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On the analogy between Hawking radiation and nonlinear optical process of subharmonic generation (waveguide model of a black hole)

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Abstract. **Observation of photon behaviour in a waveguide model of spatial electromagnetic-field localisation establishes in a thought experiment a detailed analogy between the quantum-mechanical production of photon pairs at the horizon of an evaporating black hole and the classical nonlinear phenomenon of generation of subharmonics.**

Keywords: *spatial localisation of the electromagnetic field and the waveguide model of a photon, production of photon pairs at the horizon and evaporation of a black hole, nonlinear optical generation of subharmonics.*

1. Introduction

Reproduction of some properties of black holes [1] in a series of ‘desktop’ thought experiments, summarised in [2, 3], allowed one to make a number of instructive observations that contribute to a clearer understanding of this complex space phenomenon. In the present paper we discuss the phenomenon of Hawking radiation, which consists in production of photon pairs and evaporation of a black hole, thereby preventing its immortality. [1]

The discussion, like all other thought experiments of the series [2, 3], is based on the so-called waveguide model of a photon, which is heuristically productive embodiment of the concept of physical emptiness of the image of a spatially unlimited electromagnetic wave field (in particular, an infinite plane wave).

This concept is a direct consequence of physical non-realisability of an electromagnetic wave, which is characterised by a total energy flux

$$\Phi = \int_{\infty} \Xi \mathrm{d}x\mathrm{d}y > 0, \quad (1)$$

not experiencing any restrictions, i.e., with

$$\Phi \neq \infty. \quad (2)$$

Here $\Xi = \Xi_0 f(x,y,z,t)$ is the energy flux density; $\Xi_0 > 0$ is the amplitude of the flux density of the Poynting vector; x, y are the transverse coordinates; t is the time; the wave propagates along the z axis.

Statement (1), (2) cannot be met at $f(x,y)$ = const > 0, since the improper integral (1) has a finite value only when the function $f(x,y)$ sufficiently rapidly decreases with increasing coordinates x, y, i.e., during the spatial localisation of the wave field Ξ [function Ξ is written here in Cartesian coordinates, but its transverse restriction, perpendicular to the direction of wave propagation, is obviously present in all other configurations: in cylindrical coordinates (r,ϕ,z) it is determined by the radius r, in the polar system (ϕ,ϑ,r) of a spherical wave – by the finite solid angle 4π, etc.].

As a result, the image of an electromagnetic field with a transversely unlimited wave flow (in particular, an ideal plane wave) can hardly be considered meaningful from a physical point of view (which, of course, does not preclude the use of a plane wave as a good approximation in many problems).

An important consequence of the logical inevitability of spatial localisation of the field is an indispensable presence of a stationary standing component (*‘stopped’* light) in any wave field, which makes a ‘resting’ contribution to the total energy of the field [3]. This is a prerequisite for the introduction of the concept of finite inertial and gravitational *rest mass* of a photon [2, 3].

We should, however, emphasise that the physical content of such a photon rest mass, which depends on the structure of an extraneous spatial limitation, cannot claim for absolute immanence inherent in rest masses of ordinary particles (electron, neutron, etc.) that are fundamental constants. Therefore, for a photon we use the name of the *observable rest mass* of a photon.

In the thought experiment discussed in the development of the concept of a spatially limited wave field we present a detailed analogy between the production of pairs in a ‘desktop’ waveguide model of an evaporating black hole (Hawking radiation) and a classical nonlinear optical process of generation of subharmonics [4, 5].

2. Waveguide model of a spatially localised wave field

The waveguide model [2, 3], which is one of the possible realisations of the concept of spatial localisation of the field, is an infinitely deep potential well for photons with the free wave propagation along the z axis and with a complete limitation of the field over the transverse coordinates. This model, in the case of a simple mathematical apparatus [6] not losing enough generality, adequately reflects the main properties of transform-limited wave fields; in particular, it demonstrates a complete coincidence of the mathematical expressions governing

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the photon of the waveguide mode with the corresponding formulas of relativistic kinematics and dynamics of an ordinary particle with inertial and gravitational rest mass $M = M_{\mu\nu}$, which is equal to the observable rest mass attributed to a photon, i.e., mass-like quantity

$$M_{\mu\nu} = \hbar\omega_{\mu\nu}/c^2, \quad (3)$$

where $\omega_{\mu\nu}$ is the critical frequency of the waveguide mode with integer subscripts μ and ν; c is the speed of light [2, 3]. One can see from (3) that the *transverse standing* component of the field of the mode, which determines the critical frequency, is responsible for the observable rest mass, making a *resting* contribution to the total energy.

It is important to emphasise the informal nature of the mentioned coincidences and physical meaningfulness (3): direct calculation [7] showed that the energy $\hbar\omega_{\mu\nu}$ is exactly equal to the work done by an external force against the force of radiation pressure in the compression of the field from the surrounding space in a limited volume of the waveguide mode.

In confirmation of this it is appropriate to emphasise that throughout the series of kinetic and dynamic thought experiments with photons in a waveguide [2, 3], including those reproducing a 'black hole on the bench' [2, 8, 9], we failed to detect any differences between the behaviour of a photon, characterised by the value of $M_{\mu\nu}$ (3), and an ordinary particle with inertial and gravitational mass $M = M_{\mu\nu}$.

3. 'Desktop black hole'

The thought experiment to reproduce a 'desktop black hole' [2, 8, 9] is to observe the behaviour of the wave propagation constant

$$k_{\mu\nu}^2 = \left(\frac{\omega}{c}\right)^2 - \left(\frac{\omega_{\mu\nu}}{c}\right)^2 \approx k_\infty^2\left[1 - 2\frac{\Psi}{c^2}\left(1 + \frac{2}{1 - \omega_{\mu\nu\infty}^2/\omega_\infty^2}\right)\right], \quad (4)$$

when the waves propagate vertically in a waveguide in the gravitational field with potential $\Psi < 0$, normalised to zero at infinity, $\Psi_\infty = 0$, and $|\Psi| \ll c^2$.

In accordance with (4) the cause of the behaviour of the photon is the dependences of two parameters on the potential Ψ: the speed of light $c \approx c_\infty(1 + 2\Psi/c^2)$ and the transverse dimension of the waveguide $r \approx r_\infty(1 + 2\Psi/c^2)$, which determines the critical frequency of the mode $\omega_{\mu\nu}$, and therefore (3) – the observable photon rest mass

$$M_{\mu\nu} = \frac{\hbar\omega_{\mu\nu}}{c^2} \approx M_{\mu\nu\infty}\left(1 - \frac{3\Psi}{c^2}\right). \quad (5)$$

Formula (5) coincides with the fundamental expression for the gravitational mass $M = M_\infty(1 - 3\Psi/c^2)$ of ordinary particles (subscript ∞ hereinafter denotes the parameters for $\Psi = \Psi_\infty = 0$).

It follows from (4) that when moving up ($\Delta\Psi > 0$) from point with a potential Ψ to point with a potential $\Psi + \Delta\Psi$ and under the condition $1 - (\omega_{\mu\nu}/\omega)^2 \ll 1$, reflecting a sufficient excess of the energy equivalent of the observable photon rest mass $M_{\mu\nu}$ (3) over its kinetic energy, there occurs a decrease in the propagation constant from $k_{\mu\nu}$ to $k_{\mu\nu}(\Delta\Psi)$:

$$\frac{k_{\mu\nu}^2(\Delta\Psi)}{k_{\mu\nu}^2} \approx 1 - 2\frac{\Delta\Psi}{c^2}\left(1 + \frac{2}{1 - \omega_{\mu\nu}^2/\omega^2}\right). \quad (6)$$

This decrease continues up to a complete stop of the upward movement $k_{\mu\nu}(\Delta\Psi) = 0$, when the gravitational potential increases by

$$\Delta\Psi \approx \frac{c^2}{4}[1 - (\omega_{\mu\nu}/\omega)^2]. \quad (7)$$

Thus, the vertical coordinate at which equality (7) is met, is by definition the horizon (limiting height) $H(\Delta\Psi)$.

4. Nonlinear optical equivalent of Hawking radiation

In a space black hole, quantum fluctuations of vacuum at the horizon H produce photon pairs (as well as other pairs of particles – antiparticles), one of the photons of each pair leaving the black hole (Hawking radiation) [1].

The behaviour of photons in the adopted waveguide model provides a basis for detailed visual analogy between this quantum-mechanical phenomenon and the classical nonlinear optical process of generation of subharmonics [4, 5]. The latter in the special case of frequency halving is responsible for the decay of a primary photon (that has reached the horizon) with energy ${}_{\rm p}\hbar\omega = \hbar\omega_{\mu\nu} = M_{\mu\nu}c^2$ and zero propagation constant ${}_{\rm p}k_{\mu\nu} = 0$ on a pair of secondary photons with equal energies,

$${}_{\rm s}\hbar\omega = \frac{{}_{\rm p}\hbar\omega}{2} = \frac{{}_{\rm p}\hbar\omega_{\mu\nu}}{2}, \quad (8)$$

propagation constants ${}_{\rm s}k\uparrow$ and ${}_{\rm s}k\downarrow$, equal in absolute value,

$$({}_{\rm s}k\uparrow)^2 = ({}_{\rm s}k\downarrow)^2 = ({}_{\rm p}\omega/2c)^2 - ({}_{\rm s}Mc/\hbar)^2, \quad (9)$$

but opposite in sign, ${}_{\rm s}k_\uparrow = -{}_{\rm s}k_\downarrow$ (here the parameters of primary and secondary photons are denoted by subscripts p and s, and the direction of downward and upward movement – by arrows $\downarrow$ and $\uparrow$).

Implementation of the laws of conservation of energy and momentum in a waveguide of invariable cross section requires that the primary photon and the photons of the secondary pair belong to different modes with subscripts $\mu\nu$ and $\sigma\tau$, since according to (8), (9)

$$({}_{\rm p}M_{\mu\nu})^2 = (2\,{}_{\rm s}M_{\sigma\tau})^2 + (2\hbar\,{}_{\rm s}k/c)^2. \quad (10)$$

It follows that the critical frequencies and observable masses of each photon of the pair are less than the same values of the primary wave:

$${}_{\rm s}\omega_{\sigma\tau} < {}_{\rm p}\omega_{\mu\nu}, \quad {}_{\rm s}M_{\sigma\tau} < {}_{\rm p}M_{\mu\nu}. \quad (11)$$

A significant remark applies to the structures of the fields of primary and secondary modes: their topological homogeneity or heterogeneity can promote or inhibit the generation of subharmonics (the production of photon pairs).

Useful illustration of the results obtained: let the primary photons, belonging to the TM_{02} mode with the critical frequency ${}_{\rm p}\omega_{02} = 2\pi c/1.14R$ and the mass ${}_{\rm p}M_{02} = 2\pi\hbar/1.14cR$, decay into the secondary photon pairs, belonging to the TM_{01} mode with the critical frequency ${}_{\rm s}\omega_{01} = 2\pi c/2.61R$ and the mass ${}_{\rm s}M_{01} = 2\pi\hbar/2.61cR$, in a waveguide of circular cross section with radius R. The boundary conditions that determine the critical frequencies of primary and secondary modes consist in matching the radial coordinate R with the second and the first roots of the zero-order Bessel function of first kind

[6]. The topological homogeneity of the structures of the fields is expressed in the absence of circular periodic variations in all the modes (an alternative unfavourable structural situation would consist, for example, in an attempt to combine the primary mode TM_{01} with the secondary mode TM_{11}).

Descriptiveness of the waveguide model of a black hole, which operates the photons with the only frequency ω, can be greatly increased due to abandonment of monochromaticity and due to consideration of the photon ensemble of a wide range of frequencies. In this case, all the photons of the ensemble must reach the horizon at the same height and gravitational potential increment $\Delta\Psi$, i.e., according to (7) all the photons of the ensemble must have a common ratio $\omega_{\mu\nu}/\omega = M_{\mu\nu}c^2/\hbar\omega$, which requires obvious individual choice of the mode for each photonic waveguide. As a result of this modernisation, Hawking radiation would acquire closer-to-reality broadband nature, which would open a way to estimation of the effective temperature [1] of the model of an evaporating black hole (perhaps it would be more appropriate to call it the waveguide model of *a black hole with white light*).

5. Conclusions

The result of thought experiments on the waveguide model of a photon is to establish an analogy between the quantum-mechanical production of photon pairs (Hawking radiation) at the horizon of an evaporating black hole and the classical nonlinear optical process of generation of subharmonics; some of properties of this analogy are almost equivalent.

Subject to further investigation is the unsolved nonlinear optical mechanism of generation of subharmonics when the wave reaches the vertical coordinate, which coincides with the horizon H, as well as the physical nature of the nonlinearity. It is obvious that under the applied phenomenological approach it was hardly possible to do it as thoroughly as in classical nonlinear optics [7, 8]. Nevertheless, one can predict the existence of giant electromagnetic nonlinearity in the space directly adjacent to the horizon H, which is due to the instability of the vacuum and discontinuous changes in the kinematics of the waves on a vertical segment of zero length, leading to a change in sign of the speed of its finite modulus on both sides of H. These discontinuous changes in the propagation constants are of the nature of the integral delta function with a finite difference $\Delta k = 2_{\mathrm{s}}k = 2[(_{\mathrm{p}}\omega/2c)^2 - (_{\mathrm{s}}Mc/\hbar)^2]^{1/2}$ on the interval of an argument tending to zero.

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