2\times10^{-5}$ and minimising the functional

$$\varDelta = \sum_{i=0}^{3}|\beta_i^{\mathrm{exp}} - \beta_i^{\mathrm{th}}|, \quad (15)$$

one can calculate the required parameters of the film, H_{f} and $n_{\mathrm{f}}(z)$.

Table 1. Experimentally measured and theoretically calculated effective refractive indices β_i^{exp} and β_i^{th} for TE and TM modes in the SiO film.

Mode number i	Polarisation	β_i^{exp}	β_i^{th}	
			$\varepsilon_{\mathrm{f}}(z) = n_{\mathrm{f}}^2 = \mathrm{const}$	$\varepsilon_{\mathrm{f}}(z) = n_{\mathrm{f}}^2 + a_1 z + a_2 z^2$
0	TE	1.82309	1.82232	1.82309
	TM	1.82236	1.82150	1.82236
1	TE	1.76653	1.76762	1.76653
	TM	1.75917	1.75974	1.75917
2	TE	1.67435	1.67435	1.67435
	TM	1.65530	1.65531	1.65530
3	TE	1.54102	1.54102	1.54102
	TM	1.51281	1.51281	1.51281

Let us first assume that the refractive index is constant over the film thickness. Then, by minimising functional (15) in H_{f} and n_{f}, we obtain $H_{\mathrm{f}} = 1076.1$ nm, $n_{\mathrm{f}} = 1.84033$ for TE modes and $H_{\mathrm{f}} = 1081.6$ nm, $n_{\mathrm{f}} = 1.84187$ for TM modes. The effective refractive indices of guided modes, β_i^{th}, corresponding to the values of H_{f} and n_{f} are shown in Table 1, and the minimum values $\varDelta_{\mathrm{min}}$ of the functional (15) are listed in Table 2.

Table 1 and Table 2 show that when the refractive index is assumed constant over the film thickness, the minimum values of functional (15) for any H_{f} and n_{f} do not fall below 1×10^{-3} and that there is a strong discrepancy between the experimentally measured effective refractive indices for the waveguide modes and their theoretically calculated values. The reason for this discrepancy may lie in the fact that the refractive index $n_{\mathrm{f}}(z)$ is not constant over the film thickness. Table 1 and Table 2 present also the values of β_i^{th}, $\varDelta_{\mathrm{min}}$, H_{f} and $n_{\mathrm{f}}(z)$,

Table 2. Minimal values $\Delta_{\min}$ of functional (15) and the corresponding parameters of the SiO film, calculated from the analysis of the angular positions of TE and TM modes.

Mode	Dielectric constant	$\Delta_{\min}$	H_f/nm	n_f	a_1	a_2
TE	$\varepsilon_f(z) = n_f^2 = \text{const}$	1.86×10^{-3}	1076.1	1.84033		
TM		1.44×10^{-3}	1081.6	1.84187		
TE	$\varepsilon_f(z) = n_f^2 + a_1z + a_2z^2$	$< 1\times10^{-5}$	1079.6	1.86021	-1.12×10^{-3}	1.282×10^{-6}
TM		$< 1\times10^{-5}$	1085.5	1.86027	-8.38×10^{-3}	9.4×10^{-7}

calculated under the assumption that the refractive index gradient is present in the film with a dielectric constant

$$\varepsilon_f(z) = n_f^2 + a_1z + a_2z^2, \tag{16}$$

where a_1 and a_2 are constants. The values presented in the bottom two rows of Table 2 are obtained by minimising functional (15) in H_f, n_f, a_1 and a_2. It follows from Table 1 and Table 2 that the account for the gradient $n_f(z)$ provides a substantially lower value of $\Delta_{\min}$ and the excellent agreement of the experimentally measured and theoretically calculated values of β_i. In this case, the thicknesses H_f = 1079.6 and 1085.5 nm, found from the angular positions of the TE and TM modes for the gradient film, are somewhat higher than the corresponding thicknesses obtained under the assumption n_f = const, and differ by 5.9 nm. Thus, the error in determining the thickness of the gradient film by the prism coupling method does not exceed ±0.3%.

The distribution of the refractive index $n_f(z)$ over the film thickness, at which the best agreement between experimental and theoretical values of β_i is achieved, is presented in Fig. 3.

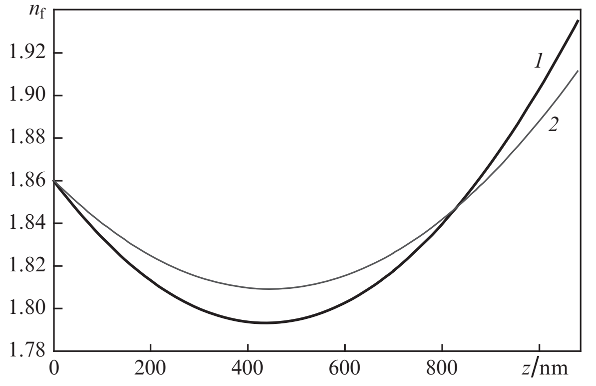

Figure 3. Distribution of the refractive index n_f over the SiO film thickness z, at which the best agreement between experimental and theoretical values of β_i is achieved for the TE (*1*) and TM (*2*) modes.

It follows from Fig. 3 that the deviation of the refractive index $n_f(z)$ from its average value 1.8309 for the TE modes and 1.8365 for the TM modes is less than 4%, i.e., the condition of small gradient [$\Delta n_f(z)/n_f \ll 1$] is fulfilled. Figure 3 also shows that the refractive index $n_f(z)$ is minimal in the central part of the film and increases towards its boundary with the substrate and air gap. In our opinion, such a profile shape of $n_f(z)$ can be caused by stress in the film, so that silicon atoms diffuse to the boundary with the substrate. This leads to a decrease in the stoichiometric parameter x of silicon oxide SiO_x and to an increase in the refractive index of the film near its boundary with the substrate. As the stress is usually maximal near this boundary, silicon atoms most strongly diffuse from the film regions that are closest to the substrate. This explains why the refractive index at the film–air interface is higher than in the middle of the film.

5. Conclusions

We have developed a method for measuring the refractive index gradient $n_f(z)$ in dielectric films. The method can be used for an arbitrary shape of $n_f(z)$ modulation over the film thickness in the limit of a small gradient [$\Delta n_f(z)/n_f \ll 1$]. According to the proposed method the dielectric constant of the film is presented in the form of a power series $\varepsilon_f(z) = n_f^2 + \sum_i a_i z^i$ with unknown coefficients a_i. Using the prism coupler we have experimentally measured the effective refractive indices for the waveguide modes, $\beta_i^{\exp}$ ($i = 0, 1, 2, \ldots$), which are then used to calculate the coefficients a_i. The method can be employed for excitation of at least three modes with the same polarisation in the film. The accuracy of determining the profile $n_f(z)$ increases with the number of waveguide modes taken into account. Use of this method to analyse the characteristics of the gradient film SiO, in which four TE and four TM waveguide modes can be excited, has shown that the accuracy of determining the film thickness is ±0.3%, and accuracy of measuring its refractive index is $\pm2\times10^{-5}$.

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