

LASER APPLICATIONS AND OTHER TOPICS IN QUANTUM ELECTRONICS

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Application of ring lasers to determine the directions to the poles of Earth's rotation

Yu.D. Golyaev, Yu.Yu. Kolbas

Abstract. **Application of a ring laser to determine the directions to the poles of Earth's rotation is considered. The maximum accuracy of determining the directions is calculated, physical and technical mechanisms that limit the accuracy are analysed, and the instrumental errors are estimated by the example of ring He–Ne lasers with Zeeman biasing.**

Keywords: *ring laser, gyroscope, gyrocompass, poles of Earth's rotation.*

1. Introduction

The problem of accurate determination of the direction and location of the poles of Earth's rotation is of great scientific and practical importance. It is particularly important in relation to the constant changes and fluctuations in the location of the magnetic poles of the Earth due to natural and human-induced magnetic anomalies. Monitoring the position of the poles of Earth's rotation also has an independent scientific value in the study of global processes.

Currently, there are fairly accurate gyroscopic systems (gyrocompasses) to determine the position of the poles of Earth's rotation [1–3]. Their operation is based on the effect of centrifugal force of a rotating Earth on a rapidly rotating rotor of the gyroscope, the axis of the rotor rotation tending to occupy a position in which it is parallel to the angular velocity vector of Earth's rotation. If a gyroscope with a negative feedback is used, it is possible to create an angular velocity sensor that will directly measure the angular velocity of Earth's rotation.

Creation of ring lasers (RLs) has opened new possibilities for fabricating systems of both applied and scientific value. Since a RL is used to measure the angular velocity, it becomes possible to determine the instability of both the magnitude and direction of the angular velocity of rotation of the Earth. Publications on the subject are sufficiently scarce [4–6], and they are devoted to specific devices.

In this paper, we consider a possible method to determine the angular velocity and the position of the poles of Earth's rotation with the help of a ring laser.

Yu.D. Golyaev, Yu.Yu. Kolbas Open Joint-Stock Company 'M.F. Stel'makh Polyus Research and Development Institute', ul. Vvedenskogo 3, 117342 Moscow; Russia;
e-mail: tigra-e@rambler.ru

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2. Measurement of the angular velocity of Earth's rotation by a RL and the model of errors

A ring laser is sensitive to the angular velocity of rotation of its platform, directed along the normal to the plane of the optical path of the laser. As is known, the dependence of the beat frequency of counterpropagating waves of such a laser, $\Delta\nu$, on the angular velocity of rotation, Ω, is given by [6]

$$\Delta\nu = \frac{4S}{\lambda L}\Omega, \quad (1)$$

where S is the area within the optical path of the laser; L is the cavity perimeter; and λ is the wavelength.

The value of $k = \lambda L/(4S)$ is called the scale factor of the RL. Note that in registering the interference pattern of counterpropagating waves with two photodetectors, located at a distance that is equal to $^1/_4$ of the width of the interference fringe, in electronic circuits not one, but four pulses of the output electrical signal can be produced per one interference fringe, recorded by photodetectors. In this case, $k = L/(16S)$, which we will use below.

If the measuring axis of the RL (normal to the plane of the RL cavity) and the direction of the angular velocity of Earth's rotation do not coincide (Fig. 1), the beat frequency of the counterpropagating waves of the laser, $\Delta\nu$, will be determined by the expression

$$\Delta\nu = k^{-1}\Omega_{\rm Earth}\cos\psi, \quad (2)$$

where $\Omega_{\rm Earth} = 15.04$ deg h^{-1} is the angular velocity of Earth's rotation; ψ is the angle between the measuring axis of the RL and the angular velocity vector of Earth's rotation.

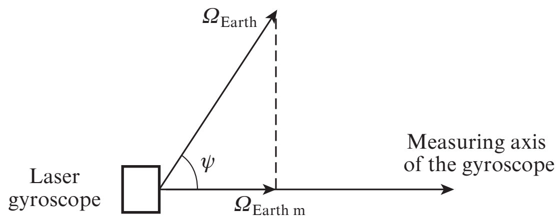

Figure 1. Scheme illustrating the measurement of the angular velocity of Earth's rotation with a ring laser.

In practice, rather than measuring the frequency $\Delta\nu$, it is convenient to estimate first the effective angle of rotation, $\theta_{\rm m}$, at the measurement time $t_{\rm m}$, and then calculate the values of $\Delta\nu$ and $\Omega_{\rm Earth}$ [7, 8]:

$$\theta_{\rm m} = \int_0^{t_{\rm m}} \Omega_{\rm Earth}(t)\,{\rm d}t = \frac{k}{\cos\psi}\int_0^{t_{\rm m}} \Delta\nu(t)\,{\rm d}t = \frac{kN}{\cos\psi}, \qquad (3)$$

where N is the number of periods of the beats of counterpropagating RL waves during the measurement time $t_{\rm m}$. If the angular velocity of the object (in this case, the Earth) during the measurement is constant, then it can be easily found from (3) by the formula

$$\Omega_{\rm Earth} = \frac{kN}{t_{\rm m}\cos\psi}. \qquad (4)$$

Obviously, if the RL support frame experiences angular oscillations and if the measurement time is comparable with their period, the measured angular velocity will include a part of the amplitudes of the angular oscillations. To eliminate their influence, one should either use a support frame rigidly anchored in the ground (vibration-isolated foundation) or to measure N in a time much longer than the period of the oscillations. In this paper the influence of such parasitic oscillations is ignored.

There are a number of physical mechanisms that lead to the emergence of a 'seeming' rotation (zero shift of the RL output characteristic, drift), and to the instability of the scale factor [7–12]. Consequence of the effect of these mechanisms is the measurement error of the true angle of rotation. In addition, when calculating the number of periods of the beats, a sampling error occurs [13], which is $0.5k$.

In view of these factors, the angle of rotation, $\theta_{\rm m}$, measured during the time $t_{\rm m}$ will be determined by the expression [8]

$$\theta_{\rm m} = (k + \Delta k)(N + \Delta\nu_{\rm dr} t_{\rm m} + 0.5)$$
$$\approx kN + \Delta kN + k\Delta\nu_{\rm dr} t_{\rm m} + 0.5k, \qquad (5)$$

where k is the error of the scale factor and $\Delta\nu_{\rm dr}$ is the RL frequency drift.

The measured values of the angular velocity of Earth's rotation, $\Omega_{\rm Earth\,m}$, can be found by the formula

$$\Omega_{\rm Earth\,m} = \frac{kN}{t_{\rm m}\cos\psi} + \frac{\Delta kN}{t_{\rm m}\cos\psi} + \frac{k\Delta\nu_{\rm dr}}{\cos\psi} + \frac{0.5k}{t_{\rm m}\cos\psi}$$
$$= \Omega_{\rm Earth} + \frac{\Delta k}{k}\Omega_{\rm Earth} + \frac{k\Delta\nu_{\rm dr}}{\cos\psi} + \frac{0.5k}{t_{\rm m}\cos\psi}. \qquad (6)$$

The first term on the right-hand side of (6) is the true angular velocity of Earth's rotation, while the second, third and fourth terms determine the error of its measurements and are related with the RL parameters, i.e., with a relative error of determination of the scale factor, by the random component of the drift and the sampling error.

In addition, the error in calculating the angular velocity of Earth's rotation also includes the inaccuracies of the measurement of the angle $\Delta\psi$ and time $\Delta t_{\rm m}$. Differentiating (4) with ψ and $t_{\rm m}$, for the corresponding components of the error we can derive the expressions:

$$\Delta\Omega_{{\rm Earth}\,\psi} = \frac{kN\sin\psi}{t_{\rm m}\cos^2\psi}\Delta\psi = \Omega_{\rm Earth}\Delta\psi\tan\psi,$$
$$\Delta\Omega_{{\rm Earth}\,t} = -\frac{kN}{t_{\rm m}^2\cos\psi}\Delta t_{\rm m} = -\frac{\Delta t_{\rm m}}{t_{\rm m}}\Omega_{\rm Earth}. \qquad (7)$$

Consider the components of the errors in equation (6). Most important is the RL frequency drift $\Delta\nu_{\rm dr}$. It includes both technical drifts, caused by the imperfection of the laser design [7–15] (lock-in of counterpropagating waves, Langmuir drift, dynamic drift), and the quantum noise of the RL frequency, caused by spontaneous emission.

According to [9], the width of the noise spectrum of the RL output signal, associated with the spontaneous emission, can be calculated by the formula

$$\Delta\nu_{\rm g} = \frac{1}{2\pi}\sqrt{D_{\rm fl}/t_{\rm m}},$$
$$D_{\rm fl} = \frac{4\hbar\omega_0\Delta\omega_{\rm cav}^2}{P}, \qquad (8)$$

where $D_{\rm fl}$ is the spectral density of fluctuations of the frequency difference of counterpropagating waves of the laser at zero frequency; c is the speed of light; $\omega_0 = 2\pi\nu_0$ is the laser frequency; $\Delta\omega_{\rm cav} = 2\pi\Delta\nu_{\rm cav} = 2\pi\delta c/L$ is the cavity bandwidth; $\hbar$ is Planck's constant; P is the laser power inside the cavity; and δ is the relative round-trip loss of light in the cavity.

For a Zeeman RL of type K-5 (OJSC 'M.F. Stel'makh Polyus Research and Development Institute') we have $L = 0.2$ m, $S = 0.0025$ m^2, $\nu_0 = 4.73\times10^{14}$ Hz, $\delta = 3.7\times10^{-4}$, $\Delta\nu_{\rm cav} = 5.6\times10^5$ Hz, $P = 5\times10^{-2}$ W, $T = 600$ s, $\Delta\nu_{\rm g} = 0.00012$ Hz, and the corresponding error of measuring the angular velocity is $\Delta\Omega_{\rm g} = k\Delta\nu_{\rm g} = 0.00032$ deg h^{-1}.

The relative error in measuring the scale factor, $\Delta k/k$, is determined by the RL parameters (primarily, backscattering resulting in lock-in of counterpropagating waves and changes in the gain of the active medium) and by the equipment parameters for evaluating the scale factor in the manufacture of laser. For a modern Zeeman RL we have $\Delta k/k \leqslant 1\times10^{-5}$. Accordingly, the additional error in measuring the angular velocity of rotation of the Earth is no more than 0.00015 deg h^{-1}.

Let us estimate the impact of the scale factor. A simple electronic circuit makes it possible to produce not one but four information pulses during a beat period of counterpropagating waves. Hence we find that for the measurement time of 600 s and geometric scale factor of 2.73″, the contribution of this factor to the error in measuring the angular velocity of rotation of the Earth is 0.001 deg h^{-1}. This value is significant, but in more complex electronic circuits, which measure not the frequency, but the signal beat period of counterpropagating waves [8], the scale factor can be reduced to 0.001 of its geometrical value. In this case, its contribution to the error is negligible.

The contribution of the errors (7) is even smaller. Widespread application of T-5 theodolite (measurement error of the angles is 30″) allows one to set the measurement axis of the RL parallel to the axis of Earth's rotation to within 90″. Accordingly, the error $\Delta\Omega_{{\rm Earth}\,\psi}$ is at most 0.000003 deg h^{-1}. The error $\Delta\Omega_{{\rm Earth}\,t}$ at relative accuracy of the time measurement of 10^{-6} will be no more than 0.000015 deg h^{-1}.

Thus, the total error of the angular velocity of Earth's rotation measured with the help of a modern Zeeman RL is less than 0.0005 deg h^{-1}.

The current state of optics and technology of optical coating allows one to fabricate ring cavities with very low losses and low backscattering for circularly polarised light waves, which can significantly improve the accuracy of measurements of the angular velocities.

To facilitate further analysis, we rewrite equation (8) in a simpler form, which is valid for a cavity with small losses:

$$\Delta\Omega_{\text{Earth}} = \frac{\pi d c^2}{S}\sqrt{\frac{\hbar}{\omega_0 P t_{\text{m}}}}. \quad (9)$$

It follows from (9) that, in order to reduce the errors it is needed to reduce losses in the cavity and to increase the area of the cavity optical path and time of measurement. The increase in power stored in the cavity is not possible because of the need to maintain constant the excess of the gain over the loss in order to ensure generation of the zero transverse mode only.

Figures 2 and 3 show the dependence of $\Delta\Omega_{\text{Earth}}$ on the relative loss and the area for a square ring He–Ne laser for the measurement time of 600 s and the 1.02-fold excess of the gain over the loss. It can be seen that in using the RL with limiting characteristics achieved up to now, the error $\Delta\Omega_{\text{Earth}}$ will not exceed 0.000015 deg h^{-1}, i.e., 0.00001 % of the Earth's rotation angular velocity.

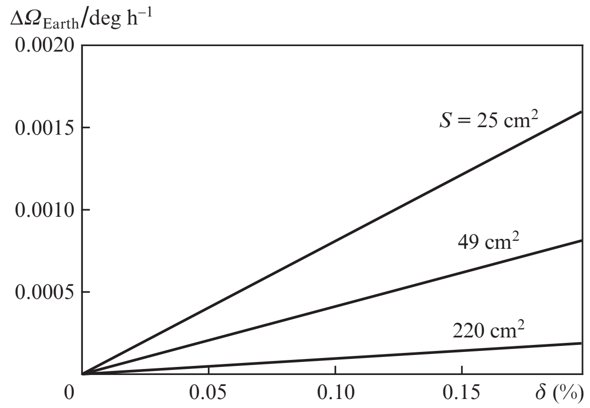

Figure 2. Dependences of the error in measuring the angular velocity of Earth's rotation on the relative loss in the cavity at different areas of its optical path.

Note also that losses in the cavity rather than the area of optical path are much more important for the measurement accuracy.

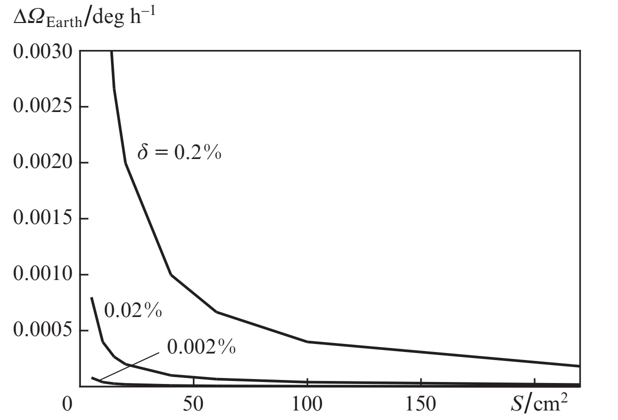

Figure 3. Dependences of the error in measuring the angular velocity of Earth's rotation on the area of the cavity optical path at different relative losses in the cavity.

3. Determination of the directions to the poles of Earth's rotation

The azimuth γ (direction to the North Pole) is determined by the projection of the angular velocity of Earth's rotation to the measuring axis of the RL, the angle between the measuring axis of the RL and the plane of the local horizon β, and the latitude of the measuring point φ (Fig. 4) by the formula [8]

$$\cos\gamma = \pm\frac{\Omega_{\text{Earth m}}/(\Omega_{\text{Earth}}\cos\varphi) - \sin\beta\tan\varphi}{\cos\beta}. \quad (10)$$

Consider the model of errors in measuring the azimuth. The total error $\Delta\gamma$, obtained from equation (10), given the fact that the value of β is small, can be written as [8]

$$\Delta\gamma \approx \left\{\left(\frac{\Delta\Omega_{\text{Earth m}}}{\Omega_{\text{Earth}}\cos\varphi}\right)^2 + \left[\left(\frac{\Delta\Omega_{\text{Earth m}}\beta}{\Omega_{\text{Earth}}\cos\varphi} + \tan\varphi\right)\Delta\beta\right]^2 + \left[\frac{\Delta\varphi}{\cos^2\varphi}\left(\frac{\Omega_{\text{Earth m}}}{\Omega_{\text{Earth}}}\sin\varphi + \beta\right)\right]^2\right\}^{1/2} = \sqrt{\Delta\gamma_{\text{m}}^2 + \Delta\gamma_\beta^2 + \Delta\gamma_\varphi^2}, \quad (11)$$

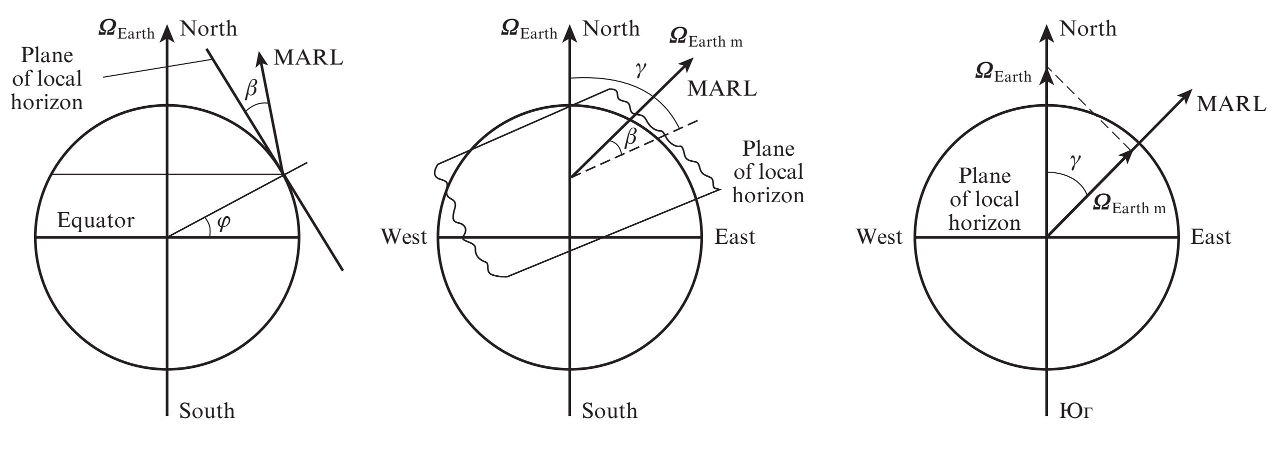

Figure 4. Schemes illustrating the determination of the direction to the North Pole (azimuth) with the help of the RL (MARL is the measuring axis of the RL).

where $\Delta\gamma_m$, $\Delta\gamma_\varphi$ и $\Delta\gamma_\beta$ are the errors due to drift of the gyroscope, uncertainties in determining the latitude and angle, respectively.

Numerical estimates of the error in determining the azimuth for the latitude 70°, depending on the relative losses in the cavity and the area of its optical path, are shown in Figs 5 and 6.

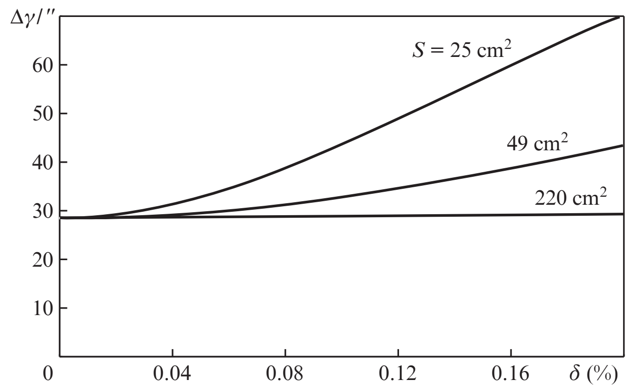

Figure 5. Dependence of the error in measuring the azimuth on the relative losses in the cavity at different areas of optical path S and parameters $\Delta\varphi = 3.3''$, $\Delta\beta = 10''$, $\beta = 30'$.

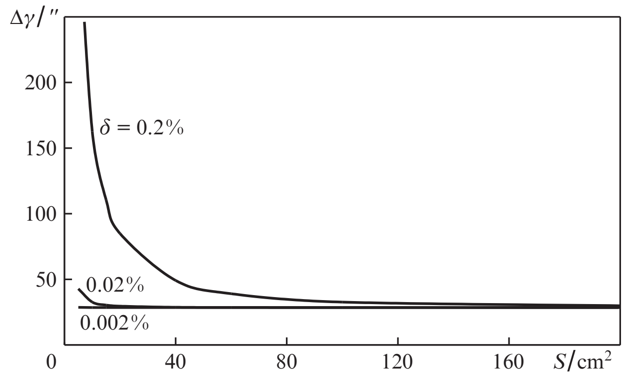

Figure 6. Dependence of the error in measuring the azimuth on the area of the cavity optical path at different relative losses in the cavity δ and parameters $\Delta\varphi = 3.3''$, $\Delta\beta = 10''$, $\beta = 30'$.

Note that for the relative loss in the cavity less than 0.02% (best foreign results), with increasing cavity area the contribution of the error associated with the RL quickly becomes less than the contribution of measurement error of inclinations. The contribution of the error in determining the latitude is negligible.

In general, the accuracy in determining the azimuth $\Delta\gamma = 28''$ for the gyrocompassing time 600 s corresponds to the accuracy of the best geodetic gyrotheodolites [16].

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