

LASER APPLICATIONS AND OTHER TOPICS IN QUANTUM ELECTRONICS

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Ionisation of a quantum dot by electric fields

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Abstract. **We have derived analytical formulas for differential and total ionisation probabilities of a two-dimensional quantum dot by a constant electric field. In the adiabatic approximation, we have calculated the probability of this process in the field of a plane electromagnetic wave and in a superposition of constant and alternating electric fields. The imaginary-time method is used to obtain the momentum distribution of the ionisation probability of a bound system by an intense field generated by a superposition of parallel constant and alternating electric fields. The total probability of the process per unit time is calculated with exponential accuracy. The dependence of the results obtained on the characteristic parameters of the problem is investigated.**

Keywords: *ionisation, quantum dot, imaginary-time method, saddle point.*

6. Introduction

The development of nanotechnology and creation of high-power lasers stimulate theoretical and experimental studies of the interaction of intense electromagnetic fields with nanostructures. Advances in the field of quantum-dot structures and prospects of these new types of heterostructures make theoretical study of ionisation of a quantum dot by intense electromagnetic fields as urgent as possible.

The purpose of this paper is to study ionisation of a two-dimensional quantum dot both by a constant electric field and by a field, which is a superposition of parallel constant and alternating electric fields.

The theoretical description of tunnelling and multiphoton ionisation of low-dimensional structures is based on the results obtained in [1–10].

Photoionisation of a bound system in the presence of a constant electric field was considered earlier in [11–16]. Semi-quantitatively, this problem was first discussed in [13]. Quantitative analysis of the phenomenon in the case of electron detachment bound by short-range forces was carried out in [12]. In the approximation of zero-range binding forces a formula was derived for the differential probability of the process in the general case, when the constant field strength is directed at an arbitrary angle to the axis along which the electric field of a linearly polarised wave oscillates. The case when the electric field strength of a linearly polarised wave is orthogonal to the constant field strength is considered in detail.

Nonperturbative, gauge-invariant approach to the description of ionisation of both one-dimensional and three-dimensional systems with a finite-range binding potential has been formulated in [11, 16], where the authors discuss the general characteristics of the process of ionisation of systems with short-range forces due to a superposition of constant and alternating electric fields.

The authors of [14] studied the decay of a weakly bound level in a superposition of a constant electric field and the electromagnetic wave. The probability of ionisation was determined by the imaginary part of the quasi-energy and calculated, as in [12], in the framework of the zero-range force potential in the case of the perpendicular fields, where the interference contribution to the ionisation probability vanishes. The analysis performed in [14] also showed that application of even a relatively weak constant field leads to a significant change in the ionisation probability of the energy level by the wave field. In this paper, to interpret the results obtained, the authors [14] proposed the mechanism of electron tunnelling from the virtual state: an electron absorbs k light quanta and with the energy $E_0 + k\omega < 0$ (ω is the wave frequency) tunnels in a constant field. This process occurs only in the presence of two fields – high-frequency and static – and cannot proceed when the constant field is switched off [14].

In the most interesting case of collinear fields, the influence of an alternating field on the quantum tunnelling of a particle through a potential barrier, interband tunnelling in a semiconductor and above-barrier reflection was considered in [15] by the method of complex trajectories [17]. The probability of a sub-barrier passage of a particle through a triangular barrier (field emission) in an alternating field was calculated with exponential accuracy. It was shown that the probability of quasi-classical tunnelling process under the influence of a time-varying perturbation sharply increased, and a succession of tunnelling regimes with increasing wave intensity was analysed.

Essential information about the process of tunnelling ionisation is obtained from the energy and angular distributions of electrons produced [18–20], which were not studied in [14, 15]. To describe operation of a photodetector with quantum wells in which photoionisation occurs in the presence of a constant electric field, knowledge of the spectral-angular distribution of photoelectrons can be of practical interest [21]. Unfortunately, the authors of [15] did not present stringent conditions for the applicability of this result, which should

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follow from the initial quantum description, and did not clarify how to calculate the pre-exponential factor in the formula for the total ionisation probability per unit time.

Given the fact that the authors of the above-mentioned work [11–16] predicted a significant increase in the probability of classically forbidden processes under the action of an electromagnetic wave, it seems relevant to study the ionisation process of bound systems by an alternating electric field with a constant component.

7. Probability of ionisation of a two-dimensional quantum dot by a constant electric field

We model the binding potential of a two-dimensional quantum dot by a potential well of the form

$$U(\rho) = \begin{cases} -U_0, \rho = \sqrt{x^2+y^2} < a, \\ 0, \rho > a, \end{cases} \tag{1}$$

where a is the radius of a quantum dot. Note that depending on the type of lateral binding potential the typical dot size varies from tens to several hundreds of nanometres, and the number of electrons in a quantum dot can be controllably varied from a few to several hundreds [22].

The probability of ionisation of the ground state of an electron in a quantum dot by a constant electric field will be calculated on the basis of the quantum-mechanical method described in [3, 4].

The solution of the stationary Schrödinger equation for the ground state of an electron with energy $E_0 = -\kappa^2/2$ in a two-dimensional potential well (1) has the form [23]

$$\psi_0(\rho, t) = \exp(\mathrm{i}\kappa^2 t/2) B \begin{cases} \dfrac{K_0(\kappa a)}{J_0(\lambda a)} J_0(\lambda\rho), \rho < a, \\ K_0(\kappa\rho), \rho > a, \end{cases} \tag{2}$$

where $J_0(x)$ and $K_0(x)$ are the Bessel and Macdonald functions of zero order;

$$\kappa = \sqrt{2|E_0|};\quad \lambda = \sqrt{2(U_0 - |E_0|)};$$
$$B = \frac{1}{\sqrt{\pi}\, a K_1(\kappa a)}\left(\frac{U_0 - |E_0|}{U_0}\right)^{1/2}. \tag{3}$$

The conditions of continuity of the wave function and its derivative at point $\rho = a$ lead to the equation

$$\frac{\lambda J_0'(\lambda a)}{J_0(\lambda a)} = \frac{\kappa K_0'(\kappa a)}{K_0(\kappa a)}, \tag{4}$$

from which the energy E_0 ($-U_0 < E_0 < 0$) of the ground state of an electron in a quantum dot is determined. The integral Schrödinger equation for the quasi-stationary regime is given by

$$\psi(\boldsymbol{r}, t) = -\mathrm{i}\int_{-\infty}^{t} \mathrm{d}t' \int \mathrm{d}\boldsymbol{r}\, G(\boldsymbol{r}, t; \boldsymbol{r}', t')\, U(\boldsymbol{r}')\psi(\boldsymbol{r}', t'), \tag{5}$$

where $G(\boldsymbol{r}, t; \boldsymbol{r}', t')$ is Green's function of the time-dependent Schrödinger equation in a constant electric field $\boldsymbol{E} = (F, 0, 0)$. If the field strength satisfies the condition

$$Fa \ll \kappa^2 < 2U_0, \tag{6}$$

then in the region of the well the electric field is a small perturbation. Considering also that outside the well $U(\boldsymbol{r}) = 0$, the function $\psi(\boldsymbol{r}', t')$ in the right-hand side of equation (5) in the first approximation can be set equal to the wave function (2) of the bound state of an electron in a quantum dot.

Describing a constant electric field by a vector potential $\boldsymbol{A}(t)$, depending only on time, Green's function $G(\boldsymbol{r}, t; \boldsymbol{r}', t')$ can be represented as [4]

$$G(\boldsymbol{r}, t; \boldsymbol{r}', t') = \frac{\theta(t-t')}{(2\pi)^2}\int_{-\infty}^{\infty} \mathrm{d}p_x \mathrm{d}p_y$$
$$\times \exp\left[\mathrm{i}\boldsymbol{\pi}(t)\boldsymbol{r} - \mathrm{i}\boldsymbol{\pi}(t')\boldsymbol{r}' - \frac{\mathrm{i}}{2}\int_{t'}^{t} \boldsymbol{\pi}^2(\tau)\mathrm{d}\tau\right], \tag{7}$$

where

$$\boldsymbol{\pi}(t) = \boldsymbol{p} - \boldsymbol{A}(t) = (p_x + Ft, p_y) \tag{8}$$

is the generalised momentum; $\boldsymbol{p} = (p_x, p_y)$ is the electron momentum, $\theta(t-t')$ is the Heaviside function.

Thus, the solution of the Schrödinger equation has the form

$$\psi(\boldsymbol{r}, t) = \frac{\theta(t-t')}{(2\pi)^2}\int_{-\infty}^{\infty} \mathrm{d}p_x \mathrm{d}p_y$$
$$\times \exp\left[\mathrm{i}\boldsymbol{\pi}(t)\boldsymbol{r} - \frac{\mathrm{i}}{2}\int_{t'}^{t} \boldsymbol{\pi}^2(\tau)\mathrm{d}\tau\right] G(\boldsymbol{\pi}(t)). \tag{9}$$

Here

$$G(\boldsymbol{\pi}(t)) = \int_{-\infty}^{t} \mathrm{d}t' g(\boldsymbol{\pi}(t'))\exp\left\{\frac{\mathrm{i}}{2}\int_{0}^{t'} [\boldsymbol{\pi}^2(\tau) + \kappa^2]\mathrm{d}\tau\right\}, \tag{10}$$

and the value of $g(\boldsymbol{\pi}(t'))$ is related to the Fourier transform of the coordinate part of the solution (2):

$$g(\boldsymbol{\pi}(t')) = \frac{1}{2}[\boldsymbol{\pi}^2(t') + \kappa^2]\psi_0(\boldsymbol{\pi}(t')), \tag{11}$$

$$\psi_0(\boldsymbol{\pi}(t')) = \int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{r}' \exp[-\mathrm{i}\boldsymbol{r}'\boldsymbol{\pi}(t')]\psi_0(\boldsymbol{r}'). \tag{12}$$

The characteristic quantity of the field dimension in this problem is the value of $F_0 = \kappa^3$. Below, along with (6), we assume that the condition

$$F \ll F_0 \tag{13}$$

to be fulfilled. In this case, the integral over the variable t' in (10) is calculated by the saddle-point method (see also [4, 21]). The saddle point, defined by the equation

$$\boldsymbol{\pi}^2(t_0') \equiv p_y^2 + (p_x^2 + Ft_0') = -\kappa^2, \tag{14}$$

is located in the complex plane of the variable t'. Calculation of the integral yields the following result:

$$G(t_0) \approx C\sqrt{\frac{2\pi}{\kappa F}}\exp\left[-\frac{1}{3}\frac{F_0}{F} - \frac{1}{2}\frac{F_0}{F}\left(\frac{p_y}{\kappa}\right)^2 - \frac{\mathrm{i}}{2}\frac{F_0}{F}\frac{p_x}{\kappa}\right], \tag{15}$$

where

$$C = (-\pi\kappa a B)[I_1(\kappa a)K_0(\kappa a) + I_0(\kappa a)K_1(\kappa a)]. \tag{16}$$

The main contribution to the integral over the variable t' is from the region of width

$$|t - t_0'| \leqslant \frac{1}{\sqrt{\kappa F}} \tag{17}$$

around the saddle point, which should be small compared with the upper limit of integration, i.e., it is assumed that

$$t \gg \frac{1}{\sqrt{\kappa F}}. \tag{18}$$

The contribution of the term proportional to $(t - t_0')^3$, can be neglected, as it was done to obtain (15), if the condition

$$\sqrt{\frac{F}{F_0}} \ll 1 \tag{19}$$

is met.

The ionisation probability per unit time is determined by the flux of electrons through a straight line (remote from the centre of the quantum dot) that is perpendicular to the x axis:

$$w = \int_{-\infty}^{\infty} \mathrm{d}y \frac{\mathrm{i}}{2}\Big[\psi(\boldsymbol{r},t)\frac{\partial\psi^+(\boldsymbol{r},t)}{\partial x} - \psi^+(\boldsymbol{r},t)\frac{\partial\psi(\boldsymbol{r},t)}{\partial x}\Big]. \tag{20}$$

First, we calculate the integral with respect to the variable p_x. Given that

$$t_0' = -\frac{1}{F}\Big(p_x - \mathrm{i}\sqrt{\kappa^2 + p_y^2}\Big), \tag{21}$$

$$0 < \mathrm{Re}\, t_0' < t, \tag{22}$$

integration with respect to p_x in (9) is performed in the interval

$$-Ft < p_x < 0. \tag{23}$$

To calculate the integral

$$I \equiv \int_{-Ft}^{0} \mathrm{d}p_x \exp\Big[\mathrm{i}x(p_x + Ft) - \frac{\mathrm{i}}{2}\frac{F_0}{F}\frac{p_x}{\kappa} - \frac{\mathrm{i}}{6F}(p_x + Ft)^3\Big] \tag{24}$$

we will use again the saddle-point method. As a result, we obtain

$$I \approx \exp\Big[-\mathrm{i}\frac{\pi}{4} + \mathrm{i}\frac{\kappa^2}{2}t + \mathrm{i}\frac{2}{3}\sqrt{2F}\Big(x - \frac{\kappa^2}{2F}\Big)^{3/2}\Big]$$
$$\times\Big\{\frac{\pi}{[x - \kappa^2/(2F)]^{1/2}}\Big(\frac{F}{2}\Big)^{1/2}\Big\}^{1/2}, \tag{25}$$

and the applicability condition of this formula has the form

$$\Big[F^{1/4}\Big(x - \frac{\kappa^2}{2F}\Big)^{3/4}\Big]^{-1} \ll 1. \tag{26}$$

Thus, when conditions (17), (19) and (26) are fulfilled, from (9) it follows that

$$\psi(\boldsymbol{r},t) \approx \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\mathrm{d}p_y \exp\Big[-\frac{1}{2}\frac{F_0}{F}\Big(\frac{p_y}{\kappa}\Big)^2 + \mathrm{i}yp_y - \frac{\mathrm{i}}{2}p_y^2 t\Big]$$
$$\times C\sqrt{\frac{2\pi}{\kappa F}}\exp\Big(-\frac{1}{3}\frac{F_0}{F}\Big)\Big\{\frac{\pi}{[x - \kappa^2/(2F)]^{1/2}}\Big(\frac{F}{2}\Big)^{1/2}\Big\}^{1/2}$$
$$\times\exp\Big[-\mathrm{i}\frac{\pi}{4} + \mathrm{i}\frac{\kappa^2}{2}t + \mathrm{i}\frac{2}{3}\sqrt{2F}\Big(x - \frac{\kappa^2}{2F}\Big)^{3/2}\Big]. \tag{27}$$

In view of (20) and (27) we find the momentum distribution of the probability of the process per unit time:

$$\frac{\mathrm{d}w}{\mathrm{d}p_y} = \frac{C^2}{4\pi\kappa}\exp\Big(-\frac{2}{3}\frac{F_0}{F}\Big)\exp\Big[-\frac{F_0}{F}\Big(\frac{p_y}{\kappa}\Big)^2\Big]. \tag{28}$$

The total probability of ionisation of a two-dimensional quantum dot by a constant electric field per unit time is obtained by integrating (28) with respect to p_y:

$$w = \frac{\sqrt{\pi}}{4}\kappa^2\Big(\frac{U_0 - |E_0|}{U_0}\Big)\Big[\frac{I_1(\kappa a)K_0(\kappa a) + I_0(\kappa a)K_1(\kappa a)}{K_1(\kappa a)}\Big]^2$$
$$\times\Big(\frac{F}{F_0}\Big)^{1/2}\exp\Big(-\frac{2F_0}{3F}\Big). \tag{29}$$

A characteristic feature of expression (29) is that the pre-exponential factor is proportional to $\sqrt{F/F_0}$. For comparison, in the case of a one-dimensional quantum well, this factor does not depend on the electric field strength, and in the formula for the probability of electron ejection from the ground state in a three-dimensional potential well this factor is proportional to the field strength [3, 4].

8. Ionisation of a quantum dot by a superposition of constant and alternating electric fields in the adiabatic approximation

If the process under consideration is formed at such a time that the phase of the wave has no time to change significantly, then the probability of this process in a periodic field is related to the probability in a constant field by the equation [4, 11, 12, 18]

$$w = \frac{2}{\pi}\int_0^{\pi/2}\mathrm{d}\psi\, w_{\mathrm{stat}}(F), \tag{30}$$

where $w_{\mathrm{stat}}(F)$ is the probability of the process in a constant field with intensity F. Using this result, we can find the probability of tunnelling ionisation of a quantum dot by an alternating electric field with intensity $F(t) = F\cos(\omega t)$ (ω is the frequency of the wave) when the Keldysh parameter is small compared with unity, i.e., when

$$\gamma = \frac{\kappa\omega}{F} \ll 1. \tag{31}$$

To do this, in (30) we replace F by $F\cos\varphi$, and averaging the resulting expression with account for (29), we obtain

$$w^{\mathrm{adiab}} = \sqrt{\frac{3}{\pi}}\Big(\frac{F}{F_0}\Big)^{1/2} w_{\mathrm{stat}}(F) = \frac{C^2\sqrt{3}}{4\pi}\frac{F}{F_0}\exp\Big(-\frac{2}{3}\frac{F_0}{F}\Big), \tag{32}$$

where the parameter C is determined by formula (16) and depends on the properties of a quantum dot.

Note that due to averaging the functional dependence of the pre-exponential factor on the field changed: in a variable field the ionisation probability is always less than in the constant field with the same intensity F. This is explained by the fact that in an alternating field only for a small part of the period when the field strength is close to the peak value, the tunnelling of a quantum dot is most effective.

Using expression (29), we obtain in the approximation of low frequencies ($\gamma \ll 1$) the ionisation probability of a quantum dot by the field, which is a superposition of constant and alternating electric fields directed along the x axis

$$F(t) = F_1 + F_2\cos(\omega t). \quad (33)$$

If conditions

$$F_1 > F_2, \quad \frac{F_0 F_2}{(F_1+F_2)^2} >> 1, \quad \frac{\kappa\omega}{F_1} << 1, \quad (34)$$

are met, then from equations (29), (30) for the probability of the process in the adiabatic approximation, we obtain the expression:

$$w = \frac{\kappa^2}{4}\sqrt{3}\left(\frac{U_0-|E_0|}{U_0}\right)\exp\left(-\frac{2}{3}\frac{F_0}{F_1+F_2}\right)$$
$$\times\left[\frac{I_1(\kappa a)K_0(\kappa a)+I_0(\kappa a)K_1(\kappa a)}{K_1(\kappa a)}\right]^2\frac{F_1+F_2}{F_0}\left(\frac{F_1+F_2}{F_2}\right)^{1/2}. \quad (35)$$

9. Probability of ionisation of the bound system under the action of parallel constant and alternating electric fields

To determine the probability of ionisation by the imaginary-time method, it is needed to calculate the imaginary part of the truncated action [4, 6, 19, 24, 25]. Using this method we calculate the probability of ionisation of a system with short-range forces by an external electric field, which is a superposition of a constant field with intensity F_1 and an alternating field with amplitude F_2 and frequency ω. The time-dependent vector potential of the field, as in [12], is written in the form

$$\boldsymbol{A}(t) = \left(-\frac{F_2\sin(\omega t)}{\omega} - F_1 t, 0, 0\right). \quad (36)$$

For definiteness, we first consider the two-dimensional case. It may be, for example, a two-dimensional quantum dot. The initial moment of time for sub-barrier motion t_0 coincides with the saddle point and is found from the equation

$$\boldsymbol{\pi}^2(t_0) \equiv [\boldsymbol{p} - \boldsymbol{A}(t_0)]^2$$
$$= \left(p_x + \frac{F_2\sin(\omega t_0)}{\omega} + F_1 t\right)^2 + p_y^2 = -\kappa^2. \quad (37)$$

The extremal sub-barrier classical trajectory that minimises the imaginary part of the truncated action $\tilde{S}$ corresponds to the situation when the particle comes from under the barrier at time $t = 0$ with zero velocity, i.e., $\dot{x}(0) = 0$. The trajectory $x = x(t)$ and truncated action $\tilde{S}$ are functions of p_x and p_y. The case $p_x = p_y = 0$ corresponds to a trajectory that minimises the value of $\mathrm{Im}\tilde{S}$, and the maximum rate of ionisation corresponds to this trajectory. Assuming also that at time t_0 the coordinate of the electron is equal to zero, we obtain that the effective barrier width is $\ell = x(0)$. Thus, the extremal classical trajectory is found from the equations

$$\ddot{x} = F_0 + F\cos(\omega t), \quad x(t_0) = 0, \quad \dot{x}(0) = 0, \quad (38)$$

and time t_0 – from the equation

$$\left(\frac{F_2\sin(\omega t_0)}{\omega} + F_1 t_0\right)^2 = -\kappa^2. \quad (39)$$

To describe illustratively the sub-barrier motion, it is convenient to pass to the real time $\tau = -\mathrm{i}t$. Then, the equation of the extremal trajectory and the equation for determining the point in time τ_0 take the form:

$$x(\tau) = -\frac{1}{2}F_1(\tau^2 - \tau_0^2) - \frac{F_2}{\omega^2}[\cosh(\omega\tau) - \cosh(\omega\tau_0)], \quad (40)$$

$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = F_1\tau + \frac{F_2}{\omega}\sinh(\omega\tau), \quad (41)$$

$$F_1\tau_0 + \frac{F_2}{\omega}\sinh(\omega\tau_0) = \kappa, \quad (42)$$

and the dependence of the effective barrier width on the characteristic parameters of the problem is given by

$$\ell = \frac{1}{2}F_1\tau_0^2 + \frac{F_2}{\omega^2}[\cosh(\omega\tau_0) - 1]. \quad (43)$$

Up to the pre-exponential factor P the probability of ionisation per unit time within the framework of the imaginary-time method is given by [4, 6, 19, 24, 25]

$$W = P\exp[-2\,\mathrm{Im}\,\tilde{S}(t_0, 0)], \quad (44)$$

where the truncated action

$$\tilde{S}(t_0, 0) = \frac{1}{2}\int_0^{t_0}\left\{\kappa^2 + p_y^2 + \left[p_x + F_1 t + \frac{F_2}{\omega}\sin(\omega t)\right]^2\right\}\mathrm{d}t. \quad (45)$$

Note that expressions (44) and (45) in the literature are also called Landau–Dykhne formulas [20, 26].

Calculating the imaginary part of the truncated action for the extremal trajectory (40)–(42), we obtain the expression for the ionisation rate:

$$W_0 = P_0\exp[-g(F_2, F_1, \kappa, \tau_0, \omega)], \quad (46)$$

where

$$g = \kappa^2\tau_0 - \frac{F_1^2\tau_0^3}{3} + \left(\frac{F_2}{\omega}\right)^2\frac{1}{\omega}\left[\frac{\omega\tau_0}{2} - \frac{\sinh(2\omega\tau_0)}{4}\right]$$
$$+ \frac{2F_1F_2}{\omega^2}[\sinh(\omega\tau_0) - \omega\tau_0\cosh(\omega\tau_0)]. \quad (47)$$

To find the spectrum of electron pulses we must take into account the contribution of classical trajectories that are close to the extremal trajectories and calculate the imaginary part of the truncated action up to quadratic terms by the deviation of such trajectories from the extremal ones. From equation (37) for $p^2 \ll \kappa^2$ we find

$$\varphi_0 \equiv \omega t_0 = \mathrm{i}\omega\tau_0 = \mathrm{i}\left[u_0 + \frac{1}{2}\left(\frac{\gamma p_x}{\kappa}\right)^2\frac{\sinh u_0}{(\cosh u_0 + F_1/F_2)^3}\right.$$
$$\left.+ \frac{\gamma}{2}\left(\frac{p_y}{\kappa}\right)^2\frac{1}{(\cosh u_0 + F_1/F_2)} + \mathrm{i}\gamma\frac{p_x}{\kappa}\frac{1}{(\cosh u_0 + F_1/F_2)}\right], \quad (48)$$

where u_0 is the solution of the equation

$$\sinh u_0 + \frac{F_1}{F_2}u_0 = \gamma. \quad (49)$$

Using (48), we expand (45) in powers of p/κ to the second order inclusive and select the imaginary part of the truncated action. As a result, we find the momentum distribution of the ionisation probability of a two-dimensional quantum dot by the field (36):

$$dW = P\exp[-g(F_2, F_1, \kappa, \tau_0, \omega) - \omega^{-1}(\alpha p_x^2 + u_0 p_y^2)]\,\frac{dp_x dp_y}{(2\pi)^2}, \quad (50)$$

where

$$\alpha = u_0 - \frac{2\gamma}{\cosh u_0 + F_1/F_2} + \frac{\gamma\cosh u_0}{(\cosh u_0 + F_1/F_2)^2} + \frac{\gamma u_0}{2(\cosh u_0 + F_1/F_2)^3}\frac{F_1}{F_2} + \frac{\gamma}{(\cosh u_0 + F_1/F_2)^2}\frac{F_1}{F_2} + \frac{u_0\sinh u_0(1-\gamma-\sinh u_0)}{(\cosh u_0 + F_1/F_2)^3}\frac{F_1}{F_2}. \quad (51)$$

In the limiting case $p_y = 0$, formulas (50), (51) describe, in the semiclassical approximation, the momentum distribution of the probability of ionisation of a one-dimensional quantum well. Note that with the exponential accuracy, the total probability of ionisation of a quantum well per unit time is also determined by (46), (47). This result in the limiting case (34) agrees with the corresponding result of [21], where it was obtained in the adiabatic approximation.

10. Discussion and conclusions

The results can be used to describe the ionisation process of real physical systems, which include not only systems with short-range forces (for example, quantum dot, quantum well and negative ion), but also a system of charged particles with long-range Coulomb forces (ionised atoms).

In the quasi-classical approximation, we have derived an analytical expression for the probability of ionisation of the quantum dot per unit time: formula (29) describes the dependence of the ionisation rate of a quantum dot on its radius, well depth and constant electric field strength.

We have calculated the rate of ionisation of a quantum dot in an alternating electric field with a constant component [Eqn (35)] in the case of relatively low frequencies, when the time of electron motion in the classically forbidden region is small compared with the period of the alternating field. This limiting case is reached in the optical and IR lasers.

We have derived formulas (50), (51), which describe the momentum distribution of the ionisation probability of a quantum dot under the influence of parallel constant and alternating electric fields.

Formulas (50), (51) (as special cases) yield a formula for the differential probability of electron detachment by the field of a linearly polarised electromagnetic wave, and a formula for the differential probability of tunnelling in a constant field, obtained Section 1. For example, when the constant electric field is switched off ($F_1 \to 0$) from (50), (51) we obtain

$$dw = P\exp\left\{-2\frac{|E_0|}{\omega}\left[f(\gamma) + \left(\frac{p_x}{\kappa}\right)^2\left(\operatorname{arsinh}\gamma - \frac{\gamma}{\sqrt{1+\gamma^2}}\right) + \left(\frac{p_y}{\kappa}\right)^2\operatorname{arsinh}\gamma\right]\right\}\frac{dp_x dp_y}{(2\pi)^2}, \quad (52)$$

where the Keldysh function [1] has the form

$$f(\gamma) = \left(1 + \frac{1}{2\gamma^2}\right)\operatorname{arsinh}\gamma - \frac{\sqrt{\gamma^2+1}}{\gamma}, \quad (53)$$

and the coefficients in the exponent have the same form as for the partial probability of the process in the field of a linearly polarised wave (see also [6, 10, 19]). Figure 1 shows the momentum distribution of the probability of process on the Keldysh parameter for different ratios of the constant field strength to the amplitude of the alternating field.

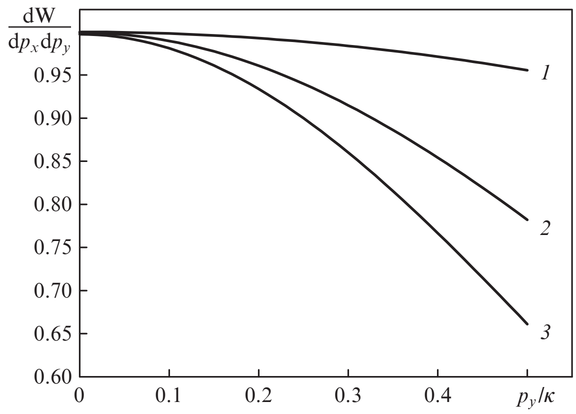

Figure 1. Momentum distribution of the probability of ionisation at the Keldysh parameter $\gamma = 1$ and $p_x/\kappa = 0.1$ for the ratio of the constant electric field strength to the alternating field amplitude $F_1/F_2 = 0.1$ (*1*), 1 (*2*) and 10 (*3*).

Formulas (46) (47), determining with an exponential accuracy the dependence of the characteristic parameters of the problem of the ionisation rate of a quantum dot and a quantum well per unit time, have been obtained in this work by the imaginary-time method. These formulas coincide with expressions (20), (21) in [15], where they were found by the method of complex classical Landau trajectories.

Dependences of $\lambda = g(F_2, F_1, \kappa, \tau_0, \omega)/g(F_2, F_1 = 0, \kappa, \tau_0, \omega)$ on the Keldysh parameter for different ratios of the constant field strength to the amplitude of the alternating field are shown in Fig. 2. One can see that this parameter takes values

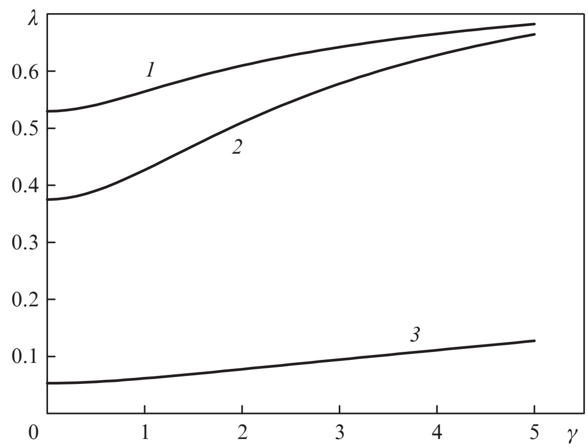

Figure 2. Dependence of the quantity λ on the Keldysh parameter for the ratio of the constant electric field strength to the alternating field amplitude $F_1/F_2 = 10$ (*1*), 1 (*2*) and 0.1 (*3*).

that do not exceed unity, which corresponds to an increase in the ionisation probability of a bound system by an alternating electric field under the action of a constant electric field.

Thus, application of even a relatively weak constant field changes significantly both the total probability of the process per unit time and the momentum distribution of the probability of the process.

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