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CONTROL OF LASER RADIATION PARAMETERS

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Parametric excitation of relaxation oscillations at the subharmonic of the external modulating signal in a ring Nd : YAG laser

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Abstract. **The dynamics of solid-state ring laser (SSRL) radiation with periodic pump modulation is studied experimentally and simulated numerically in the case of parametric excitation of relaxation oscillations at the subharmonic of a modulating signal. Parametric processes are investigated by modulating the pump in two regimes of the SSRL operation: steady-state regime of unidirectional lasing and self-modulation regime of the first kind. Significant differences in the dynamics of radiation for these regimes are found. It is established that when the laser operates in the self-modulation regime, a first-order parametric instability can result in the appearance of the dynamic chaos regime.**

Keywords: *nonlinear dynamics, solid-state ring laser, self-modulation regime of the first kind, dynamic chaos, bifurcation.*

1. Introduction

Studies in the dynamics of solid-state ring lasers (SSRLs) are important for investigating the general properties of nonlinear dynamics of various systems. Currently, the dynamics of SSRLs with periodic modulation of parameters is studied in detail both theoretically and experimentally in the case of resonant excitation of relaxation oscillations at the frequency of the external modulating signal (see, for example, [1, 2] and review [3]). Relaxation oscillations can be excited in the resonant interaction with the frequency of the modulating pump signal f_p, close to the relaxation oscillation frequency f_r, and in the parametric resonance, when the subharmonic of the modulating signal $f_p/2$ is close to the relaxation oscillation frequency f_r. To our knowledge, the dynamics of solid-state lasers with parametric excitation of relaxation oscillations has not been previously investigated. It is expected that the bifurcations occurring in parametric excitation of relaxation oscillations will depend on the regime in which the laser operated in the absence of the modulating signal.

The aim of this paper is to study the dynamics, temporal and spectral characteristics of a solid-state ring laser (unidirectional and bidirectional) in the case of parametric excitation of relaxation oscillations by the subharmonic of the modulating signal.

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2. The experimental setup

The ring chip laser under study represented a monolithic block in the form of a prism with one spherical (radius of curvature of 50 mm) face and three flat total-internal-reflection faces. The geometrical perimeter of the resonator was about 2.8 cm. The nonplanarity angle of the resonator was equal to 80°. As shown in our previous work [4], the generation regime of this laser can be effectively controlled by an external magnetic field generated by a permanent magnet. In the absence of an external field the laser under study operated in the self-modulation regime of the first kind, and in the presence of an external field – in the steady-state regime of unidirectional lasing. The transition from the self-modulation regime of the first kind to the unidirectional lasing regime was caused by the fact that when a magnetic field was applied in a ring resonator, considerable amplitude nonreciprocity appeared.

The laser was pumped by a semiconductor laser diode at $\lambda = 0.810$ μm. The power circuit of the diode had a generator of periodic oscillations, which modulated the pump power of the laser in question in the frequency range 50 – 220 kHz. The characteristics of the radiation were recorded with a TektronixTDS 2014 digital oscilloscope.

Studies were performed (in the absence of pump modulation) in two SSRL operation regimes: steady-state regime of unidirectional lasing and self-modulation regime of the first kind. In the case of pump modulation the excess of the pump $\eta(t)$ over the threshold can be expressed as

$$\eta(t) = \eta_0 + h\sin(2\pi f_p t), \qquad (1)$$

where η_0 is the excess of the pump over the threshold in the absence of pump modulation; h and f_p are the depth and frequency of pump modulation, respectively.

3. Parametric excitation of relaxation oscillations in the steady-state regime of unidirectional lasing

Experimental studies and numerical simulation of the SSRL dynamics in the steady-state regime of unidirectional lasing were performed at a constant excess of the pump over the threshold, $\eta_0 = 0.1$. The relaxation oscillation frequency was $f_r = 89$ kHz. The modulation frequency f_p varied in the range 50 – 220 kHz, and the modulation depth h was constant and equal to 0.05.

In this regime, the periodic modulation of the pump with a small depth h leads, in a wide range of modulation frequencies (except for the regions of parametric resonance), to the sinusoidal modulation of the radiation intensity with frequency f_p. If the subharmonic $f_p/2$ of the modulating signal approaches the relaxation oscillation frequency f_r, a parametric instability occurs in the SSRL, leading to excitation of relaxation oscillations at the frequency of this subharmonic. In the parametric instability region the response of the laser to the modulation signal significantly increases: modulation of the radiation becomes pulse periodic and the period-doubling bifurcation occurs, the pulse repetition period T being equal to two periods of modulation T_p ($T = 2T_p = 2/f_p$).

Typical oscillograms of the radiation intensity in the parametric instability region are shown in Figs 1a and b for two values of the frequency of the modulation signal. Figure 1c shows the experimentally obtained spectrum of the laser intensity. It is seen that in the parametric instability region there appear spectral components at the frequency $f_p/2$ and its multiple frequencies.

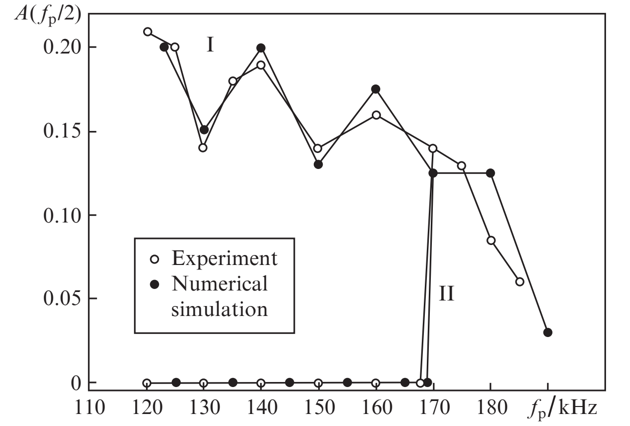

Figure 2. Dependence of the spectral component amplitude at the subharmonic frequency $A(f_p/2)$ on the pump modulation frequency f_p.

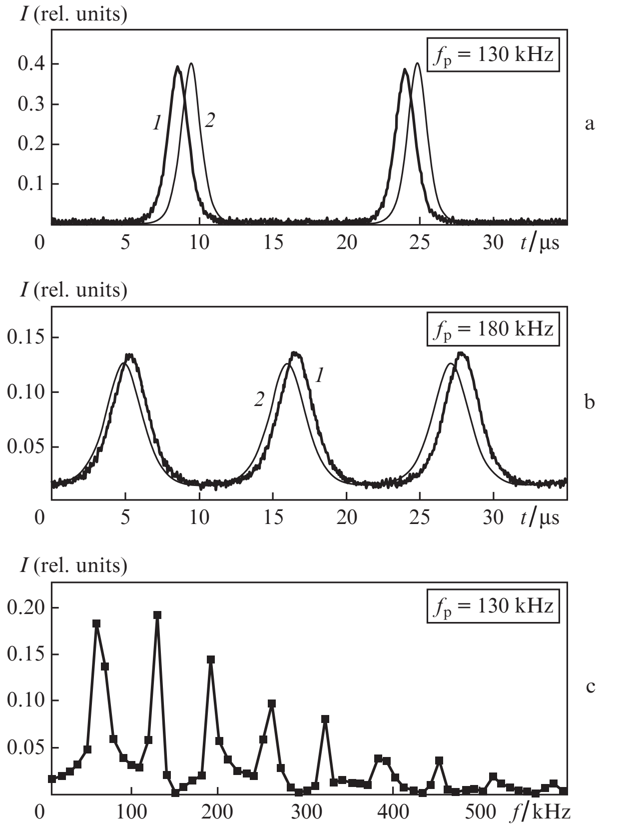

Figure 1. Time (a, b) and spectral (c) characteristics of radiation in the parametric instability region at modulation frequencies of 130 and 180 kHz [(*1*) experiment, (*2*) numerical simulation].

Figure 2 shows the experimentally measured and numerically simulated dependences of the amplitude of the spectral components $A(f_p/2)$ on the pump modulation frequency f_p. One can see from Fig. 2 that there are two regions (I and II), in which relaxation oscillations are parametrically excited. When entering the region of parametric instability at higher frequencies (decrease in the modulation frequency; region I), the parametric excitation is observed for values of f_p from 190 to 120 kHz. With increasing the modulation frequency the parametric instability region is much smaller (region II) – from 170 to 190 kHz.

We have also simulated numerically the dynamics of the SSRL with periodic pump modulation in the regime of unidirectional lasing. To describe the dynamics of unidirectional lasing, use was made of a system of rate equations for the radiation intensity I and the population inversion N

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -\frac{\omega}{Q}I + \frac{\sigma l}{T}NI,$$
$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{1}{T_1}\{N_{\text{th}}[1+\eta(t)] - N - aIN\}\,. \tag{2}$$

Here, ω/Q is the linewidth of the resonator; $T = L/c$ is the round-trip transit time in the resonator; T_1 is the longitudinal relaxation time, equal to 240 μs in the Nd:YAG laser; l is the length of the active element; $a = T_1 c\sigma/(8\hbar\omega\pi)$ is the saturation parameter; σ is the cross section of the laser transition. The pump rate is presented in the form $N_{\text{th}}[1 + \eta(t)]/T_1$, where N_{th} is the threshold population inversion. As in the case of the experiment, in the numerical simulation of the SSRL dynamics the values of η_0 and h were constant: $\eta_0 = 0.1$, $h = 0.05$. The relaxation oscillation frequency, defined by

$$f_r = \frac{1}{2\pi}\sqrt{(\omega/Q)\eta_0/T_1},$$

at $\eta_0 = 0.1$ was set equal to experimentally measured value (89 kHz). During the numerical simulation in the parametric instability region we have calculated the response $I(t)$ of the laser to the modulation signal and found the spectrum of this signal. Figure 1 shows the experimentally measured dependences of $I(t)$ compared with those obtained by numerical simulation, whereas in Fig. 2 the dependence (obtained by numerical simulation) of the amplitude $A(f_p/2)$ on the pump modulation frequency f_p is compared with that measured experimentally. As can be seen from these figures, the experimental results agree well with the numerical simulations.

4. Parametric excitation of relaxation oscillations in the self-modulation regime of the first kind

In the bidirectional lasing regime we have performed the experiments on the parametric excitation of relaxation oscillations at the subharmonic $f_p/2$ of the pump modulation signal, similar to those described above in the case of unidirectional lasing. The modulation frequency was varied in the range 50–220 kHz, and modulation depth h was varied from 0.01 to 0.15. The excess of the pump over the threshold η_0 was varied from 0.1 to 1.1.

In the absence of pump modulation the laser operated in self-modulation regime of the first kind, which is characterised by the following parameters: at $\eta_0 = 0.1$ the self-modulation oscillation frequency is $f_m = 209$ kHz, and the main relaxation frequency is $f_r = 89$ kHz. As noted in [4], in the investigated monolithic SSRL, when the temperature of the monolithic block changes, it is possible to change the ratio of the average intensities of the counterpropagating waves, I_1/I_2. In the present experimental studies the temperature of the monolithic block was such that $I_1/I_2 = 1.2$. Typical oscillograms of the intensities of the counterpropagating waves and the spectrum of the intensity of one of the waves in this regime are shown in Fig. 3. In the intensity spectrum shown in Fig. 3b, there is only one (major) spectral component corresponding to the self-modulation oscillation frequency f_m. The spectral component at the relaxation oscillation frequency f_r is not visible in Fig. 3b, since its intensity is three orders of magnitude less than that of the main component.

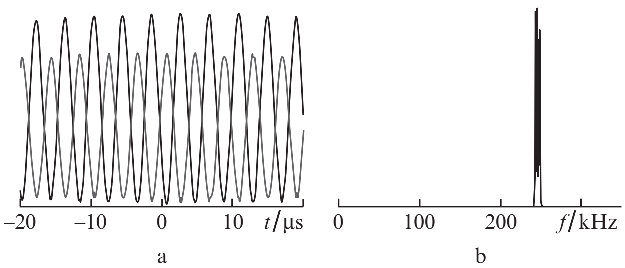

Figure 3. Typical oscillogram (a) and spectrum of the radiation intensity (b) in the self-modulation regime of the first kind ($f_m = 255$ kHz, $f_r = 102$ kHz, $\eta_0 = 0.155$).

The pump modulation leads (outside the region of the parametric excitation of relaxation oscillations) to the appearance of a quasi-periodic lasing regime. Typical oscillograms and the spectrum of the radiation intensity in this region are shown in Fig. 4. One can see that at a small modulation depth ($h = 0.075$) the spectrum exhibits a component at the modulation frequency f_p.

Within the region of parametric resonance, in contrast to the unidirectional lasing regime, the situation becomes more

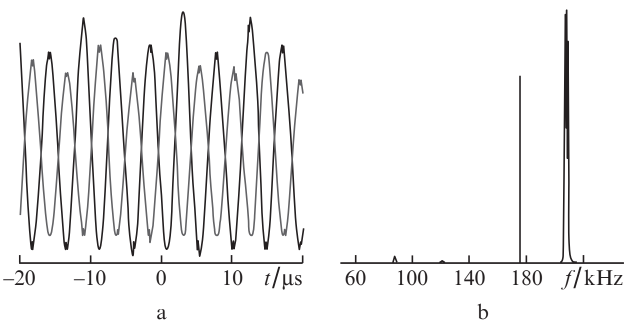

Figure 4. Typical oscillogram (a) and spectrum of the radiation intensity (b) outside the region of parametric resonance ($f_p = 178$ kHz, $f_m = 210$ kHz, $f_r = 98$ kHz, $\eta_0 = 0.085$).

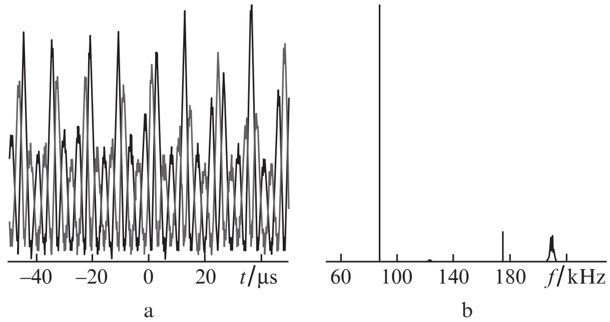

Figure 5. Typical oscillogram (a) and spectrum of the radiation intensity (b) in the region of parametric resonance for the regime of quasi-periodic oscillations ($f_p = 176$ kHz, $f_m = 210$ kHz, $f_r = 98$ kHz, $\eta_0 = 0.085$).

complicated. Depending on the pump modulation depth, both the regime of quasi-periodic oscillations (Fig. 5) and the regime of the dynamic chaos (Fig. 6) can appear. Figure 5 shows typical oscillograms and the spectrum of the radiation intensity (when the laser is exposed to the modulation signal) for the quasi-periodic regime. In this case, the intensity spectrum, as in the unidirectional lasing regime, exhibits an additional spectral component at the subharmonic $f_p/2$, and the spectrum is discrete. Figure 6 presents the oscillograms and the spectrum of the radiation intensity in the regime of dynamical chaos. As can be seen from the figure, in this case, in some neighbourhood of the relaxation frequency f_r spectrum becomes continuous.

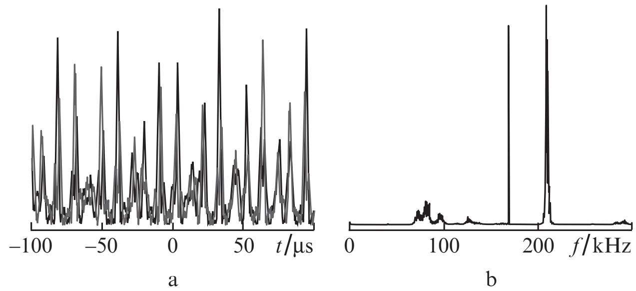

Figure 6. Typical oscillogram (a) and spectrum of the radiation intensity (b) in the region of parametric resonance for the regime of dynamic chaos ($f_p = 174$ kHz, $f_m = 210$ kHz, $f_r = 98$ kHz, $\eta_0 = 0.085$).

The boundaries of the regions of the quasi-periodic regime and the regime of dynamical chaos as functions of the pump modulation depth are shown in Figure 7. One can see that at small modulation depths only the quasi-periodic regime is observed. Unlike the case of unidirectional lasing, the region of existence of this regime is much narrower (few kHz) and narrows with increasing modulation depth: at $h = 0.04$ the width of this region is 4 kHz, and at $h = 0.08$ it is only 1 kHz. The region of existence of the dynamical chaos regime, however, increases with increasing modulation depth. In the chaos region (for a fixed modulation depth h) the region of the continuous spectrum varies significantly depending on the modulation frequency f_p. Figure 8 shows the evolution of the continuous spectrum with changing f_p. As can be seen from the figure, by varying the modulation frequency, we can effectively control the region of the continuous spectrum in the vicinity of the main relaxation frequency.

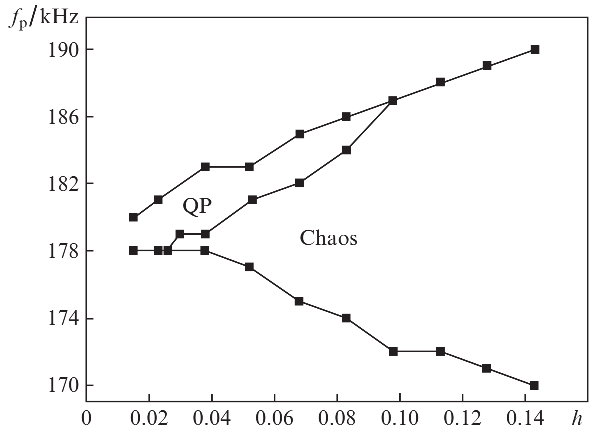

Figure 7. Boundaries of the quasi-periodic (QP) regime and the regime of dynamical chaos as functions of the pump modulation depth h.

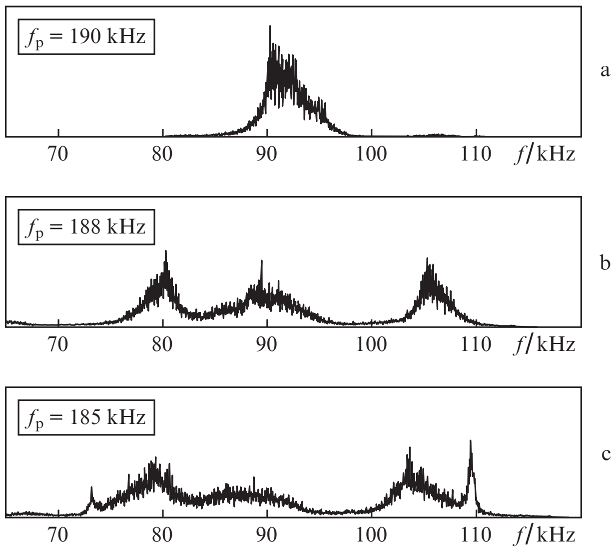

Figure 8. Evolution of a region of the continuous spectrum with changing the modulation frequency f_p and at $\eta_0 = 0.015$.

5. Conclusions

Thus, we have studied the parametric excitation of relaxation oscillations at the subharmonic of the pump modulation signal in the SSRL. We have found that the bifurcations emerging in parametric resonance are significantly different in the regime of unidirectional and bidirectional lasing. In the unidirectional lasing regime, the period-doubling bifurcation takes place in the parametric instability region. In the case of bidirectional lasing, in the parametric resonance region the periodic self-modulation lasing regime is replaced by either the quasi-periodic regime or the regime of dynamic chaos.

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