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# Supersensitive SQUID/magnetostrictor detecting system

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***Abstract.*** **It is shown that using the state-of-the-art quantum interferometer (SQUID) with the resolution $10^{-6}$ $\Phi_0$ Hz$^{-1/2}$ = $2.07\times10^{-21}$ Wb Hz$^{-1/2}$, coupled to a magnetostrictor, playing the role of tensomagnetic transducer, it is possible to construct a system for detecting pressure variations with the ultimate sensitivity of $10^{-13}$ Pa Hz$^{-1/2}$ and for measuring specific elongation with the sensitivity of $10^{-24}$ Hz$^{-1/2}$. The analysis of physical grounds of the inverse magnetostriction effect demonstrates concrete ways to essentially higher efficiency of tensomagnetic conversion. The estimates performed demonstrate the possibility of using the SQUID/magnetostrictor system as a detector of gravitational waves. Other possibilities of using this system for solving both fundamental and applied problems are also considered.**

***Keywords:*** *quantum interferometer, magnetostrictor, gravitational waves.*

## 1. Introduction

Using the state-of-the-art superconducting quantum interferometer (SQUID [1, 2]) with resolution $\langle\delta\Phi\rangle$ Hz$^{-1/2}$ = $10^{-6}\Phi_0$Hz$^{-1/2}$ coupled to a magnetostrictor that plays the role of transducing the pressure $p$ and/or elongation $\Delta l/l$ into a magnetic signal, one can design a system for measuring $p$ and $\Delta l/l$ with the sensitivity of $\sim10^{-13}$ Pa Hz$^{-1/2}$ and $\sim0.5\times10^{-24}$ Hz$^{-1/2}$, respectively. The above-mentioned resolution of the quantum interferometer, equal to one millionth part of the flux quantum $\Phi_0 = \pi\hbar/e = 2.07\times10^{-15}$ Wb ($e$ is the electron charge) in the unit frequency bandwidth is considered as high, but not record-breaking for a state-of-the-art SQUID. In the 1990s the low-noise two-step direct-current SQUIDs (DC SQUIDs) were developed to support the operation of Weber-type gravitational antennas, in which the elongation is detected via the mechanical displacement of the magnetic flux, frozen in the reversibly-deformable superconducting circuit. In such a two-cascade cryoelectronic circuit the second DC SQUID plays the role of a low-noise amplifier of electric signals, coming from the first DC SQUID [3, 4]. In this case the resolution $\langle\delta\Phi\rangle$ Hz$^{-1/2}$ = $(2-5)\times10^{-7}\Phi_0$ Hz$^{-1/2}$ is achieved [5 – 8]. In particular cases in the devices of such type the obtained resolution appeared to be greater than formally allowed by the Heisenberg uncertainty relation $(\langle\delta\Phi\rangle/\sqrt{\Delta f})^2/(2L) \geqslant \hbar/2$ ($\Delta f$ is the operating frequency bandwidth, $L$ is the inductivity of the operating ring of the SQUID with Josephson tunnel junctions) [9, 10]. Note, that in these cases no special methods of quantum squeezing [11, 12] were used to ‘overcome’ the quantum limit. In alternating-current SQUIDs (RF SQUIDs), the condition $LI_J < \Phi_0/(2\pi)$ being satisfied allows transition to the anhysteretic regime, in which the direct channel of energy dissipation is completely absent ($I_J$ is the critical tunnelling current of the Josephson junction). The unhysteretic RF SQUID [13 – 15] will possess nonzero noise temperature only due to being coupled to ‘external dissipative electronics’, and its resolution is estimated as being able to reach $\sim10^{-9}\Phi_0$ Hz$^{-1/2}$.

The operation of the pressure or elongation transducer, which is supposed to be used together with the SQUID, is based on the inverse magnetostriction effect, or Villari effect (discovered by E. Villari in 1865). This effect manifests itself in the change of magnetisation under the action of a mechanical stress (pressure) and/or deformation of the magnetostrictive sample. In turn, the ‘direct’ magnetostriction effect, or ‘common’ magnetostriction (discovered by J. Joule in 1842) manifests itself in the change of magnetostrictive sample dimensions when being magnetised [16]. To demonstrate the relation between the direct and inverse effect, let us write the expression for the variation of thermodynamic potential density $\wp$ of the magnetostrictor, depending on the variables $U$, $T$, $S$, $p$, $H$:

$$\mathrm{d}\wp(U,T,S,p,H) = \mathrm{d}\wp_0(U,T,S) + \frac{l(H)}{l_0}\mathrm{d}p + B(p)\mathrm{d}H.$$

Here $U$, $T$, and $S$ (‘thermal variables’) are the internal energy, temperature, and entropy, respectively; $p$ is the pressure, caused by the magnetostrictor deformation; $H$ is the magnetic field strength; $l$ and $l_0$ are the values of the magnetostrictor length in the presence of the field and without it; and $B$ is the magnetic field induction inside the magnetostrictor. The last two terms characterise the contributions to d$\wp$ from the energy of deformation and magnetisation. Then the partial derivatives of the density $\wp$ are calculated as

$$\frac{\partial\wp}{\partial p} = \frac{l(H)}{l_0} \text{ и } \frac{\partial\wp}{\partial H} = B(l),$$

and from the equality of mixed derivatives it follows that

$$\frac{1}{l_0}\frac{\partial l}{\partial H} = \frac{\partial^2\wp}{\partial H\partial p} = \frac{\partial^2\wp}{\partial p\partial H} = \frac{\partial B}{\partial p}.$$

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Then, expressing the elongation in terms of the pressure variation, in correspondence with Hook's law $dp = E dl/l_0$, we arrive at the quantitative relation between the direct (the constant $\Lambda$) and inverse [the constant $\Lambda^{(-1)}$] magnetostriction effects:

$$\Lambda = \frac{\partial p}{\partial B} = \frac{E}{\mu_0 \mu}\frac{\partial B}{\partial p} = \frac{E}{\mu_0\mu}\Lambda^{(-1)},$$

where $E$ is the Young modulus; and $\mu_0$ and $\mu$ are the magnetic permeability of vacuum and the deformed body, respectively. Since for the alloy, consisting of Pt (54%) and Fe (46%) and possessing the magnetostriction parameters far from record-breaking [17] ($\Lambda/E \approx 6\times10^{-3}$ T$^{-1}$), the inverse effect will be characterised by the constant $\Lambda^{(-1)} \approx \mu_0\mu\Lambda/E \approx 7\times10^{-6}$ T Pa$^{-1}$ (here $\mu \approx 1000$).

## 2. SQUID/magnetostrictor system

The idea of using the inverse magnetostriction effect for strain measurements can be hardly considered as original. The descriptions of such systems (without using a SQUID) are not only presented in special reviews [18], but also can be found in handbooks on general physics [19]. From the point of view of applications, the Villari effect is characterised by the magnetostriction sensitivity, which coincides with the constant $\Lambda^{(-1)} = \partial B/\partial p$ and demonstrates quantitative relation between the variation of magnetic induction and the variation of elastic stress, causing the former in a particular material [20].

Let us estimate the ultimate capabilities of this classical method in detecting the magnetic response by means of a superconducting quantum interferometer under the condition $\Delta\Phi = \delta\Phi$, i.e., when the magnetic signal is equal to the resolution of the SQUID. The variation of the flux of the magnetic induction $B$ through the section $S_{ms}$ of the magnetostriction cylinder is determined by the relation $S_{ms}\Delta B = S_{ms}\Lambda^{(-1)}\Delta p$, from where the ultimate sensitivity of the SQUID/magnetostrictor system used for pressure measurement is expressed in terms of the resolution of the used quantum interferometer in the form $\delta p = \delta\Phi/(S_{ms}\Lambda^{(-1)})$. If $\langle\delta\Phi\rangle$ Hz$^{-1/2} = 10^{-6}\Phi_0$ Hz$^{-1/2}$ = $2.07\times10^{-21}$ Wb Hz$^{-1/2}$, $S_{ms} = 0.003$ m$^2$ and $\Lambda^{(-1)} \approx 7\times10^{-6}$ T Pa$^{-1}$, then the ultimate sensitivity of the system is $\langle\delta p\rangle$ Hz$^{-1/2}$ = $10^{-13}$ Pa Hz$^{-1/2}$. This pressure, which in principle can be still detected by the system in the unit bandwidth, corresponds to the ultimate detectable elongation of $\langle\delta l/l\rangle$ Hz$^{-1/2} = 10^{-24}$ Hz$^{-1/2}$ in a magnetostrictor with the typical value of the Young modulus $E = 100$ GPa.

Thus, preliminary estimates, obtained without explicit accounting for the intrinsic noise of the magnetostrictor, demonstrate the possibility of using the SQUID/magnetostrictor system for detecting gravitational waves (Fig. 1) with the intensity $\pi c^3 f^2 |\delta g_{ij}^{\perp}|^2/(2\gamma) \approx 6.5\times10^{-7}$ W m$^{-2}$, which at the frequency $f = 1$ kHz corresponds to the perturbation amplitude of the transverse metric tensor components $|\delta g_{ij}^{\perp}|^2 \approx 10^{-24}$ (the universal gravitation constant is $\gamma = 6.67\times10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$).

It is not necessary to make the entire antenna of a magnetostrictive material. In Fig. 2 the schematic diagram is presented, where the elongation of the working body having the shape of a 'large' frame is transferred onto a 'little' ($l_{ms} \ll l_{fr}$) magnetostrictive body having the shape of a tablet via pressure concentrators. The stress, corresponding to the elongation of the frame under the action of gravitational wave, $\delta p_{fr} = E_{fr}\delta l_{fr}/l_{fr} = E_{fr}|\delta g_{ij}^{\perp}|$, is 'concentrated' at the faces of the tablet: $\delta p_{ms} = (S_{fr}/S_{ms})\delta p_{fr} = (S_{fr}/S_{ms})E_{fr}|\delta g_{ij}^{\perp}|$, providing the magnetic response $\delta\Phi = S_{ms}\Lambda^{(-1)}\delta p_{ms} = S_{fr}E_{fr}\Lambda^{(-1)}|\delta g_{ij}^{\perp}|$, which is then detected by the SQUID ($S_{ms}$ and $S_{fr}$ are the areas of the magnetostrictor and the frame, respectively). The transverse size of the tablet is a free parameter that does not enter the last part of the expression for $\delta\Phi$, which allows the use of a low-inductance input loop of the superconducting flux transformer (encircling the tablet), which facilitates the increase in the coefficient of magnetic response transmission $\delta\Phi$ into the operating ring of the SQUID.

However, the ratio of the frame length $l_{fr}$ and the tablet thickness $l_{ms}$ now cannot be arbitrary, because from the condition of continuity of the mechanical link $\delta l_{ms} = \delta l_{fr}$, Newton's third law $S_{ms}\delta p_{ms} = S_{fr}\delta p_{fr}$ and Hook's law $\delta p = E\delta l/l$ it follows that $l_{fr}/l_{ms} = (S_{fr}/S_{ms})(E_{fr}/E_{ms})$. Then the resulting incre-

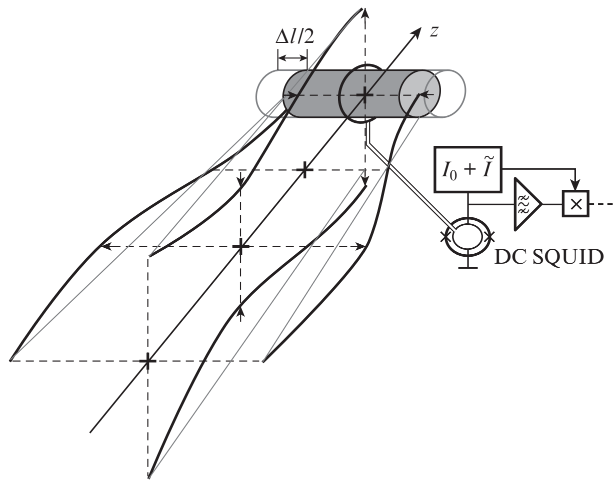


**Figure 1.** Qualitative relation between the geometry of gravitational wave propagation and the position of magnetostrictive antenna, the change of magnetic flux in which is detected by the SQUID. In the foreground the plane-polarised gravitational wave is schematically shown. Arrows point at the regions of decreasing and increasing dimensions of a virtual test body, caused by the influence of the gravitational wave. The proportion between the wavelength $\lambda$ and the dimension $l$ of the cylindrical antenna is deliberately distorted (really $\lambda$ is greater than $l$ by nearly 5 orders of magnitude); for clarity also exaggerated (by 20 orders of magnitude) is the variation of geometric dimensions of the test body in the field of the gravitational wave. Here and in Figs 2 and 3 the electric circuit includes a DC SQUID and the elements of service electronics [1], the crosses in the operating ring of the DC SQUID mark Josephson tunnel junctions – the principal elements of the superconducting quantum interferometer [2].

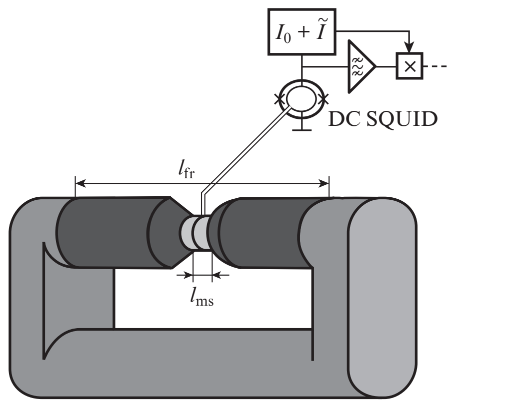


**Figure 2.** Schematic diagram of a 'frame' gravitational antenna with compact sensitive magnetostrictive element ('grey' tablet between the faces of the 'black' concentrator of mechanical stresses).

ment of the magnetic flux $\delta\Phi = S_{\rm fr}E_{\rm fr}\Lambda^{(-1)}|\delta g_{ij}^{\perp}| = (l_{\rm fr}/l_{\rm ms})\times S_{\rm ms}E_{\rm ms}\Lambda^{(-1)}|\delta g_{ij}^{\perp}|$ appears to be $l_{\rm fr}/l_{\rm ms}$ times greater than the flux increment in the absence of the frame, $\delta\Phi_{\rm ms} = S_{\rm ms}E_{\rm ms}\times \Lambda^{(-1)}|\delta g_{ij}^{\perp}|$. It is assumed that the measurements of $\delta\Phi$ are performed at the eigenfrequency of the frame vibration, different from the resonance vibration frequencies of the concentrator arms, which allows considering the latter as incompressible.

Now let us explicitly allow for the contribution of the magnetostrictor $\delta\Phi_{\rm ms}$ into the total fluctuations of the magnetic flux $\delta\Phi_{\rm ms} + \delta\Phi_{\rm SQUID}$ ($\delta\Phi_{\rm SQUID}$ are the intrinsic noises of the SQUID) that limit the total sensitivity of the system. To make the increment of pressure $\Delta p$ and/or elongation $\Delta l/l$, estimated above, actually detectable, it is also necessary to satisfy the condition that the magnetic self-fluctuations of the magnetostrictive rod $\delta\Phi_{\rm ms}$ are small compared to the Nyquist noise of the SQUID, i.e., $\delta\Phi_{\rm ms} \ll \delta\Phi_{\rm SQUID}$. The estimate of $\delta\Phi_{\rm ms}$ may be obtained using an analogy with the simplest derivation of the Nyquist formula $\delta U = \sqrt{4\pi kTR\Delta f}$ that allows calculation of the amplitude of noise voltage $\delta U$, produced by the resistor $R$ at the temperature $T$, within the frequency bandwidth $\Delta f$. In fact, this formula follows from the condition that within the unit frequency bandwidth the power $W$ is continuously dissipated, which corresponds to the equilibrium value of the energy $kT$ per one degree of freedom: $kT = W\tau = (\delta U/\sqrt{2})^2/(2\pi R\Delta f)$, where $\tau = \Delta f^{-1}$. The analogous condition for a paramagnetic in the absence of external magnetic biasing, when $\langle\delta\Phi_{\rm ext}\rangle = 0$, has the form

$$kT = W\tau = (\delta E_{\rm pm}/\tau_{\|})\,\frac{1}{2\pi\Delta f} = \left[\frac{l_{\rm pm}}{\mu\mu_0 S_{\rm pm}}\left(\frac{\delta\Phi_{\rm pm}}{\sqrt{2}}\right)^2\tau_{\|}^{-1}\right]\frac{1}{2\pi\Delta f}\,,$$

from which it follows that $\delta\Phi_{\rm pm} = [4\pi kT(\mu\mu_0 S_{\rm pm}\tau_{\|}/l_{\rm pm})\Delta f]^{1/2}$, where $\tau_{\|}$ is the longitudinal relaxation time of the spin system. For a ferromagnetic with external magnetic biasing the mean flux is nonzero ($\langle\delta\Phi_{\rm ext}\rangle = B_{\rm ext}S_{\rm fm}$), and then $kT = [l_{\rm fm}B_{\rm ext}\times \delta\Phi_{\rm fm}/(\mu\mu_0)]\tau_{\|}^{-1}/(2\pi\Delta f)$, from where we get $\delta\Phi_{\rm fm} = 2\pi kT\times [\mu\mu_0\tau_{\|}/(l_{\rm fm}B_{\rm ext})]\Delta f$. The longitudinal relaxation time can be estimated based on 'reverse' width of spin resonance $\Delta f_{\rm sr}^{-1}$. Thus, for ferromagnetic resonance in ferroalumoyttrium garnet ($f_{\rm sr} = 3.3\times10^9$ Hz for $B = 0.11$ T, $\Delta f_{\rm sr} = 5\times10^5$ Hz) the longitudinal relaxation time amounts to $\sim 2\times10^{-6}$ s. Using these parameters to estimate the self-noise of the magnetostrictive core 0.1 m long at the liquid helium temperature ($T = 4.2$ K) in the saturation field $B_{\rm ext} = 1$ T, we conclude that their amplitude is limited from above by the value $\delta\Phi_{\rm ms}(\Delta f) \approx \delta\Phi_{\rm fm}(\Delta f) = 2\pi kT[\mu\mu_0\tau_{\|}/(l_{\rm fm}B_{\rm ext})]\Delta f < 10^{-23}$ Wb, and within the unit frequency bandwidth by the value $\delta\Phi_{\rm ms}(\Delta f = 1\ \text{Гц}) < 2\times10^{-29}$ Wb. Hence, the magnetic self-fluctuations of the magnetostrictive rod $\delta\Phi_{\rm ms}$ appear to be essentially (by eight orders of magnitude) smaller than the amplitudes of the Nyquist noise of the SQUID $\delta\Phi_{\rm SQUID} = 10^{-6}\Phi_0$ Hz$^{-1/2}$ = $2.07\times10^{-21}$ Wb Hz$^{-1/2}$.

The above estimate of the ultimate sensitivity was obtained without accounting for the transmission coefficient of the flux transformer. In the SQUID the superconducting flux transformer functions as a coupling and matching element. By means of this element the magnetic signal $\Delta\Phi$ from the macroscopic region, where, in correspondence with experimental conditions, the magnetic flux measurements are to be performed, is transferred into the input circuit of the interferometer, which, in accordance with the requirement $LI_{\rm J} \lesssim \Phi_0$ that excludes unambiguity of performance characteristic, must have microscopic dimensions. The magnetic flux transformer consists of a couple of loops, closed to form a single superconducting loop by means of a low-inductance twin-wire line (see Figs 1 and 2). The first loop is coupled to the macroscopic region and the second one is coupled to the operating circuit of the SQUID. Due to the conservation of the total magnetic flux, passing through the closed superconducting circuit, the variation $\Delta\Phi$ is transmitted from the first loop into the second one in correspondence with the condition $\Delta\Phi_1 = -\Delta\Phi_2$. The maximal transmission coefficient $\Delta\Phi_{\rm SQUID}/\Delta\Phi$ is proportional to $\sqrt{L/L_1}$ and usually takes the value within the limits 0.005–0.05 [21]. In long-term measurements such a reduction of sensitivity is easily compensated by increasing the signal acquisition time up to $\sim 10^4$ s. However, in detection of separate events such as a pulsed gravitational wave, when the artificial narrowing of the spectrum is not acceptable, it is required to increase the transmission coefficient of the flux transformer by means of efficient reduction of the inductance $L_1$of its coupling turn. For this purpose the loop should be shielded with an additional external superconducting ring (Fig. 3). An alternative way to increase the transmission coefficient of the flux transformer consists in artificial increase in the inductance of the SQUID input circuit. In order to bypass the limitation $LI_{\rm J} \lesssim \Phi_0$, the authors of Ref. [22] proposed to replace single Josephson tunnelling junctions in the SQUID with chains of $n$ successively connected junctions with nearly equal $I_{\rm J}$. In this case the limitation becomes less strict and takes the form $LI_{\rm J} \lesssim n\Phi_0$.

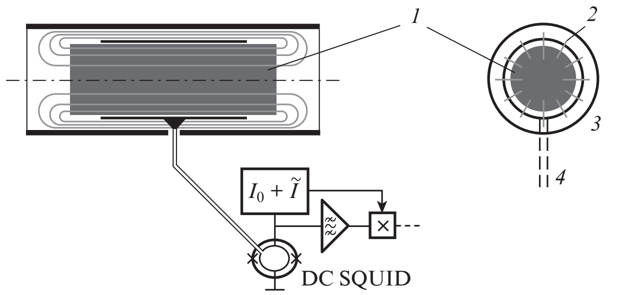


**Figure 3.** Schematic diagram of a superconducting magnetic flux concentrator:
(*1*) magnetostrictor; (*2*) tubular loop of the superconducting flux transformer; (*3*) external superconducting tube (superconducting flux concentrator); (*4*) twin-wire connection link of the flux transformer.

## 3. Physical grounds of magnetostriction and criteria for the optimal test body choice in the SQUID/magnetostrictor system

In connection with mentioned above, it is of certain interest to seek for ways of improving magnetostrictive properties of the substance, used as a test body in the SQUID/magnetostrictor system.

### 3.1. Phenomenological description of magnetostriction

In accordance with the simplest phenomenological scheme, describing magnetostriction effects, the common expansion of the free energy [23] $F(T,M) = F_0 + a(T - T_{\rm K})M^2 + bM^4$ ($T_{\rm K}$ is the Curie temperature) should be completed with a 'mixed' term, proportional to the product of the magnetic moment $M$ of the system and the relative elongation $\Delta l/l = \varepsilon$; then $F(T,M,\varepsilon) = F_0 + a(T - T_{\rm K})M^2 + bM^4 + c_1M\varepsilon$ [24]. Such a mixed term corresponds to the direct linear and inverse linear magnetostriction effects, i.e., $\varepsilon(M) \propto M$ and $\Delta M(\varepsilon) \propto \varepsilon$, respectively. The minimum of free energy in a ferromagnetic, possessing no magnetostriction properties, corresponds to the zero of the partial derivative

$$\frac{\partial F(T,M)}{\partial M} = 2a(T - T_K)M + 4bM^3 = 0,$$

from which the common expression for the spontaneous magnetization $M_0 = \sqrt{a(T_K - T)/(2b)}$ follows. The analogous condition for the free energy minimum with the mixed term taken into account,

$$\frac{\partial F(T,M)}{\partial M} = 2a(T - T_K)M + 4bM^3 + c_1\varepsilon = 0,$$

leads to a cubic equation with respect to $M$, the approximate solution of which includes the correction, responsible for deformation, $M \approx M_0 + \Delta M(\varepsilon) = M_0 + c_1\varepsilon/[8a(T_K - T)]$. It is seen that the magnetisation due to Villari effect used in the system, described in the present paper, grows as $T$ approaches the Curie temperature.

The mixed term, having the form $c_1 M\varepsilon$, corresponds to linear magnetodeformation effects, observed either in piezomagnetics (antiferromagnetics, where under the influence of deformation the equilibrium of oppositely polarised sublattices is broken), or under the conditions of 'common' quadratic magnetostriction effects, when an external magnetic field is applied to the sample (in this case the proportionality coefficient becomes dependent upon the external field, $c_1 = c_1(B_{ext}) \propto B_{ext}$) [24]. In the absence of the external magnetic biasing the quadratic effects should be described using the mixed term of the form $c_2 M^2\varepsilon$ (here, for simplicity, we do not take into account the tensor nature of the coefficients $c_1$ and $c_2$, reflecting the anisotropy of the crystal [25–27]). Such a mixed term corresponds to the direct quadratic and inverse linear magnetostriction effects, i.e., $\varepsilon(M) \propto M^2$ and $\Delta M(\varepsilon) \propto M\varepsilon$, respectively. As above, the expression of the magnetic moment, corresponding to the minimum of free energy of the ferromagnetic with the term $c_2 M^2\varepsilon$ taken into account, includes the correction, depending on the deformation, $M \approx M_0 + \Delta M(\varepsilon) = M_0 - (c_2\varepsilon/4)\sqrt{2b/[a(T_K - T)]}$, which is just the essence of the inverse magnetostriction effect. However, now the temperature dependence of the response to deformation is seen to appear weaker, i.e., $\Delta M(\varepsilon) \propto (T_K - T)^{-1/2}$ instead of $\Delta M(\varepsilon) \propto (T_K - T)^{-1}$. At the same time it appears that both in linear and quadratic cases the effect, used in the described SQUID/magnetostrictor system, is unlimitedly enhanced, when $T \to T_K$.

Thus, in order to increase the sensitivity, one should make the operating temperature of this measuring system to be as close to $T_K$ as possible. The Curie temperature of common magnetostrictive materials on the basis of iron or nickel amounts to hundreds of kelvins and, therefore, the detector using such substances under the condition $T \to T_K$ cannot be considered as low-noise detectors (and even less super-low-noise). This circumstance makes the search for cryogenic magnetostrictive materials, i.e., magnetostrictors with low (from a few kelvins to parts of a kelvin) Curie temperatures to be an extremely urgent problem.

It is also worth noting, that a magnetostrictor, as well as any ferromagnetic, possesses magnetic viscosity [25–27], characterising the scale of energy dissipation in the magnetic subsystem. Obviously, this viscosity limits the $Q$-factor of the loaded oscillatory system, including magnetostrictive elements. Therefore, in the case when it is required to provide a high $Q$-factor, it is probably reasonable, at the expense of an insignificant reduction of the recording system sensitivity, to replace the magnetostrictor with a piezomagnetic, in which, like in a single-domain antiferromagnetic, the losses, corresponding to the magnetic viscosity, appear to be essentially smaller.

### 3.2. Microscopic description of magnetostriction

An attempt of simple consideration of magnetostriction from the point of view of microscopic positions from the very beginning meets the necessity of direct accounting for collective effects of spin–spin interaction of electrons. Starting from the microscopic mechanism that implies no collective effects in the interaction of each electron with each electron, we find that the calculated sensitivity of the magnetostriction-based detector appears to be nearly zero. From the microscopic position, the 'ferromagnetic collectivism' manifests itself in self-consistent interaction of each spin with total magnetisation of all the rest electrons or holes, belonging to the appropriate energy zone, but with explicit accounting for the exchange spin– spin interaction. The spin polarisation due to the exchange interaction and/or under the action of the external magnetisation leads to the splitting of the initial energy zone into two subzones. The difference of the carrier numbers in the spin-split subzones $\Delta n_{\uparrow\downarrow}$ is expressed in terms of the product of the Fermi density of states $N_F$ and the difference of the 'subzone' energies $\Delta n_{\uparrow\downarrow} = (n_\uparrow - n_\downarrow)/2 = N_F \Delta E_{\uparrow\downarrow}$. The formation of subzones leads to the growth of the kinetic energy of the system:

$$\Delta E_{kin} = \frac{n_\uparrow - n_\downarrow}{2}\Delta E_{\uparrow\downarrow} = \frac{1}{N_F}\left(\frac{n_\uparrow - n_\downarrow}{2}\right)^2 = \frac{(\Delta n_{\uparrow\downarrow})^2}{N_F}.$$

Meanwhile, the energy of spin–spin interaction, expressed in terms of the exchange integral $J_{ss}$, will decrease:

$$\Delta E_{ex} = J_{ss} n_\uparrow n_\downarrow - J_{ss}\left(\frac{n_\uparrow + n_\downarrow}{2}\right)^2 = J_{ss}(\Delta n_{\uparrow\downarrow})^2.$$

Thus, the expression for the total change of energy with the polarisation of spins caused by the external field $B_{ext}$ taken into account may be written in the form $\Delta E = \Delta E_{kin} + \Delta E_{ex} - 2(\mu_B \Delta n_{\uparrow\downarrow})B_{ext} = (1/N_F - J_{ex})(\Delta n_{\uparrow\downarrow})^2 - 2\mu_B B_{ext}\Delta n_{\uparrow\downarrow}$ ($\mu_B$ is the Bohr magneton), and the condition of energy minimum of the system at $T \approx 0$ takes the form $\partial(E_0 + \Delta E)/\partial(\Delta n_{\uparrow\downarrow}) = 0$ (the closeness of the temperature to absolute zero makes it possible not to take the entropy into account). From the condition of minimum the equation $(1/N_F - J_{ss})\Delta n_{\uparrow\downarrow} = \mu_B B_{ext}$ follows, the solution of which allows determination of the ferromagnetic susceptibility

$$\chi = \frac{m_{int}}{B_{ext}} = \frac{\mu_B \Delta n_{\uparrow\downarrow}}{B_{ext}} = \frac{\mu_B^2 N_F}{1 - J_{ss}N_F}.$$

It is seen that at $T \approx 0$ the dependence of the internal specific magnetisation $m_{int}$ on the exchange integral of spin–spin interaction becomes stronger as the Stoner factor $J_{ss}N_F$ approaches unity: $\partial m_{int}/\partial J_{ss} = N_F m_{int}/(1 - J_{ss}N_F)$. Expressing the ferromagnetic susceptibility $\chi$ in terms of the Pauli susceptibility of the gas of non-interacting spins $\chi_P$, we arrive at the formula $\chi = \chi_P/[1 - (J_{ss}/\mu_B^2)\chi_P]$, analogous in structure to the known expression for the gain of a system with the feedback taken into account, $K = K_0/(1 - \beta K_0)$. Therefore, it appears

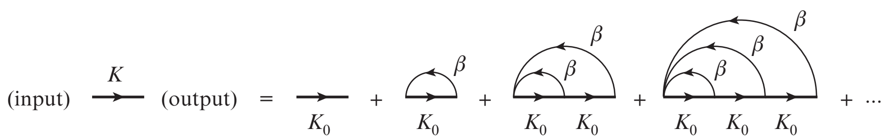

**Figure 4.** A series of diagrams, demonstrating the formation of the gain in a system with feedback. Straight lines in individual diagrams correspond to 'unperturbed' amplification ($K_0$), the arcs correspond to feedback channels with the transmission coefficient $\beta$, the straight lines are directed from input to output, and the arcs are directed from output to input. In the case of a magnetic system the straight-line segments correspond to Pauli susceptibility of non-interacting spin gas $\chi_P$ and the arcs represent the constant of exchange spin–spin interaction $J_{ss}/\mu_B^2$.

that the exchange interaction constant $J_{ss}/\mu_B^2$ plays the role of a positive feedback coefficient $\beta$, promoting the amplification of the internal polarisation, which is a response to the impact in the form of external magnetic field $B_{ext}$. The gain may be presented in the form of a power series, converging at $|\beta K_0| < 1$, $K = K_0(1 + \beta K_0 + \beta^2 K_0^2 + \beta^3 K_0^3 + ...)$, to which the series of diagrams presented in Fig. 4 corresponds. Successive summation of this series corresponds to taking the feedback effect into account in higher and higher orders of approximation: the first term describes the gain without feedback, the second describes the amplification of a part of already amplified response, passed from the output to the input via the feedback channel with the transmission coefficient $\beta$, the third term corresponds to the amplification of a part of the doubly-amplified response, etc.

Under the deformation, the relative variation of the distance between the spins $\Delta l_{ss}/l_{ss}$ will be approximately equal to the relative elongation of the entire magnetostrictive sample, which, in turn, implies the presence of essential dependence of the exchange integral of the spin–spin interaction $J_{ss}$ on the relative elongation of the crystal $\Delta J_{ss}/J_{ss} \simeq \Delta l_{ss}/l_{ss} = \Delta l/l$. When the Stoner factor approaches unity ($J_{ss}N_F \approx 1$), the strong dependence of the magnetisation $m_{int}$ on the exchange integral

$$\frac{\Delta m_{int}}{m_{int}} = \frac{J_{ss}N_F}{1 - J_{ss}N_F}\frac{\Delta J_{ss}}{J_{ss}} \approx \frac{J_{ss}N_F}{1 - J_{ss}N_F}\frac{\Delta l}{l} \approx \frac{1}{1 - J_{ss}N_F}\frac{\Delta l}{l}$$

makes it possible to reach high sensitivity when measuring the relative elongation with the SQUID/magnetostrictor system: $\Delta\Phi \approx [\mu_0\chi S_{ms}B_{ext}/(1 - J_{ss}N_F)]/(\Delta l/l)$, where $S_{ms}$ is the cross-section of the magnetostrictor.

Thus, while within the framework of the phenomenological ferromagnetism model one should seek for the high sensitivity of a magnetostrictive detector near the Curie temperature (see Section 3.1), from the position of the microscopic theory the highest sensitivity may be obtained when the Stoner factor approaches unity (which is implemented, e.g., in palladium).

## 4. Possible applications of the SQUID/magnetostrictor system and alternative scheme of detecting gravity fields with a SQUID

Apart from detecting gravitational waves, the SQUID/magnetostrictor system may be useful as a supersensitive accelerometer, as well as for performing high precision measurements of local variations of the free fall acceleration. The first of these facilities (measuring the inertia fields) is urgent in the problems of orientation and control of object motion, the second one (measuring gravity field of the Earth) is important for geophysics, geological exploration, etc.

If the test body (magnetostrictor) having the mass $m = \rho_{ms}S_{ms}l_{ms}$ bears on the base having the area $S_{ms}$, then the recorded acceleration $a$ will be related to the detected pressure $p$ via the formula $a = F/m = pS_{ms}/m = p/(\rho_{ms}l_{ms})$. Substituting into this relation the minimal detectable pressure $\delta p \sim 10^{-13}$ Pa Hz$^{-1/2}$, obtained in Section 2, and the typical density $\rho_{ms} \approx 10$ kg m$^{-3}$ and length $l \approx 0.1$ m of a magnetostrictor, we get the acceleration, detected at the ultimate sensitivity, $\delta a \sim 10^{-13}$ m s$^{-2}$ Hz$^{-1/2}$ = 0.01 nGal Hz$^{-1/2}$. Such high sensitivity in measuring the free fall acceleration may appear useful for solving different geophysical problems [28], associated with the search for gravity abnormalities, corresponding to variations of the Earth crust density $\Delta\rho_E$ in the regions of mineral deposits. For example, the salt domes, which are one of the fingerprints of oil-bearing bed presence, are accompanied by the variation of the rock density $\Delta\rho_E \approx -25$ kg m$^{-3}$. Basing on the abovementioned sensitivity, during the acquisition time $\sim$100 s such variation may be detected at the depth of 5 km, if the dome volume amounts to at least 150 m$^3$, which in geological measure corresponds to a small-scale inhomogeneity of the Earth crust. With the same sensitivity of the SQUID/magnetostrictor system and the same acquisition time ($\sim$100 s) at the surface of the Earth, it is possible to detect the perturbation of the gravity field, produced by a light tank (with the mass $\sim$15 t), at the distance of 10 km. Such high sensitivity of the system not only can be useful for solving applied problems of geological exploration, but also will allow continuation of the studies of weak deviations from the law of universal gravitation, the possibility of which in the 'near-field' zone was reported in a number of papers [29, 30]. If the study of such deviations will be performed at the distance of a few metres from a body having the mass $m_0 = 15$ kg, then the sensor, detecting the gravitational field strength (i.e., the acceleration of free fall on the body with the mass $m_0$) with the abovementioned sensitivity of $10^{-13}$ m s$^{-2}$ Hz$^{-1/2}$, during the time of $\sim$100 s will be able to detect a deviation from the Newton gravitation law, if it amounts to at least 0.001 %.

Choosing the test body in the form of a dumbbell with the masses $m_1$ and $m_2$ at the ends, linked with a magnetostrictive rod, which is coupled to a SQUID via a superconducting flux transformer, one can construct a supersensitive gradiometer of the gravitation field. With magnetostrictive link, having the cross section $S_{ms} = 10^{-4}$ m$^2$, the length $l_{12} = 0.1$ m and the end masses $m_1 = m_2 = m = 5$ kg, such measuring instrument with the ultimate sensitivity $\Delta p_{12} = \delta p \sim 10^{-13}$ Pa Hz$^{-1/2}$ will be able to detect the gradient of the gravitation field strength $|\nabla \boldsymbol{a}| = S_{ms}\Delta p_{12}/(2ml_{12}) = 10^{-17}$ s$^{-2}$ Hz$^{-1/2}$. In comparison with the gravimeter, the gradiometer possesses better noise resistance, because the impact of the noise sources located at the distance much greater than $l_{12}$ appears to be strongly suppressed. The tensor character of the gradient of the gravitation field strength ($\nabla \boldsymbol{a} = \nabla_i a_j$) indicates its direct relation with the Ricci curvature tensor, arising due to gravitational distortion of the 'plane' Euclidean metric. The gradiometric scheme described above allows the measurement of diagonal elements of the curvature tensor only, however, detecting the magnetic response that corresponds to shear deformations of a piezomagnetic, will probably allow the detection of non-diagonal components as well.

The energy density 1 keV $cm^{-3}$ corresponds to the internal pressure $p_{int} \approx 1.6\times10^{-10}$ Pa. In correspondence with the estimates, this is just the density that should be possessed by the dark energy [31–35], the everywhere presence of which explains the additional acceleration in recession of galaxies, as compared to Hubble's law $V = HR$ (the velocity of recession $V$ is proportional to the distance $R$ to the observed objects, $H$ being Hubble's constant). One of the popular hypotheses about the nature of dark energy actually identifies it with the vacuum of quantised electromagnetic field. In principle, this model allows implicit registration of dark energy density variations by performing long-term observations of the Casimir effect, which consists in appearance of small difference of pressures, exerted by virtual photons from inside and outside the gap between two closely spaced parallel mirrors. The calculation, accounting for the resonance factors in the statistics of virtual photons that are created and annihilated in the quantum-field vacuum, shows that the pressure difference $\Delta p_{qv} = \pi^2\hbar c/(240d^4) \approx 1.2\times10^{-27}/d^4$ [36–39]. If $d$ = 50 μm is the width of the gap that plays the role of a resonator for virtual photons, then this difference will, on average, amount to $\sim2\times10^{-10}$ Pa. Thus, the SQUID/magnetostrictor system, used as a sensor for detecting the variations of the internal pressure $p_{int}$ or the variations $\Delta p_{qv}$, relevant to the Casimir effect, with the sensitivity $\sim10^{-13}$ Pa $Hz^{-1/2}$, will allow laboratory studies of periodical variations of dark energy, corresponding to complex polycyclic motion of the Earth in the space.

And, finally, let us consider the possible scheme of 'direct' registration of gravity fields, in which it is proposed to apply the superconducting interferometer without using the magnetostrictive sensor. Relativistic effects are closely connected to the natural anisotropy, possessed by the motion and gravity. In the special relativity theory this anisotropy manifests itself in the difference of Lorenz transformations along and across the motion direction, while in the general relativity theory it is present in the form of anisotropic influence of gravity on the appropriate components of the metric tensor, responsible for linear scale along and across the vector of gravity field strength (it is reasonable to note that in modern cosmological theories, relating the observed distribution of matter in the Universe to the Finsler metric [40–42], the anisotropy acquires vital significance).

The natural anisotropy of relativistic effects is reflected in the geometry of the classical Michelson–Morley experiment, too. In this experiment the role of the velocity of the laboratory coordinate frame is played by the solar escape velocity, with which the Earth moves around the Sun (30 km $s^{-1}$). In this case, with respect to the Sun the interferometer was in the zero-gravity state. However, if an interferometer with mutually perpendicular arms, one of which is directed towards the centre of the gravitation field, will be actually at rest with respect to this centre, then the variation of the gravitational potential $\Delta\varphi_g$, corresponding to slow displacement of the device up or down by the distance $\Delta h$, will cause the displacement of the resulting interference pattern. The effect will be, in principle, observable, but obviously hard to detect, since its order of magnitude will be

$$\frac{\Delta\varphi_g}{c^2} = \frac{1}{c^2}\frac{\partial\varphi_g}{\partial z}\Delta h,$$

where the strength of the gravitational field at the surface of the Earth is $\partial\varphi_g/\partial z \approx 9.8$ m $s^{-2}$. The required sensitivity may be provided by quantum interferometers, registering the phase change $\Delta\Theta$, which is proportional to the effective elongation $\Delta l$, expressed in the units of the operating wavelength, i.e., $\Delta\Theta = 2\pi\Delta l/\lambda$, and the operating wavelength in the superconducting ring of a SQUID, depending on the accumulated magnetic flux $\Phi$, can be essentially smaller than the typical one in common optical systems, $\lambda_{SQUID} = 2\pi r\Phi_0/\Phi$ ($r$ is the ring radius, $\Phi_0 = \pi\hbar/e$).

Consider a flux transformer, incorporating two superconducting loops, closed on each other, with the inductances $L_\perp$ and $L_\parallel$ and the lengths $l_\perp$ и $l_\parallel$, the planes of which are oriented horizontally and vertically with respect to the direction of the gravitation field strength. Assume that $N$ flux quanta are stored in the transformer, then $N\Phi_0 = \Phi_\parallel + \Phi_\perp = (L_\perp + L_\parallel)I$ ($I$ is the current in the flux transformer) and, therefore, $\Phi_\perp = L_\perp I = N\Phi_0 L_\perp/(L_\perp + L_\parallel)$. Expressing the inductance $L$ of the long rectangular loop with the length $l$, much greater than its width, in terms of its specific value, $L = (\partial L/\partial l)l$, assuming the specific inductances to be equal in both loops, $\partial L_\parallel/\partial l_\parallel = \partial L_\perp/\partial l_\perp$, and neglecting the contribution of the ends, we get: $\Phi_\perp = N\Phi_0 l_\perp/(l_\perp + l_\parallel)$ and $\Delta\Phi_\perp = -N\Phi_0 l_\perp\Delta l_\parallel/(l_\perp + l_\parallel)^2$. Assuming that with no influence of gravitation taken into account the loops have equal lengths ($l_\perp = l_\parallel = l_0$) and using the law of linear scale transformation $l_\parallel = l_0\sqrt{g_{zz}(z)} = l_0\times\sqrt{(1+\varphi_g/c^2)^{-1}}$ и $l_\perp = l_0\sqrt{g_{\perp\perp}} = l_0$, describing the relativistic gravity effects in terms of the appropriate components of the metric tensor, we arrive at the expression for the flux variation in the horizontal loop $\Delta\Phi_\perp$, arising when the superconducting interferometer is lifted by the height $\Delta h$

$$\Delta\Phi_\perp = -\frac{N\Phi_0}{8c^2}\frac{\partial\varphi_g}{\partial z}\Delta h.$$

For quantitative estimation let us substitute the Earth gravitation parameter $\partial\varphi_g/\partial z \approx 9.8$ m $s^{-2}$ into the obtained formula and assume that the superconducting loops, in which $10^9$ flux quanta are stored, are lifted up the height 800 m. Then the flux will be redistributed between the loops so that $\Delta\Phi_\perp \approx 10^{-6}\Phi_0 = -2.07\times10^{-21}$ Wb. If the elevation was performed with the velocity 0.5 m $s^{-1}$, then this decrease of the flux will be registered by the SQUID in the frequency band $\Delta f = 1/\tau$ = 0.5 m $s^{-1}$ / 800 m = $6.25\times10^{-3}$ Hz, so that $\sqrt{\Delta f}$ = 0.025 $Hz^{-1/2}$. Thus, to make the registration of the effect possible, the flux fluctuations in the quantum interferometer should not exceed $10^{-6}\Phi_0/0.025$ $Hz^{1/2} = 4\times10^{-5}\Phi_0/Hz^{1/2}$, whereas the noise amplitude in modern SQUIDs is almost two orders of magnitude smaller.

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