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A self-consistent regime of generation of terahertz radiation by an optical pulse with a tilted intensity front

A.N. Bugay, S.V. Sazonov, A.Yu. Shashkov

Abstract. **We derived a self-consistent system of nonlinear wave equations describing the terahertz generation in dielectric uniaxial crystals by optical pulsed radiation with a tilted wavefront. The numerical analysis of the system of equations showed that the generation of a broadband one-period terahertz signal is accompanied by a red shift of the carrier frequency of the optical pulse, the magnitude of the shift being proportional to the pulse intensity. The generation efficiency with respect to energy reached a maximum at a certain distance of propagation in the crystal, after which the efficiency decreased. A satisfactory agreement was obtained between theoretical calculations and experimental data of other investigations.**

Keywords: *terahertz radiation, optical rectification, tilted wavefronts.*

1. Introduction

Of great interest today are the studies devoted to the techniques for generating terahertz radiation. Sensitivity of vibrational, rotational, vibrational–rotational and quantum tunnelling transitions to terahertz radiation determines the prospects of the development of terahertz spectroscopy. In addition, this type of radiation finds nowadays use in image processing, security systems, astronomy, biology, and many other fields [1–4].

The optical method for generating broadband terahertz radiation in quadratic nonlinear media, based on optical rectification, is one of the most effective [5]. In this case, the generation is of Cherenkov nature [6].

Interest in the Cherenkov mechanism is caused by the possibility of using femtosecond laser pulses to generate terahertz radiation. Because of a large spectral width of the femtosecond pulse, it contains Fourier components at the frequency difference of which a broadband terahertz signal is generated, comprising approximately one period of electromagnetic oscillations.

The idea that a terahertz pulse with duration of the order of one period of electromagnetic oscillations can be generated by using a femtosecond optical pulse was put forward for the first time in theoretical paper [7]. After a while, the Cherenkov pulse was recorded experimentally [8, 9].

The phase-matching condition for the terahertz generation has the form

$$\cos\theta = \frac{v_{\rm ph}}{v_{\rm g}}, \tag{1}$$

where $v_{\rm ph}$ is the phase velocity of the generated terahertz signal; $v_{\rm g}$ is the group velocity of the input optical pulse; and θ is the angle between the directions of propagation of these pulses.

When femtosecond and generated terahertz pulses propagate noncollinearly, the generation efficiency with respect to energy is low and barely reaches $\sim 10^{-6}$.

Using optical pulsed radiation with a tilted wavefront, Hebling et al. [10] and Kitaeva [11] managed to increase significantly (by two-three orders of magnitude) the generation efficiency for media with velocities, which are markedly different in optical and terahertz regions. In this case, θ means the angle of the front tilt of an optical beam (Fig. 1).

It should be noted that in recent years the method of front-tilting is quite popular. The method is efficiently used in dielectric crystals, where the ohmic loss can be neglected and where the group velocity in the optical range is much greater than the phase velocity corresponding to terahertz frequen-

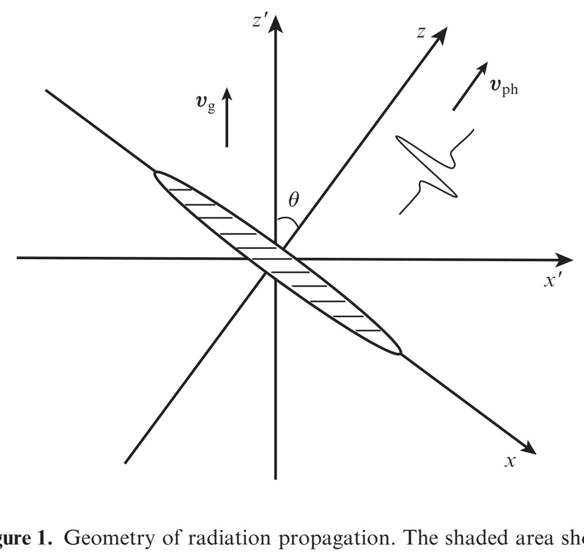

Figure 1. Geometry of radiation propagation. The shaded area shows tilted-wavefront optical pulsed radiation propagating along the axis z'. The broadband terahertz signal is generated in the direction of the z axis.

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cies. A typical representative of such media is a uniaxial crystal of lithium niobate ($LiNbO_3$).

Many studies have been devoted to the techniques aimed at improving the generation efficiency and reducing the wavefront distortion of the pump pulse that is transferred to the generated terahertz signal [12–14]. Of importance are such parameters as the crystal length and temperature, the length of the input optical pump pulse and the beam diameter. Due to focusing, the electric field of the terahertz signal with the energy of the order of a microjoule can reach values of 1–100 V cm^{-1}, and the beam diameter at the waist can reach the values of 1–10 μm [13–15]. In this case, the pump pulse duration is 500 fs [14, 16], and the generation efficiency with respect to energy reaches 0.25% in the experiment [16]. The conversion efficiency of optical photons into terahertz photons is almost 100% (i.e., in a nonlinear medium each optical photon decays into two photons, the frequency of one of which lies in the terahertz range), and the energy of terahertz pulses is 50 μJ [17].

Even in pioneering works of the 1970s, the theoretical description of the terahertz radiation generation through the mechanism of optical rectification was carried out in the given field approximation. This situation has changed only recently [12–14, 18].

At the same time, in the case of sufficiently efficient terahertz generation, the reverse effect of the generated pulse on the initial optical signal comes to the fore. First of all, this effect leads to a red shift of the optical pulse carrier frequency (or the spectrum as a whole). This shift is easily explained by the decay of the photons of the input optical pulse in a quadratic nonlinear medium (see, for example, [19]). The corresponding self-consistent problem was solved theoretically in [20–23], where the optical pulse fronts were assumed untilted ($\theta = 0$).

Terahertz generation by femtosecond pulsed radiation with a tilted wavefront was registered experimentally in a 2-mm-thick $LiNbO_3$ crystal [24]. It is important to emphasise that the value of the red shift, as evident from the experimental data, is proportional to the intensity of the optical pulse.

To our knowledge, there are currently no theoretical works, which consider self-consistently the dynamics of the optical pulsed radiation with a tilted wavefront and of the generated terahertz signal.

The present paper is devoted to a theoretical study of the self-consistent generation regime of terahertz signals in dielectric uniaxial crystals, using femtosecond pulsed radiation with a tilted wavefront.

2. Basic equations

Let tilted-wavefront laser radiation at the input be polarised in the plane of the extraordinary wave and propagate along the axis z', which is perpendicular to the optical axis x' of the uniaxial crystal. In such a geometry, only the extraordinary wave exists in the crystal. The generated terahertz signal in this case propagates along the axis z, and its electric field $\boldsymbol{E}_{\mathrm{t}}$, as the optical pulse field $\boldsymbol{E}_\omega$, propagates along the axis x (Fig. 1). The primed and unprimed coordinates are related with each other by the rotation conversion

$$z' = z\cos\theta + x\sin\theta, \qquad x' = x\cos\theta - z\sin\theta. \tag{2}$$

We represent the polarisation response P as a sum of its linear and nonlinear parts:

$$P = \int_0^\infty \chi_1(\tau)E(t-\tau)\mathrm{d}\tau + \int_0^\infty E(t-\tau_2)\mathrm{d}\tau_2 \times \int_0^\infty \chi_2(\tau_1,\tau_2)E(t-\tau_1)\mathrm{d}\tau_1, \tag{3}$$

where $E = E_\omega + E_{\mathrm{t}}$ is the total electric field; $\chi_1(\tau)$ and $\chi_2(\tau_1,\tau_2)$ are the temporary susceptibilities of the first and second orders, respectively. Here, we have taken into account only the main, quadratic, nonlinearity. For the optical component we will have the expression

$$E_\omega = \psi(z',x',t)\exp[\mathrm{i}(\omega t - kz')] + \mathrm{c.\,c.}$$

$$= \psi(z,t)\exp[\mathrm{i}(\omega t - k_z z - k_x x)] + \mathrm{c.\,c.}, \tag{4}$$

where $k_z = k\cos\theta$ and $k_x = k\sin\theta$ are components of the wave vector $\boldsymbol{k}$; and ψ is a slowly varying envelope.

In this case, the polarisation response can be expressed as the sum of two components – the optical P_ω (at the carrier frequency ω) and terahertz P_{t} (without the carrier frequency):

$$P_\omega = \exp[\mathrm{i}(\omega t - kz')]\int_0^\infty \chi_1(\tau)\exp(-\mathrm{i}\omega\tau)\psi(t-\tau)\mathrm{d}\tau$$

$$+ \exp[\mathrm{i}(\omega t - kz')]\Big[\int_0^\infty E_{\mathrm{t}}(t-\tau_1)\mathrm{d}\tau_1\int_0^\infty \chi_2(\tau_1,\tau_2)$$

$$\times\exp(-\mathrm{i}\omega\tau_2)\psi(t-\tau_2)\mathrm{d}\tau_2 + \int_0^\infty E_{\mathrm{t}}(t-\tau_2)\mathrm{d}\tau_2$$

$$\times\int_0^\infty \chi_2(\tau_1,\tau_2)\exp(-\mathrm{i}\omega\tau_1)\psi(t-\tau_1)\mathrm{d}\tau_1\Big] + \mathrm{c.c.},$$

$$P_{\mathrm{t}} = \int_0^\infty \chi_1(\tau)E_{\mathrm{t}}(t-\tau)\mathrm{d}\tau + \int_0^\infty E_{\mathrm{t}}(t-\tau_1)\mathrm{d}\tau_1$$

$$\times\int_0^\infty \chi_2(\tau_1,\tau_2)E_{\mathrm{t}}(t-\tau_2)\mathrm{d}\tau_2 + \int_0^\infty E_{\mathrm{t}}(t-\tau_2)\mathrm{d}\tau_2$$

$$\times\int_0^\infty \chi_2(\tau_1,\tau_2)E_{\mathrm{t}}(t-\tau_1)\mathrm{d}\tau_1 + \int_0^\infty \exp(-\mathrm{i}\omega\tau_1)\psi(t-\tau_1)\mathrm{d}\tau_1$$

$$\times\int_0^\infty \chi_2(\tau_1,\tau_2)\exp(\mathrm{i}\omega\tau_2)\psi(t-\tau_2)\mathrm{d}\tau_2$$

$$+\int_0^\infty \exp(-\mathrm{i}\omega\tau_2)\psi(t-\tau_2)\mathrm{d}\tau_2\int_0^\infty \chi_2(\tau_1,\tau_2)\exp(\mathrm{i}\omega\tau_1)\psi(t-\tau_1)\mathrm{d}\tau_1.$$

Here we have neglected the terms that oscillate at the doubled carrier frequency of the optical pulse, assuming that the phase-matching conditions for second harmonic generation are not met.

Below we assume the variance of the optical and terahertz components to be relatively weak; therefore, in the linear parts of both responses we will use the expansions

$$\psi(t-\tau) \approx \psi(t) - \tau\frac{\partial\psi}{\partial t} + \frac{\tau^2}{2}\frac{\partial^2\psi}{\partial t^2},$$

$$E_{\rm t}(t-\tau) \approx E_{\rm t}(t) - \tau\frac{\partial E_{\rm t}}{\partial t} + \frac{\tau^2}{2}\frac{\partial^2 E_{\rm t}}{\partial t^2},$$

and in nonlinear parts we will set

$$\psi(t-\tau) \approx \psi(t), \quad E_{\rm t}(t-\tau) \approx E_{\rm t}(t).$$

Then,

$$P_\omega = \exp[{\rm i}(\omega t - kz')]\bigg[\chi^{(1)}_\omega\psi - {\rm i}\frac{\partial\chi^{(1)}_\omega}{\partial\omega}\frac{\partial\psi}{\partial t} - \frac{1}{2}\frac{\partial^2\chi^{(1)}_\omega}{\partial\omega^2}\frac{\partial^2\psi}{\partial t^2} + 2\chi^{(2)}_{\omega 0}E_{\rm t}\psi\bigg] + {\rm c.c.} \quad (5)$$

$$P_{\rm t} = \chi^{(1)}_0 E_{\rm t} - {\rm i}\frac{\partial\chi^{(1)}_\omega}{\partial\omega}\bigg|_{\omega=0}\frac{\partial E_{\rm t}}{\partial t} - \frac{1}{2}\frac{\partial^2\chi^{(1)}_\omega}{\partial\omega^2}\bigg|_{\omega=0}\frac{\partial^2 E_{\rm t}}{\partial t^2} + \chi^{(2)}_{00}E_{\rm t}^2 + 2\chi^{(2)}_{\omega-\omega}|\psi|^2, \quad (6)$$

where the linear and nonlinear frequency susceptibilities are determined by the formulas

$$\chi^{(1)}_\omega = \int_0^\infty \chi_1(\tau)\exp(-{\rm i}\omega\tau)\,{\rm d}\tau,$$

$$\chi^{(2)}_{\omega_1\omega_2} = \int_0^\infty \exp(-{\rm i}\omega_2\tau_2)\,{\rm d}\tau_2\int_0^\infty \chi_2(\tau_1,\tau_2)\exp(-{\rm i}\omega_1\tau_1)\,{\rm d}\tau_1.$$

In expression (6) the linear susceptibility $\chi^{(1)}_0$ and the component of the nonlinear susceptibility $\chi^{(2)}_{00}$ are formally considered at zero frequency, which actually corresponds to the terahertz range.

Guided by the fact that the spectrum of terahertz radiation can lie both in the transparency region and in the absorption region, for the corresponding linear complex susceptibility we write expression [25]

$$\chi^{(1)}_0 = \chi_{\rm e} + \chi_{\rm i} + \frac{\omega_{\rm pi}^2/(4\pi)}{\omega_{\rm i}^2 + 2{\rm i}\Gamma\omega - \omega^2}, \quad (7)$$

where $\chi_{\rm e}$ and $\chi_{\rm i}$ are the instantaneous parts of the electron and ion susceptibilities, respectively; $\omega_{\rm pi}$ is the ion plasma frequency, which is proportional to the density of ions with the eigenfrequency $\omega_{\rm i}$ of resonance absorption, corresponding to optical phonons; and Γ is a characteristic decay constant. The first two terms in this expression take into account the instantaneous contributions of optoelectronic transitions and nonresonant phonon modes with frequencies far from $\omega_{\rm i}$.

In (7), when $\Gamma \ll \omega_{\rm i}$, we find that

$$\chi^{(1)}_0 = \chi_{\rm e} + \chi_{\rm i} + \frac{\omega_{\rm pi}^2}{4\pi\omega_{\rm i}^2},$$
$${\rm i}\frac{\partial\chi^{(1)}_\omega}{\partial\omega}\bigg|_{\omega=0} = \Gamma\frac{\omega_{\rm pi}^2}{2\pi\omega_{\rm i}^4}, \quad \frac{\partial^2\chi^{(1)}_\omega}{\partial\omega^2}\bigg|_{\omega=0} = \frac{\omega_{\rm pi}^2}{2\pi\omega_{\rm i}^4}. \quad (8)$$

Thus, the second term on the right side of (6) describes the absorption of terahertz waves, and the third term describes their dispersion.

As for the carrier frequency of the optical pulse, we assume that it is in the transparency window of the crystal, where we can neglect absorption. In this case, for the susceptibility $\chi^{(1)}_\omega$ in the optical frequency range we have an expansion of the form [26]

$$\chi^{(1)}_\omega = \chi\bigg[1 + a\frac{\omega^2}{\omega_{\rm e}^2} + b\frac{\omega^4}{\omega_{\rm e}^4} + \ldots - \left(\frac{\omega_{\rm pi}}{\omega_{\rm pe}}\right)^2 a'\frac{\omega_{\rm e}^2}{\omega^2} - \ldots\bigg]. \quad (9)$$

Here, χ is the instantaneous part of optical susceptibility; $\omega_{\rm pe}$ is the electron plasma frequency; $\omega_{\rm e}$ is the characteristic frequency of electron-optical transitions; a, b,…, a'.. are the empirical constants of the order of unity. The group of terms with the sign '+' on the right-hand side of (9) describes the contribution of the electron response to $\chi^{(1)}_\omega$, and with the sign '–' describes the contribution of the ion response.

Now, taking into account the fact that the Laplace operator is invariant with respect to transformation (2), we write the wave equations for the optical and terahertz components in the form

$$\frac{\partial^2 E_\omega}{\partial z'^2} + \frac{\partial^2 E_\omega}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2 E_\omega}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 P_\omega}{\partial t^2}, \quad (10)$$

$$\frac{\partial^2 E_{\rm t}}{\partial z^2} + \frac{\partial^2 E_{\rm t}}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_{\rm t}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 P_{\rm t}}{\partial t^2}, \quad (11)$$

where c is the speed of light in vacuum.

After substituting (5) into (10), for the optical components we can use the standard slowly varying envelope approximation (SVEA) [5]. Then, moving from the primed coordinates to the unprimed coordinates, according to (2), we obtain

$${\rm i}\left(\cos\theta\frac{\partial\psi}{\partial z} - \sin\theta\frac{\partial\psi}{\partial x}\right) = \beta E_{\rm t}\psi - k_{2\theta}\frac{\partial^2\psi}{\partial T^2} + \frac{c}{2n_\omega\omega\cos^2\theta}\left(\frac{\partial^2\psi}{\partial x^2} - \frac{2\sin\theta}{v_{\rm g}}\frac{\partial^2\psi}{\partial x\partial T}\right), \quad (12)$$

where $T = t - z/(v_{\rm g}\cos\theta)$; $v_{\rm g}^{-1} = \partial k/\partial\omega$; $k = n_\omega\omega/c$; $n_\omega = (1 + 4\pi\chi^{(1)}_\omega)^{1/2}$ is the refractive index at the frequency ω; $\beta = 4\pi\omega\chi^{(2)}_{\omega 0}/(cn_\omega)$;

$$k_{2\theta} = k_2 - \frac{c}{\omega n_\omega v_{\rm g}^2}\tan^2\theta, \quad (13)$$

and $k_2 = \partial^2 k/\partial\omega^2$ is the group velocity dispersion (GVD) coefficient.

As can be seen from (13) and (12), the wavefront tilt effectively changes the GVD, additively adding to k_2 the term depending on θ and corresponding to the anomalous GVD. This situation was mentioned in [27].

Using expression (9) the dispersion dependence of the refractive index on the frequency can be represented in the form

$$n_\omega = n\left(1 + \frac{A}{3}\omega^2 + \frac{B}{5}\omega^4 - \frac{D}{\omega^2}\right),$$

$$\frac{1}{v_{\rm g}} = \frac{n}{c}\left(1 + A\omega^2 + B\omega^4 + \frac{D}{\omega^2}\right), \quad (14)$$

$$k_2 = \frac{\partial^2 k}{\partial \omega^2} = 2\frac{n}{c}\left(A\omega + 2B\omega^3 - \frac{D}{\omega^3}\right),$$

where $n = \sqrt{1 + 4\pi\chi}$ is the instantaneous part of the optical refractive index;

$$A = \frac{6\pi\chi a}{n^2\omega_e^2}; \quad B = \frac{10\pi\chi b}{n^2\omega_e^4}; \quad D = \frac{2\pi\chi a'}{n^2\omega_e^2}\left(\frac{\omega_{pi}}{\omega_{pe}}\right)^2.$$

We now turn to the transformation of equation (12) for the terahertz pulse. Substituting (6) and (8) into (11), we obtain

$$\frac{\partial^2 E_t}{\partial z^2} - \frac{n_t^2}{c^2}\frac{\partial^2 E_t}{\partial t^2} = -\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\left(\Gamma\frac{\omega_{pi}^2}{2\pi\omega_i^4}\frac{\partial E_t}{\partial t} + \frac{\omega_{pi}^2}{4\pi\omega_i^4}\frac{\partial^2 E_t}{\partial t^2} - \chi_{00}^{(2)}E_t^2 - 2\chi_{\omega-\omega}^{(2)}|\psi|^2\right) - \frac{\partial^2 E_t}{\partial x^2}, \quad (15)$$

where $n_t = \sqrt{1 - 4\pi\chi_0^{(1)}}$ is the instantaneous part of the terahertz refractive index.

The right-hand side of (15) contains relatively small terms associated with attenuation, dispersion, nonlinearity, and dependence of E_t on the transverse coordinate x. Therefore, we can use the slowly varying profile approximation (SVPA) [28] (not to be confused with the SVEA). To do this, we represent the field of the terahertz pulse in the form $E_t = E_t(T, \zeta)$, where $\zeta = \mu z$. The coefficient $\mu \ll 1$ makes it possible to take into account the small terms on the right-hand side of (15). Then, neglecting the term proportional to μ^2, we find

$$\frac{\partial^2}{\partial z^2} \approx \frac{1}{v_g^2\cos^2\theta}\frac{\partial^2}{\partial T^2} - \frac{2\mu}{v_g\cos\theta}\frac{\partial^2}{\partial\zeta\partial T}, \quad \frac{\partial^2}{\partial t^2} = \frac{\partial^2}{\partial T^2}.$$

Accordingly, after integrating (15) in T at zero values of the field and its derivatives at infinity, we obtain

$$\frac{\partial E_t}{\partial z} + \delta\frac{\partial E_t}{\partial T} - \eta\frac{\partial^2 E_t}{\partial T^2} - \sigma\frac{\partial^3 E_t}{\partial T^3} + \beta_t\frac{\partial(|\psi|^2)}{\partial T} = \frac{v_g\cos\theta}{2}\frac{\partial^2}{\partial x^2}\int_{-\infty}^{T} E_t \mathrm{d}T', \quad (16)$$

where

$$\delta = \frac{v_g\cos\theta}{2}\left(\frac{n_t^2}{c^2} - \frac{1}{v_g^2\cos^2\theta}\right); \quad \eta = \Gamma\frac{\omega_{pi}^2 v_g\cos\theta}{c^2\omega_i^4};$$

$$\sigma = \frac{\omega_{pi}^2 v_g\cos\theta}{2c^2\omega_i^4}; \quad \beta_t = \frac{4\pi\chi_{\omega-\omega}^{(2)}v_g\cos\theta}{c^2}.$$

In deriving expression (16) we have neglected the eigen-nonlinearity of the terahertz component proportional to E_t^2, because usually $E_t^2 \ll |\psi|^2$ (i.e., the intensity of the optical pulse is much greater than the intensity of the terahertz signal) [20, 21, 23].

We emphasise that unlike the optical component, we did not apply the SVEA to the terahertz component of radiation, because the terahertz pulse can consist of an arbitrarily small number of oscillations (up to one). In (12) and (16) ψ is the envelope of the optical pulse, and E_t is the electric field of the terahertz signal.

Assuming the detuning δ to be zero in (16), we obtain equality (1), which, in this case, represents the condition of the most efficient tilted wavefront terahertz generation.

3. Numerical analysis

Further theoretical study is based on the analysis of nonlinear systems (12), (16), and condition (1). Using the second expression in (14) and setting $v_{ph} = c/n_t$, we obtain from (1) the relation

$$\cos\theta = \frac{n}{n_t}\left(1 + A\omega^2 + B\omega^4 + \frac{D}{\omega^2}\right) \quad (17)$$

for the front tilt angle. Consider the generation of a terahertz signal in the $LiNbO_3$ crystal. According to Ref. [29], $A = 4.83\times10^{-33}$ s^2, $B = 1.25\times10^{-64}$ s^4, $D = 4.88\times10^{27}$ s^{-2}, $n = 2.14$, $\chi_{\omega-\omega}^{(2)} \approx \chi_{00}^{(2)}$ 7.9×10^{-8} CGSE units. For the terahertz range we have $n_t = 5.099$, $\omega_{pi}/(2\pi) = 29.76$ THz, $\omega_i/(2\pi) = 7.44$ THz, $\Gamma/(2\pi) = 1.15$ THz [18].

Substitution of these values into (17) gives the tilt angle $\theta \approx 64.4°$. The account for the dispersion in the terahertz range, described by the fourth term on the left-hand side of (16), shows that the tilt angle should be a little larger. As the boundary conditions for $z = 0$, we choose the relations

$$\psi = \psi_0\exp\left[-\ln 2\left(\frac{2T^2}{\tau_p^2} + \frac{x^2}{2R^2}\right)\right], \quad E_t = 0,$$

where ψ_0, τ_p and R are, respectively, the initial amplitude, duration, and transverse radius of the laser beam.

Let the input optical pulse have a centre frequency $\omega = 2.36\times10^{15}$ s^{-1}, which corresponds to the wavelength $\lambda = 800$ nm. The pulse duration is considered to be 100 fs, and its energy – 70 μJ. For the transverse beam diameter of 0.5 mm, the initial intensity of such a pulse is $I_0 \sim 10^{11}$ W cm^{-2}. The tilt angle θ is taken equal to ~64.46°.

Results of the numerical simulation have shown that the maximum optical-to-terahertz energy conversion efficiency is ~10^{-4} for the terahertz signal energy of 11.5 nJ at a propagation distance $z = 0.3$ mm. At a longer distance the optical pump pulse and terahertz signal begin to diverge significantly in space. Although, as seen from Figs 2 and 3, a partial 'drag' of the terahertz signal by the optical pulse does occur, the 'centre of gravity' of the latter increasingly shifts to the region of positive x. At a distance $z = 0.9$ mm the terahertz signal energy reduces by about 1.5 times, accounting for 7 nJ. The optical pulse experiences an increasing dispersion spreading. Clearly visible is the red shift of the optical pulse spectrum (Fig. 4), reaching ~2.5 THz in the centre of the beam cross section and gradually decreasing in the periphery. Note also that the spectrum of the generated terahertz signal gradually shifts to the region of lower frequencies (Fig. 5). This can be explained as follows. For the values of the parameters listed above, the effective group dispersion coefficient is $k_{2\theta} > 0$. Thus, the red shift of the optical pulse frequency is accompanied by an increase in its group velocity. For condition (1) to be valid, it is necessary to increase the phase velocity in the terahertz range, where the dispersion is positive and taken into account by the fourth term on the left side of (16). Therefore, this increase

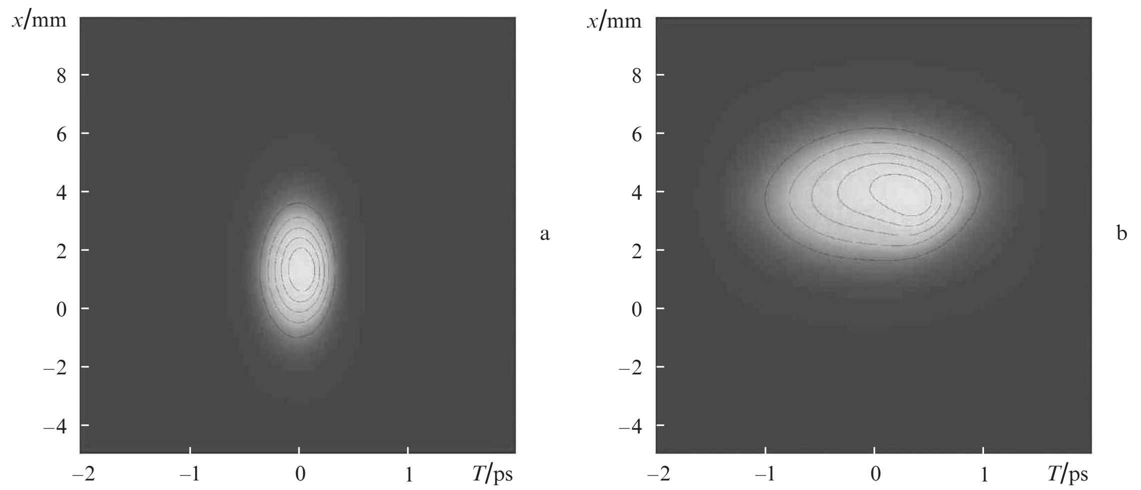

Figure 2. Evolution of the optical pulse envelope at $z = 0.3$ (a) and 0.9 mm (b). Closed isolines correspond to the same absolute values of the field.

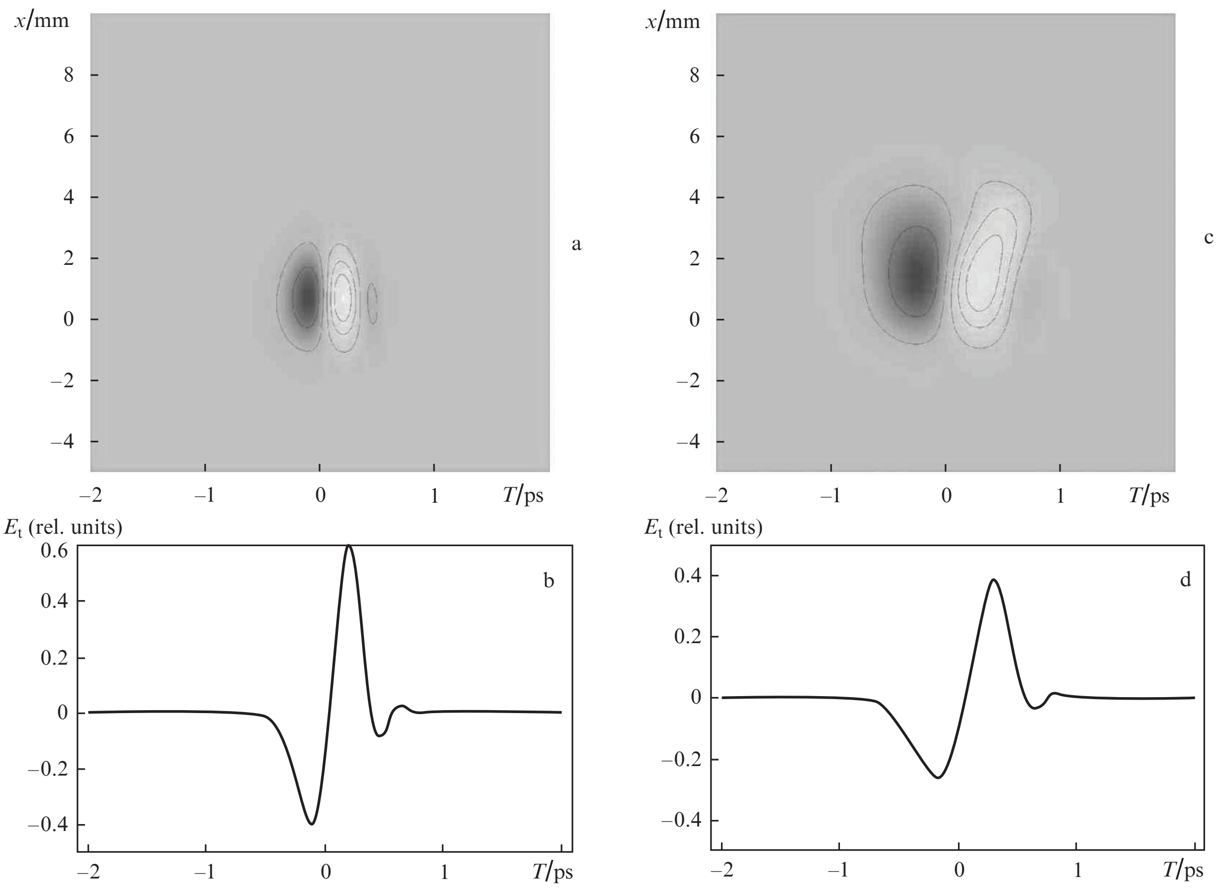

Figure 3. Evolution of the field of a one-period terahertz pulse at $z = 0.3$ (a, b) and 0.9 mm (c, d). Isolines correspond to the same values of the electric field strength E_t. Figures 3b and d show the cross sections of the field distribution E_t at $x = 1$ (a) and 1.8 mm (b).

is possible if the terahertz radiation frequency will synchronously decrease with the red shift of the optical pulse frequency.

In addition, we have compared the data of the corresponding experiments [30] with those obtained from the analysis of a 150-fs input optical pulse with the transverse beam size of 1 mm. The numerical analysis of systems (12) and (16) predicts a somewhat higher generation efficiency than that observed in the experiment. In our case, the efficiency increases monotonically from 0 to 8×10^{-4} with a continuous increase in the optical pulse energy from 0 to 400 μJ. In the same experimental work [30], the maximum efficiency is $\sim5\times10^{-4}$. Apparently, this discrepancy is due to the fact that in the framework of our theoretical analysis we do not take into account the losses caused by the escape of radiation from the crystal.

The red shift of the optical pulse spectrum, as follows from our analysis, increases proportionally to the input inten-

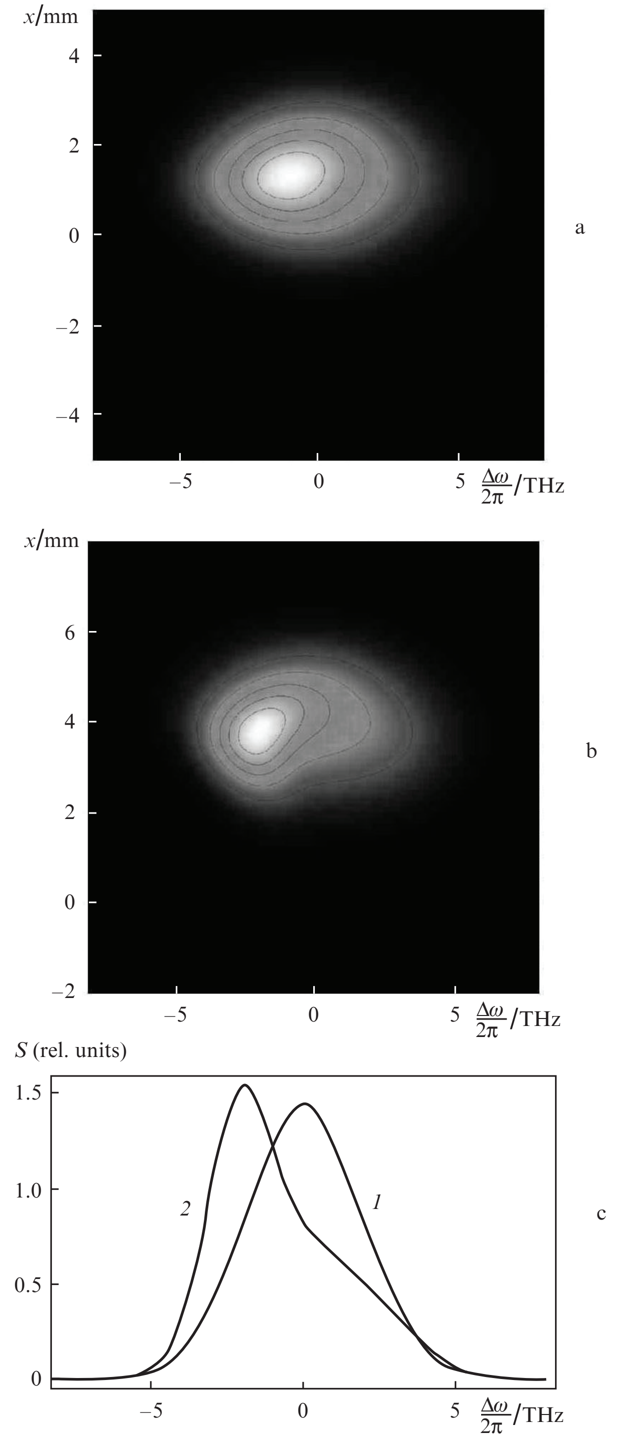

Figure 4. Evolution of the optical pulse spectrum at $z = 0.3$ (a) and 0.9 mm (b). Isolines correspond to the same values of the spectral density S ($\Delta\omega$ is the frequency shift with respect to the carrier frequency of the input pulse). Figure 4c shows the cross sections of the spectral density distribution at $x = 1.5$ mm, $z = 0.3$ mm (*1*) and $x = 4$ mm, $z = 0.9$ mm (*2*).

sity of the same pulse, in accordance with the experimental results [24]. This can be explained with the help of systems (12) and (16), using the formal analogy with quantum mechanics. If in this system we neglect in the zero approximation the dependence of the field on the transverse coordinate x, then equation (12) formally coincides with the quantum mechanical Schrödinger equation, in which as time, coordinate and potential energy use is made of the parameters $z/\cos\theta$, T and $\beta E_{\rm t}$, respectively. On the other hand, according to (16), the depth of the 'potential well' $-\beta E_{\rm t}$ increases with increasing $|\psi|^2$. From the quantum-mechanical analogy it becomes clear that the energy of optical photons trapped by the 'potential well' should decrease. The reduction the more noticeable, the greater the intensity of the optical component itself. On the peripheral (coordinate x) segments of the pulse, its intensity is smaller. As noted above, on these segments the frequency shift to the red is also less than in the centre of the optical pulse.

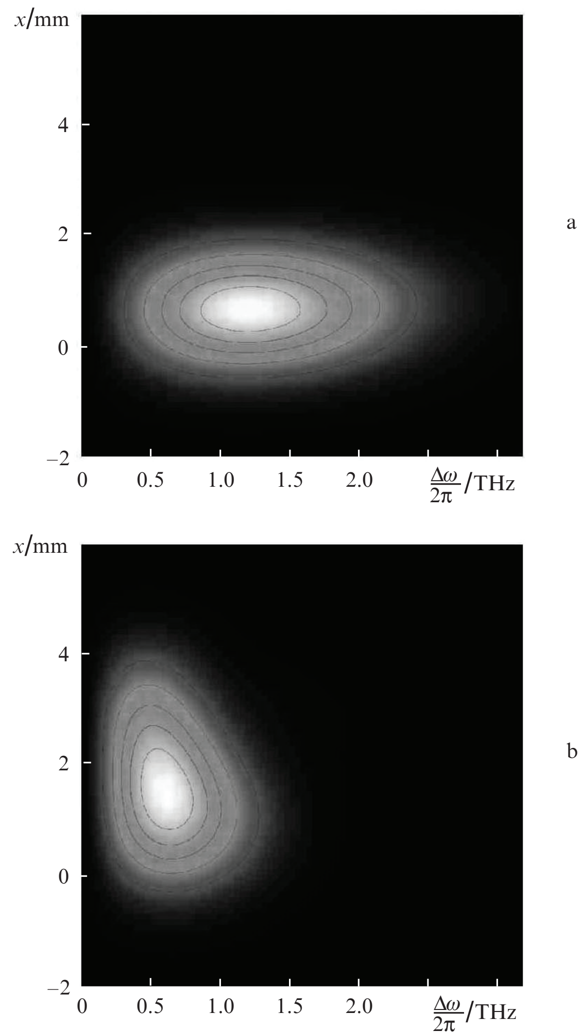

Figure 5. Evolution of the terahertz signal spectrum at $z = 0.3$ (a) and 0.9 mm (b). Isolines correspond to the same values of the spectral density.

4. Conclusions

The system of equations (12) and (16) derived in the present study can describe the self-consistent dynamics of optical tilted wavefront radiation and terahertz generation. This system describes satisfactorily the known experiments. In particular, it suggests that the generated terahertz signal has a reverse effect on the optical parent pulse. As a result, the spectrum of the latter is shifted to the red, and the magnitude of this shift is proportional to the intensity of the input optical pulse.

On the other hand, these equations do not take into account losses due to the escape of radiation from the crystal. At laser pulse intensities that are higher than those considered in this paper, it is necessary to take into account the cubic nonlinearity, which should significantly affect the dynamics of the optical pulse, including the processes of self-focusing.

The angular addition to the GVD [see (13)] can change the sign of the latter, which can qualitatively affect the dynamics of the optical (as well as of terahertz) pulse. The sign of the GVD in the $LiNbO_3$ crystal does not change; therefore, in the future it is reasonable to consider other uniaxial crystals, where such a change is possible.

With decreasing input optical pulse duration down to a few tens of femtoseconds, dispersion effects of nonlinearity, which we have neglected in this paper, can be significant. The analysis shows that for the studied durations of 100 fs and higher, the dispersion of nonlinearity does not play an important role.

In the future, we plan to modify the system of equations (12) and (16), and conduct a research taking into account the comments made above.

Apart from using the tilted front, condition (1) can be satisfied by introducing into the crystal the resonance impurities, which can significantly reduce the group velocity of the optical pulse [31, 32]. It is possible that the combined use of these two methods can lead to an increase in the terahertz generation efficiency.

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