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SURFACE WAVES

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Profiles of optical surface waves formed at the metal – photorefractive crystal interface

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Abstract. **Photorefractive surface waves propagating in a stationary regime along the surface of the metal – photorefractive crystal (SBN-75) interface are considered. The transverse structure of the optical field distribution is calculated in the near- and far-field zones, depending on the angle of incidence of the exciting wave. The calculation results are in good agreement with the published experimental results. It is shown that the photorefractive effect leads to a splitting of the spectrum of a surface plasmon polariton excited at the metal – photorefractive crystal interface.**

Keywords: *photorefractive crystal, photorefractive surface wave, surface plasmon polariton.*

1. Introduction

Study of optical waves propagating along the surface of photorefractive crystals (PRCs) is one of the important trends in modern optics. Interest in such waves, called photorefractive surface waves (PR SWs), is explained by both fundamental and applied reasons. From the point of view of possible applications, especially important is the fact that to excite a PR SW does not require prior formation of the waveguide, because the optical wave is self-localised in the surface region of the crystal due to the photorefractive effect, i.e., the dependence of the refractive index in the crystal on the intensity of propagating optical radiation. In PRCs this effect may begin to appear at relatively low optical intensities (~1 W cm^{-2}), which is particularly important from the practical point of view [1].

Interest in PR SWs is demonstrated in a number of articles, for example [2–5]. These papers appeared after the first works on the subject [6, 7]. Nevertheless, to date there are few studies which consider in detail the experiments on the excitation of PR SWs and their registration, as well as measure the efficiency of their excitation. Therefore, it is necessary to single out the works of Sychugov, Ivleva and their co-authors [1, 8], who experimentally demonstrated the possibility of excitation of PR SWs at the interface with a metal, registered a record-high efficiency of excitation of PR SWs in a strontium barium niobate (SBN) single crystal bordering a metal, and gave a detailed layout of the experiment, allowing further detailed study and analysis of PR SWs. In particular, the authors of paper [8] observed interesting characteristics of the distributions of optical fields of PR SWs excited in the SBN – metal system in the near-and far-field zones.

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The SBN – metal system is especially promising, because it allows one to control the parameters of PR SW excitation with the help of an electric field. At the same time, it remains unclear how the experimentally recorded characteristics of the optical field distribution fit into the currently used theoretical model based on a system of Kukhtarev equations [9] developed, in particular, in papers [1, 3, 4, 6, 7, 10]. The validation of this model for calculating the optical field distribution is particularly important for conducting further research on optimisation of the processes of PR SW excitation and registration. One of the validation stages is to compare the reliable experimental data [8] with the results of the corresponding calculations.

The goal of this paper is to calculate the distributions of PR SW optical fields in the PR SW – metal system and compare them with previously published experimental results [8].

2. Theoretical model

In this paper, we rely on the experimental results obtained in [8], and therefore consider a similar optical scheme of PR SW propagation and observation. It is assumed that a SBN single crystal occupies a region $x > 0$, the optical axis is parallel to the x axis, and a region $x < 0$ corresponds to a metal with a complex refractive index n_1.

An extraordinary TM-polarised PR SW propagates along the z axis in the surface region of the PRC. A PR SW is generated due to the interference of an optical wave, incident on the interface ($z = 0$) from the SBN at an angle θ, and an optical wave reflected from the interface. The key role belongs to the photorefractive effect, thereby creating a volume phase grating, which prevents the penetration of the interference optical field into the crystal [2].

As was already noted, the model that allows a quantitative description of these processes was first proposed in [6] and is based on the theory of the optical interaction in the PRC, previously proposed by Kukhtarev and co-authors [9]. The basis of this model is a system of equations, which includes the kinetic equation, the Gauss equation, the expression for the charge density, the continuity equation and the equation for the current density. Assuming that the electric fields of the space charges and currents in the problem under study are directed along the x axis and depend only on the coordinate x, the basic system of the equations can be written as [10]

$$\frac{\partial N_{\mathrm{D}}^{+}}{\partial t} = s(I_{\mathrm{d}} + I)(N_{\mathrm{D}} - N_{\mathrm{D}}^{+}) - \sigma N_{\mathrm{e}} N_{\mathrm{D}}^{+}, \quad (1)$$

$$\varepsilon\frac{\partial E_{sc}}{\partial x} = \rho, \quad (2)$$

$$\rho = q(N_D^+ - N_A - N_e), \quad (3)$$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial j}{\partial x}, \quad (4)$$

$$j = q\mu N_e E_{sc} + k_B T\mu\frac{\partial N_e}{\partial x}, \quad (5)$$

where $N_D(x, t)$ is the concentration of donors in the crystal; $N_e(x, t)$ is the concentration of free electrons; $N_A(x,t)$ is the concentration of acceptors; $N_D^+(x,t)$ is the concentration of ionised donors; $j(x,t)$ is the electric current density; $I_d(x,t)$ is the equivalent dark intensity; $I(x,t)$ is the optical intensity; $\rho(x,t)$ is the charge density; ε is the dielectric constant; s is the photoionisation cross section; q is the elementary charge; μ is the electron mobility; σ is the recombination constant; E_{sc} is the space charge field; and T is the temperature.

This system of equations makes it possible to determine the electric field E_{sc}, created by space charges inside the crystal. Intracrystalline electric field E_{sc}, in turn, causes a change in the refractive index due to the linear electrooptic effect, which ultimately leads to a redistribution of the optical field and formation of the PR SW. The corresponding distributions of optical fields are the solutions of the wave equation derived from Maxwell's equations [4]. The solution of the wave equation for the optical field of the PR SW is expressed in the form stationary optical modes. For a TM-polarised surface wave the y-component of the magnetic field is $H(x, z) = A(x)\exp(i\beta z)$, where β is the mode propagation constant, and $A(x)$ is the amplitude of the magnetic field with the boundary condition at infinity $A(x)|_{x=\infty} = 0$. For the selected polarisation of the optical wave the SBN crystal can be characterised by a refractive index n_2, defined by extraordinary refractive index of the crystal and the radiation intensity. The corresponding analytical expressions are given below.

Taking into account the condition of stationarity, i.e. the time independence of the solution, and the boundary condition $j|_{x=\infty} = 0$, we obtain from (1)–(5) the expression for the intracrystalline field of space charges:

$$E_{sc}(x) = -\frac{k_B T}{q(I_d + I)}\frac{dI(x)}{dx}. \quad (6)$$

Thus, the intracrystalline field created by space charges and directed along the x axis, which coincides with the optical axis of the crystal, is determined by the field intensity, temperature, elementary charge and Boltzmann constant.

The field E_{sc} changes the extraordinary refractive index n_2 [11]:

$$n_2(E_{sc}) = n_e - n_e^3 r_{33}\frac{E_{sc}}{2}, \quad (7)$$

where r_{33} is the electrooptic tensor component and n_e is the extraordinary refractive index in the absence of the field.

Substitution of (7) for the changed extraordinary refractive index, taking into account (6) and the above expression for the mode field, into the wave equation leads to the nonlinear differential equation for the distribution of the amplitude of the magnetic field of the mode, $A(x)$ [12]:

$$\frac{d^2A(x)}{dx^2} + \gamma\frac{A^2(x)}{A^2(x) + 2I_d/(n_e Z_0)}\frac{dA(x)}{dx}$$
$$+(k_0^2 n_e^2 - \beta^2)A(x) = 0, \quad (8)$$

where $k_0 = 2\pi/\lambda_0$ is the wave number in vacuum; Z_0 is the vacuum impedance; and $\gamma = 2k_0 n_e^4 r_{33} k_B T/q$ is the damping coefficient.

We restrict ourselves to the case $I(x) \gg I_d(x,t)$, which corresponds, in particular, to the conditions of the experiment described in [8]. As a result, the nonlinear equation (8) is simplified and transforms into a linear differential equation of second order:

$$\frac{d^2A(x)}{dx^2} + \gamma\frac{dA(x)}{dx} + (k_0^2 n_e^2 - \beta^2)A(x) = 0. \quad (9)$$

Note that the process of PR SW formation and propagation is essentially nonlinear, because it is determined by the dependence of the refractive index on the intensity of the optical wave. This is reflected in a nonzero value of the damping coefficient ($\gamma \neq 0$), which determines a decrease in the amplitude of the optical wave along the x coordinate and allows the boundary condition at infinity, $A(x)|_{x=\infty} = 0$, to be fulfilled. In the linear case, $\gamma = 0$, and the solution in the form of surface waves, i.e., satisfying the boundary condition $A(x)|_{x=\infty} = 0$, is absent. Next, we consider the solutions of equation (9), corresponding to different regimes of PR SW excitation and various types of excited modes.

3. Optical modes in the PRC at the boundary with a metal

Usually, in the theory of oscillations [13] three major domains of solutions of equation (9) are considered, which, if we introduce the parameter $\alpha^2 = k_0^2 n_2^2 - \beta^2$, will have the form:

$\alpha^2 - \gamma^2/4 > 0$ (domain I, oscillation regime),

$\alpha^2 - \gamma^2/4 = 0$ (domain II, critical regime), (10)

$\alpha^2 - \gamma^2/4 < 0$ (domain III, aperiodic regime).

In the studied system, various domains of the solutions should correspond to different optical modes; therefore, the domain

$\alpha^2 \leqslant 0$ (domain IV)

should be considered as well. In the linear case this domain for certain parameters of the metal and dielectric corresponds to the propagation of surface plasmon polaritons. In the case of a nonlinear metal–PRC system we can suggest the possible existence of a hybrid plasmon-polariton photorefractive mode.

Domain I. This type of mode has been discussed in detail in Refs [3, 4, 6–8] for the PRC–dielectric system. In the case of a metal, the refractive index is a complex quantity, which can lead to a significant change in the distribution of the field, particularly, at the interface. To this end, $\beta^2 < k_0^2 n_2^2 - \gamma^2/4$ and the surface wave amplitude in the PRC region $x > 0$ can be written as

$$A(x) = A_0\exp(-\gamma x/2)\cos[(k_0^2 n_2^2 - \beta^2 - \gamma^2/4)^{1/2}x + \varphi]. \quad (11)$$

In a metal, which corresponds to the region $x < 0$, the surface wave amplitude is

$$U(x) = U_0 \exp[(\beta^2 - k_0^2 n_1^2)^{1/2} x]. \quad (12)$$

The boundary conditions at the metal–PRC interface have the usual form

$$U(x)\big|_{x=0} = A(x)\big|_{x=0}, \; \frac{1}{\varepsilon_1}\frac{dU(x)}{dx}\bigg|_{x=0} = \frac{1}{\varepsilon_2}\frac{dA(x)}{dx}\bigg|_{x=0} \quad (13)$$

(where $\varepsilon_1 = n_1^2$, $\varepsilon_2 = n_2^2$), and lead to the expressions for the amplitude factors and the phase:

$$U_0 = A_0 \cos\varphi,$$
$$\varphi = \arctan\left\{\frac{\varepsilon_2}{(k_0^2 n_2^2 - \beta^2 - \gamma^2/4)^{1/2}}\left[\frac{(\beta^2 - k_0^2 n_1^2)^{1/2}}{\varepsilon_1} + \frac{\gamma}{2\varepsilon_2}\right]\right\}. \quad (14)$$

As stated in [8], this relation is similar to the dispersion relation for thin-film waveguides. But here, it defines the initial phase of the surface wave as a function of the propagation constant β. The spectrum of this constant is continuous, and in magnitude it coincides with the projection (on the z axis) of the wave vector of the optical wave that excites the considered PR SW, i.e., $\beta = k_0 n_2 \sin\theta$ [3, 4, 8].

The amplitude parameter A_0 is free, but hereafter, it is convenient to normalise it if we set, for each considered mode, the value of the transferred power per unit length along the y coordinate. This is equivalent to the introduction of the normalisation condition $\int_{-\infty}^{\infty} |A(x)|^2 dx = \text{const}$.

Domain II (i.e., $\beta^2 = k_0^2 n_2^2 - \gamma^2/4$) corresponds to the critical damping of the oscillations of a harmonic oscillator with friction. It is well known that the solution of (9) in this case is given by

$$A(x) = (A_0 + B_0 x)\exp(-\gamma x/2). \quad (15)$$

The expression for the field in the metal is still given by (13). The amplitude factors are easily found from the boundary conditions (11):

$$U_0 = A_0, \; B_0 = A_0\left[(\beta^2 - k_0^2 n_1^2)^{1/2}\frac{\varepsilon_2}{\varepsilon_1} + \frac{\gamma}{2}\right]. \quad (16)$$

Domain III (i.e., $\beta^2 \geqslant k_0^2 n_2^2 - \gamma^2/4$) is characterised by a slower decay of the amplitude of the optical wave along the x axis as compared to domains I and II, but nevertheless up to an angle of grazing incidence the wave amplitude vanishes at infinity ($x = \infty$). This is a necessary condition for the stability of solutions and allows the experimental observation of the corresponding mode [4].

In the limiting case of grazing incidence ($\beta^2 = k_0^2 n_2^2$), the situation changes. Equation (9) transforms into the equation

$$\frac{d^2 A(x)}{dx^2} + \gamma\frac{dA(x)}{dx} = 0, \quad (17)$$

which has a solution in the general form

$$A(x) = A_0 \exp(-\gamma x) + B_0. \quad (18)$$

In a metal, the field is determined by expression (12). Taking into account the boundary conditions, we obtain the relations for the amplitude factors:

$$B_0 = \frac{U_0}{1 + C}, \quad A_0 = \frac{U_0 C}{1 + C},$$

$$C = -\left[1 + \frac{\varepsilon_1}{\varepsilon_2}\gamma\frac{1}{(\beta^2 - k_0^2 n_1^2)^{1/2}}\right]^{-1}.$$

For the metal–SBN system in question, $C \neq \infty$, the field amplitude in the crystal does not vanish at infinity ($x = \infty$) and the solution of (9) is similar to the profile of the field distribution of a dark soliton [14]. Experimental observation of such a mode may be prevented by the modulation instability [2].

Domain IV. As was noted above, by analogy with the surface plasmon modes, in linear metal–dielectric systems we should take into account the domain $\alpha^2 \leqslant 0$ for completeness or, which is the same, the domain $\beta^2 \geqslant k_0^2 n_2^2$. In this case, the solution of (9) in a metal still has the form (12), and in a PRC –

$$A(x) = A_0 \exp(\lambda_1 x) + B_0 \exp(\lambda_2 x), \quad (19)$$

where

$$\lambda_{1,2} = -\frac{\gamma}{2} \mp \sqrt{\frac{\gamma^2}{4} + (\beta^2 - k_0^2 n_2^2)}. \quad (20)$$

Because $\lambda_2 > 0$, the condition $A(x)|_{x=\infty} = 0$ can be satisfied only if $B_0 = 0$. Consequently,

$$A(x) = A_0 \exp\left\{\left[-\frac{\gamma}{2} - \sqrt{\frac{\gamma^2}{4} + (\beta^2 - k_0^2 n_2^2)}\right]x\right\}, \quad (21)$$

which in view of (12), (13) leads to the dispersion equation

$$\frac{1}{\varepsilon_1}(\beta^2 - k_0^2 n_1^2)^{1/2} = \frac{1}{\varepsilon_2}\left[-\frac{\gamma}{2} - \sqrt{\frac{\gamma^2}{4} + (\beta^2 - k_0^2 n_2^2)}\right]. \quad (22)$$

From equation (22) we obtain an expression for the propagation constant:

$$\beta = \left(k_0^2 \varepsilon_1 + \left\{\gamma\frac{\varepsilon_1 \varepsilon_2}{2(\varepsilon_1^2 - \varepsilon_2^2)} \pm \sqrt{\left[\gamma\frac{\varepsilon_1 \varepsilon_2}{2(\varepsilon_1^2 - \varepsilon_2^2)}\right]^2 - k_0^2\frac{\varepsilon_1^2}{\varepsilon_1 + \varepsilon_2}}\right\}^2\right)^{1/2}. \quad (23)$$

The mode corresponding to (23) can be interpreted as a hybrid plasmon-polariton photorefractive mode. The presence of two signs before the radical indicates the splitting of the spectrum compared to the case of excitation of a surface plasmon-polariton wave in a conventional linear metal–dielectric system. It is easy to see that if $\gamma = 0$, i.e., in the linear case, expression (23) transforms into the well-known dispersion Sommerfeld–Zenneck equation for a surface plasmon polariton:

$$\beta = k_0\sqrt{\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}}.$$

Note that the condition $\beta^2 > k_0^2 n_2^2$ could not be fulfilled when exciting a PR SW with the help of the technique used in [1, 8]; however, in principle, it can be done and the corresponding mode can be excited by the use of, for example, tunnel or lattice excitation. Next, we will calculate the field modes corresponding to different regimes of the experiment of Ref. [8].

4. The results of calculations of the mode fields and discussion

In Refs [1, 8], whose results we use in this paper, it was shown experimentally that the PR SW can propagate not only along the PRC–dielectric interface, but also along the PRC–metal interface. In this case, the authors noticed a number of interesting features in the distribution of the PR SW optical field as a function of angle of incidence, θ, of the exciting wave from the crystal. In particular, it was found that the PR SW in this case is characterised by a 'broad bright stripe in the region of the crystal, not adjacent to the metal layer'. In addition, it was noted that as the angle θ is decrease, a shift of the stripe into the crystal is observed, which most clearly manifests itself in a range $\theta \sim 0 - 2°$. The authors of paper [8] suggested that these features are the result of changes in the distribution of electrostatic fields of volume charges in the illuminated area of the crystal, caused by the presence of the metal coating. These effects are obviously to be considered in the theory developed in Sections 2 and 3, but this has not been verified numerically in [1, 8]. To test this assumption, we set the following values of the basic physical parameters of the system that need to be substituted in the analytical expressions for the PR SW optical fields: $r_{33} = 750$ pm V^{-1}; $n_1 = 0.04 - 2.56i$; $n_e = 2.36$; $\lambda_0 = 0.44$ µm; $\theta = 4°$, $2°$, $0.5°$ and $0.2°$ (θ_c, critical damping), or $0.02°$ (grazing incidence); and $T = 300$ K ($k_B = 1.38\times10^{-23}$ J K^{-1}, $q = 1.60\times10^{-19}$ Q).

It should also be noted that the authors of paper [8] were the first to propose an original method for registering the distribution of the PR SW optical field directly at the end face of the crystal, and in the far-field zone. This was achieved by using an additional optional lens, introduced in the optical scheme. Thus, there were obtained the distribution patterns of optical fields in the near- and far-field zones as functions of the angle of incidence of the exciting optical wave. However, the corresponding numerical calculations, which could explain these distribution patterns, have not been performed. Therefore, in this paper, apart from the calculation of the mode fields in the near-field zone, i.e., directly in the SBN crystal, we have also calculated the amplitude of the Fourier spectra of these fields, corresponding to the distributions of the optical field in the far-field zone, obtained in the experiment [8]. The Fourier spectra of the optical fields, $F(k)$, were obtained from the analytical expressions that have been found from the expression for the Fourier transform: $F(k) = \int_{-\infty}^{\infty} A(x)\exp(-ikx)\,dx$, where k is the spatial frequency. In this case, both positive and negative optical frequencies, corresponding to the positive and negative angles of diffraction in the far field, were taken into account.

The results of calculations of the distributions of optical fields and squares of the moduli of the Fourier amplitudes are shown in Figs 1 and 2. In Ref. [8] one can find the corresponding experimental photographs. The comparison suggests a good agreement of the theory with the experiment. In particular, well traced are a wide bright stripe, separated from the metal by a dark region, and its shift into the crystal with decreasing angle of incidence. With decreasing angle from 4° to 0.02°, the size of the dark region preceding the first bright stripe, increases from 1 to 10 µm, which is in good agreement with the experimental data. In the experiment, a gradual narrowing of the intensity distribution in the far field with decreasing angle θ was also observed. As a result, the intensity distribution with two distinct peaks was replaced by the distribution with one clear peak. This also agrees with the calculations performed (see Fig. 2).

Overall, the analysis of the results of calculations presented in Figs 1 and 2 allows one to make a conclusion that the main parameter determining the distribution of the optical field is the angle of incidence, θ_c, corresponding to the case of critical damping and equal to 0.2° for the selected system parameters. In the case of $\theta > \theta_c$ (Figs 1a, b and 2a, b), the mode field is of oscillatory nature, and its Fourier spectrum contains two peaks. The case of critical damping [curve (*2*) in Figs 1a and 2b] is characterised by a peak in the transverse distribution of the field and a peak in its Fourier spectrum. At $\theta \geqslant \theta_c$, the characteristic width $1/\gamma$ of the field distribution along the x axis, i.e., the attenuation depth, is ~30 µm, while at $\theta < \theta_c$, it gradually increases to infinity at $\theta = 0$. The above properties are fully confirmed by the experimental data presented in [8].

The inset in Fig. 1a clearly shows a change in the sign of the derivative of the optical field at the metal–crystal interface. This effect is associated with a negative sign of the real part of the dielectric constant of the metal and determines the possibility of the excitation of surface plasmon polaritons. As was already mentioned above, the experimental setup used in [8] made it impossible to excite the plasmon mode. It should also be noted that the experimental observation of the splitting of the spectrum, which follows from (23), is quite a challenging task because of the short propagation length of the surface plasmon and its broad spectrum. At the same time, comparison of Figs 1d and 2d with the photographs of the optical fields in the case of grazing incidence ($\theta = 0$) [8] suggests that in the experiment the dark soliton was excited or this excitation regime was nearly achieved. This issue requires further study. In general, good agreement of our model with the experimental data suggests the feasibility and advisability of its use for calculating the distribution of the PR SW optical fields with TM polarisation at the metal–PRC interface.

5. Conclusions

Thus, we have considered the theoretical model describing the propagation of surface waves in the PRC–metal system. We have analysed the main types of TM-polarised optical waves for this system. It is shown that the photorefractive effect can lead to the splitting of the spectrum of surface plasmon polaritons excited at the metal–PRC interface. Good agreement between the calculated distributions of optical fields with the published experimental results [8] suggests that this approach is quite promising in the calculation of distributions of surface photorefractive waves.

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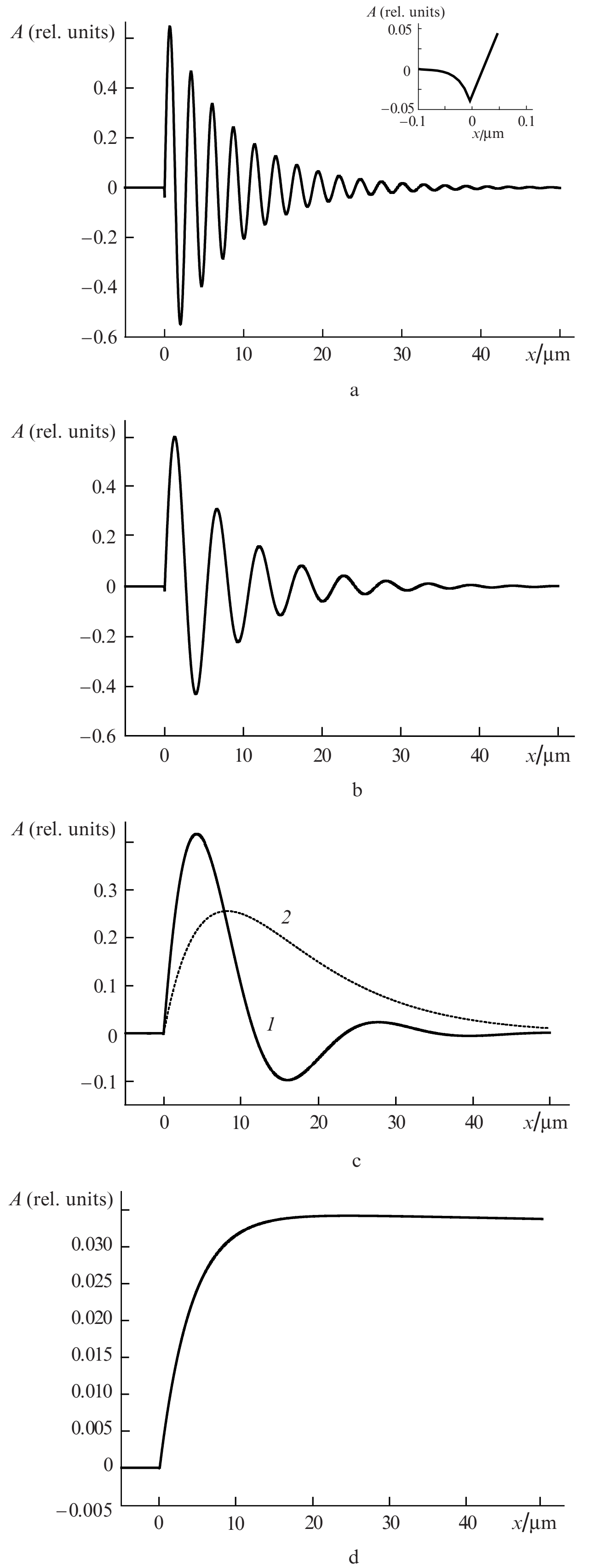

Figure 1. Transverse distribution of the amplitude of the PR SW optical field with the TM polarisation at an angle of incidence of the exciting wave $\theta = 4°$ (a), $2°$ (b), $0.5°$ (*1*) and $0.2°$ (*2*) (c) and $0.02°$ (d).

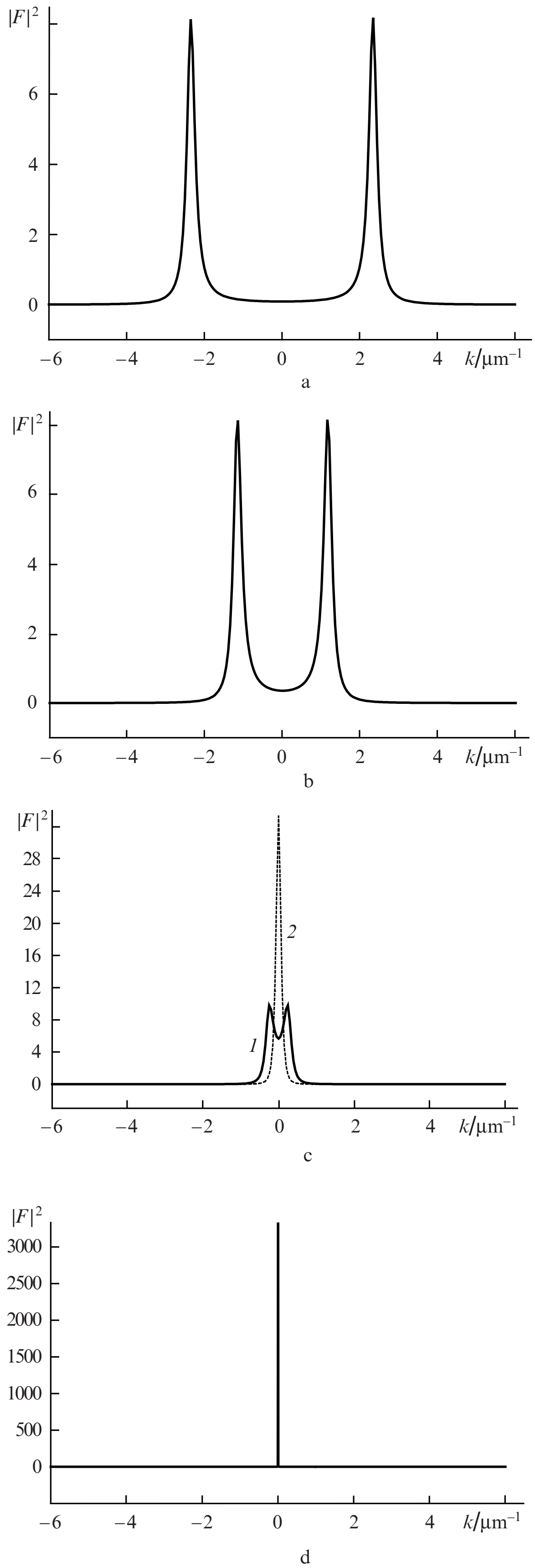

Figure 2. Square of modulus of the Fourier spectrum of the transverse distribution of the amplitude of the PR SW optical field with the TM polarisation at $\theta = 4°$ (a), $2°$ (b), $0.5°$ (*1*) and $0.2°$ (*2*) (c) and $0.02°$ (d).

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