

# Cascade amplification of self-similar frequency-modulated pulses in normal group velocity dispersion active fibres

I.O. Zolotovskii, O.G. Okhotnikov, D.I. Sementsov, A.A. Sysolyatin, A.A. Fotiadi

**Abstract.** This paper examines the possibility of efficient amplification of self-similar frequency-modulated wave packets in longitudinally inhomogeneous active fibres. We analyse the dynamics of parabolic pulses with a constant frequency modulation rate and derive algorithms for optimising the group velocity dispersion profile in order to ensure self-similar propagation of such pulses. We demonstrate that the use of a cascade scheme can ensure efficient amplification of individual subpicosecond pulses of this type.

**Keywords:** similariton, amplification, cubic nonlinearity, frequency modulation, group velocity dispersion, active fibres.

## 1. Introduction

The generation of stable self-similar frequency-modulated (FM) pulses, referred to as similaritons, in a normal group velocity dispersion (GVD) active (gain) medium opens up a wide variety of possibilities in comparison with classical approaches to pulse amplification [1–10]. This is because, in the case of pulse amplification in an anomalous-dispersion medium, it is difficult to prevent the development of various instabilities and the undesirable influence of stimulated Raman scattering (SRS), which significantly distorts the shape of the wave packet. However, some difficulties arise as well in a normal GVD medium. In particular, known techniques for pulse amplification in homogenous fibres are sensitive to ‘unintentional’, random variations in fibre diameter (which always occur in practice) [11, 12]. In view of this, the use of long (>100 m) fibre amplifiers with a low gain per unit length,  $\gamma \ll 0.1 \text{ m}^{-1}$ , appears inefficient.

An effective approach to this problem is to use compact, inhomogeneous fibre amplifiers having a normal GVD that increases along the length of the fibre and a gain per unit length  $\gamma > 0.1 \text{ m}^{-1}$  (with good control over other performance

parameters – fibre diameter, dispersion, and nonlinearity – due to the relatively small amplifier length).

In this paper, we analyse the dynamics of a parabolic FM wave packet in a longitudinally inhomogeneous amplifier and examine conditions for stable (self-similar) propagation of the wave packet in the form of a similariton. We propose using combinations of active and passive fibre segments, which makes it possible not only to scale up the output power by placing many stages in series but also to considerably increase the amplified pulse compression efficiency in the output compressor. Raman fibres are shown to have considerable potential for use in such systems.

## 2. Condition for the existence of similaritons

Consider the dynamics of an optical pulse in an inhomogeneous gain medium. The field of a wave packet propagating along such a fibre can be represented in a standard way:

$$E(t, r, z) = \frac{1}{2} e U(r, z) A(t, z) \exp[i(\omega_0 t - \int_0^z \beta'(\xi) d\xi)] + \text{c.c.}, \quad (1)$$

where  $e$  is a unit vector along the light polarisation direction;  $U(r, z)$  is the radial field distribution in the fibre;  $\omega_0$  is the carrier frequency of the wave packet; and  $\beta'(z)$  is the real part of the complex propagation constant. The temporal profile of the pulse,  $A(t, z)$ , meets a nonlinear Schrödinger equation (NSE) with coefficients varying along the length of the fibre [1–5]:

$$\frac{\partial A}{\partial z} - i \frac{D(z)}{2} \frac{\partial^2 A}{\partial \tau^2} + i R(z) |A|^2 A = g(z) A, \quad (2)$$

where  $\tau = t - \int_0^z d\xi/u(\xi)$  is the time in a moving frame of reference;  $u(z) = (\partial\beta(z)/\partial\omega)^{-1}$  and  $D(z) = (\partial^2\beta(z)/\partial\omega^2)_0$  are the group velocity and GVD of the fibre; and  $R(z)$  is the Kerr (cubic) nonlinearity coefficient. The dependence of the fibre parameters on the longitudinal coordinate  $z$  results largely from their dependence on the mode area  $S_m(z)$ . In particular, the modal gain is related to the material gain  $\gamma(z)$  and mode area by

$$g(z) = \gamma(z) - \frac{1}{2S_m(z)} \frac{dS_m(z)}{dz}. \quad (3)$$

Here, the mode area is determined by the radial mode field distribution in the fibre:

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Received 4 July 2012

Kvantovaya Elektronika 42 (9) 822–827 (2012)

Translated by O.M. Tsarev

$$S_m(z) = 2\pi \int_0^\infty |U_m(r, z)|^2 r dr. \quad (4)$$

A distinctive feature of similariton propagation is that, independent of its original profile, the pulse envelope asymptotically approaches a parabolic shape and the parabola grows as the coordinate of the pulse increases [4–9]. The temporal pulse profile, which is a solution to the relevant NSE, can be represented in the following form:

$$A(\tau, z) = A(z) Y_{\text{par}}(z, \tau) \exp[i(\phi(z) + \alpha(z)\tau^2)], \quad (5)$$

where  $\phi(z)$  is the pulse phase and  $\alpha(z)$  is the modulation frequency change rate, constant for a given similariton. The function  $Y_{\text{par}}(z, \tau)$  defines the shape of the pulse envelope. In the case of a parabolic pulse, it has the form

$$Y_{\text{par}} = \begin{cases} \sqrt{1 - \tau^2/\tau_s^2(z)}, & \tau \leq \tau_s(z), \\ 0, & \tau > \tau_s(z), \end{cases} \quad (6)$$

where  $\tau_s(z)$  is the pulse duration.

It is known that the condition for the formation of a stable similariton, under which the NSE has an exact solution in the form of a parabolic FM pulse, can be represented as [6–9]

$$g_{\text{eff}} - 3\alpha_0 D_0 = 0. \quad (7)$$

Here, the effective gain per unit length of the inhomogeneous fibre,  $g_{\text{eff}}$ , is related not only to its material gain coefficient  $\gamma(z)$  but also to the mode area  $S_m(z)$ , GVD  $D(z) = (\partial^2 \beta / \partial \omega^2)_0 = d(z) D_0$  and the nonlinearity coefficient by

$$g_{\text{eff}}(\xi) = \frac{\gamma(\xi)}{d(\xi)} - \frac{\partial S_m / \partial \xi}{2S_m} - \frac{\partial d(\xi) / \partial \xi}{2d(\xi)} + \frac{\partial R / \partial \xi}{2R(\xi)}, \quad (8)$$

where  $\xi = D_0^{-1} \int_0^z D(z) dz$ ,  $\alpha_0$  and  $D_0$  are the values of the corresponding parameters at the fibre input. In the case of a homogenous active fibre, this condition can be written in an extremely simple form:  $\gamma_0 - 3\alpha_0 D_0 = 0$ .

It should be noted here that available picosecond pulse sources that can be used to generate FM similaritons typically ensure frequency modulation rates no higher than  $10^{23}$  to  $10^{24} \text{ s}^{-2}$  (after additional dispersive components are passed). In the active fibres in common use (e.g., those doped with  $\text{Er}^{3+}$ ), which support single-mode propagation, the GVD does not exceed  $3 \times 10^{-26} \text{ s}^2 \text{ m}^{-1}$  [6–8, 13–15]. As a consequence, the gain per unit length of an inhomogeneous fibre, which ensures the generation of an FM similariton, should be considerably lower than  $0.1 \text{ m}^{-1}$  (a more detailed analysis yields  $\gamma \leq 0.01 \text{ m}^{-1}$ ). Therefore, to increase the pulse energy by more than one order of magnitude, the fibre length should considerably exceed 100 m. On the other hand, experimental data for erbium- and bismuth-doped fibres [7, 8] and numerical simulation results [11, 12] demonstrate that self-similar FM pulses are rather sensitive to gain fluctuations and variations in fibre diameter. Because of this, a significant increase in pulse energy (by more than one order of magnitude) is difficult to achieve through the use of long (>100 m) active fibres.

One possible solution to this problem is to use a short piece of fibre (considerably shorter than 10 m) with a high gain per unit length ( $\gamma > 0.2 \text{ m}^{-1}$ ) and a steep rise in GVD along its length. In what follows, we examine the possibility

of an efficient, ‘rapid’ amplification of parabolic FM pulses in a longitudinally inhomogeneous fibre.

With relation (4) for the effective gain per unit length condition (3) takes the form

$$\frac{\partial D}{\partial z} - f(z) D(z) + 6\alpha_0 D^2(z) = 0, \quad (9)$$

where  $f(z) = 2\gamma(z) - S_m^{-1}(\partial S_m / \partial z) + R^{-1}(\partial R / \partial z)$ . This is a Bernoulli equation [16], and its solution can be used to find the GVD profile that ensures self-similar propagation of parabolic FM pulses. The general formula of the solution is

$$D(z) = \frac{D_0 F(z) \exp\left[2 \int_0^z \gamma(z') dz'\right]}{1 + 6\alpha_0 D_0 \int_0^z F(z') \exp\left[2 \int_0^{z'} \gamma(z'') dz''\right] dz'}, \quad (10)$$

where  $F(z) = R(z) S_{m0} / R_0 S_m(z)$ . Below, we consider so-called W-profile fibres (with a W-shaped radial refractive index profile) [13, 14]. The nonlinearity and mode area of such fibres are weak functions of the outer fibre diameter. We can therefore take with high accuracy  $F(z) = 1$ . For a W-profile active fibre, we then have

$$D(z) = \frac{D_0 \exp\left(2 \int_0^z \gamma(z') dz'\right)}{1 + 6\alpha_0 D_0 \int_0^z \exp\left(2 \int_0^{z'} \gamma(z'') dz''\right) dz'}. \quad (11)$$

With the effective gain per unit length

$$G_{\text{eff}}(z) = \exp\left(2 \int_0^z \gamma(z') dz'\right), \quad (12)$$

equal to the relative increase in pulse energy over length  $G_{\text{eff}}(z) = W(z) / W(0)$ , Eqn (11) can be written in the form

$$D(z) = \frac{D_0 G_{\text{eff}}(z)}{1 + 6\alpha_0 D_0 \int_0^z G_{\text{eff}}(z') dz'}. \quad (13)$$

Figure 1 shows the GVD profiles obtained using Eqn (13) that ensure self-similar propagation of a parabolic FM pulse at fixed  $\tau_0 = 10^{-12} \text{ s}$  and  $D_0 = 2 \times 10^{-27} \text{ s}^2 \text{ m}^{-1}$  and varied parameters  $\alpha_0 = 10^{24} \text{ s}^{-2}$  and  $\gamma = 0.05, 0.1, 0.2$  and  $0.4 \text{ m}^{-1}$  in Fig. 1a and  $\gamma = 0.1 \text{ m}^{-1}$  and  $\alpha_0 = (0.1, 1, 2 \text{ and } 4) \times 10^{23} \text{ s}^{-2}$  in Fig. 1b. It is seen that, with increasing fibre length, the GVD approaches the level  $D_\infty = \gamma / 3\alpha_0$ , which rises with increasing nonlinearity  $\gamma$  at a constant initial frequency modulation rate  $\alpha_0$  and drops with increasing  $\alpha_0$  at constant  $\gamma$ . At small fibre lengths ( $\gamma z \ll 1$ ), the sought GVD profile is almost exponential:  $D(z) \approx D_0 \exp(2\gamma z)$ .

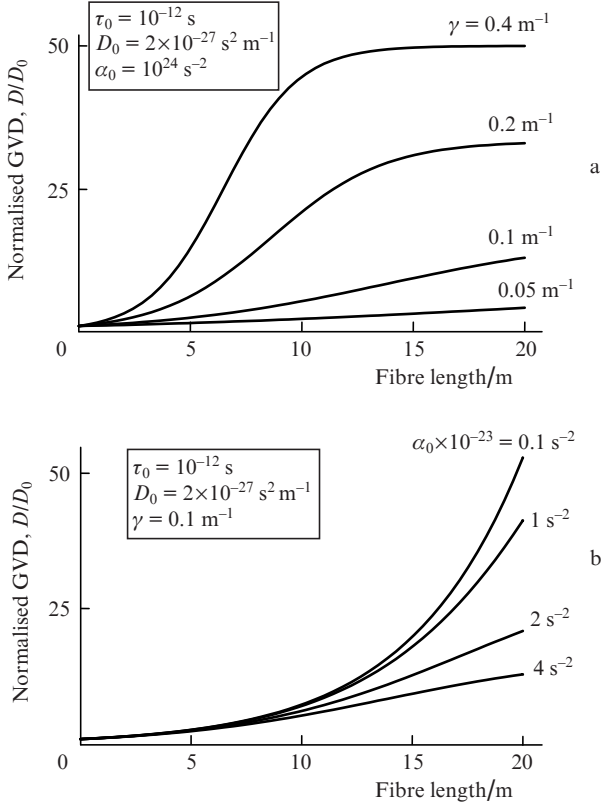
The similariton pulse duration is in general given by [9]

$$\tau_s(z) = \tau_0 \exp\left(2\alpha_0 \int_0^z D(z') dz'\right). \quad (14)$$

Substituting (13), we obtain

$$\tau_s(z) = \tau_0 \left(1 + 6\alpha_0 D_0 \int_0^z G_{\text{eff}}(z') dz'\right)^{1/3}. \quad (15)$$

If a fibre can, to a good approximation, be considered passive, i.e.  $\gamma = 0$ , we obtain a known relation for the pulse duration:  $\tau_s(z) = \tau_0 (1 + 6\alpha_0 D_0 z)^{1/3}$ .

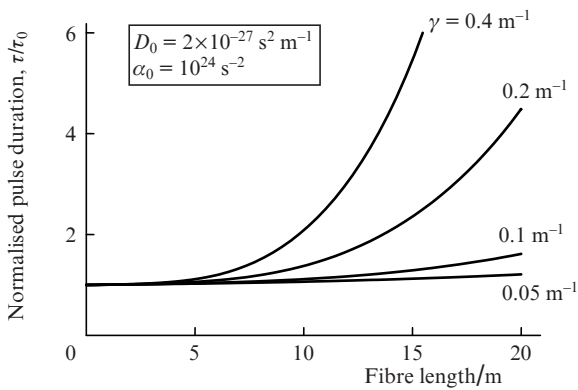


**Figure 1.** Normalised GVD profiles that ensure self-similar pulse propagation at: (a)  $\alpha_0 = 10^{24} \text{ s}^{-2}$  and  $\gamma = 0.05, 0.1, 0.2, 0.4 \text{ m}^{-1}$ ; (b)  $\gamma = 0.1 \text{ m}^{-1}$  and  $\alpha_0 = (0.1, 1, 2, 4) \times 10^{23} \text{ s}^{-2}$ .

Figure 2 shows the normalised similariton pulse duration as a function of distance travelled at the following parameters:  $D_0 = 2 \times 10^{-27} \text{ s}^2 \text{ m}^{-1}$ ,  $\alpha_0 = 10^{24} \text{ s}^{-2}$ ,  $\gamma = 0.05, 0.1, 0.2$  and  $0.4 \text{ m}^{-1}$ . As seen in Fig. 2, the pulse duration at the fibre output increases with distance travelled and the slope of the curves increases with gain.

Note also that, in general, the formation energy of a parabolic similariton is given by [9]

$$W_{s0} = 8D_0\alpha_0^2\tau_0^3/3R_0, \quad (16)$$



**Figure 2.** Normalised pulse duration as a function of distance travelled at  $D_0 = 2 \times 10^{-27} \text{ s}^2 \text{ m}^{-1}$ ,  $\alpha_0 = 10^{24} \text{ s}^{-2}$  and  $\gamma = 0.05, 0.1, 0.2, 0.4 \text{ m}^{-1}$ .

which demonstrates that it strongly depends on  $\alpha_0$ . When a pulse propagates along an inhomogeneous active fibre, its energy increases and can, in general, be represented as

$$W_s(z) = W_{s0} \exp\left(2 \int_0^z \gamma(z') dz'\right). \quad (17)$$

Note that the generation of a parabolic FM pulse is also possible when the input pulse energy differs slightly from  $W_{s0}$ . One condition for similariton generation is that the initial energy  $W_0$  fall in the range  $(0.75-1.25)W_{s0}$  with high accuracy.

Of particular interest for raising the energy of FM (chirped) pico- and subpicosecond pulses are Raman amplifiers, capable of ensuring a uniform gain profile in a wide frequency range (so-called broadband flat gain) [17–19]. In the case of a high pump power  $P_p$ , high conversion efficiency from the pump of frequency  $\omega_p$  to a pulse train and, as a consequence, a relatively small amplifier length ( $<100 \text{ m}$ ), the intrinsic losses of the Stokes and pump waves can be neglected. In the undepleted-pump approximation, a simple relation can then be written for the Stokes pulse gain [17–20]:

$$G_{\text{eff}}(z) = \exp(G_R z), \quad (18)$$

Here,  $G_R = g_R(\omega_p)P_{p0}/S_{\text{eff}}(\omega_p)$ ;  $g_R$  is the Raman gain coefficient and  $S_{\text{eff}}$  is the effective mode area, which depends on the longitudinal coordinate in the case of inhomogeneous fibre:

$$S_{\text{eff}}(z) = \frac{2\pi \left( \int_0^\infty |U(r,z)|^2 r dr \right)^2}{\int_0^\infty |U(r,z)|^4 r dr}. \quad (19)$$

In the case of germanium-doped fibres [20], the Raman gain coefficient satisfies the inequality  $g_R(\omega_p)/S_{\text{eff}}(\omega_p) > 0.1 \text{ m}^{-1} \text{ W}^{-1}$ . The total pump power may then be within several watts, ensuring a Raman gain per unit length  $G_R \geq 0.2 \text{ m}^{-1}$ , which is sufficient for a tenfold amplification of subpicosecond pulses over lengths no greater than 10 m. The appreciable difference between the pump power and average pulse power (which is typically within 10 mW) in the Raman amplifier under consideration allows pump depletion to be completely neglected. Because of the small amplifier length, the influence of the intrinsic loss of the pump wave can also be neglected.

Thus, in the case of undepleted pump, which is of most practical interest, relations (13) and (15) take the simple form

$$D(z) = \frac{D_0 \exp(G_R z)}{1 + \delta(\exp(G_R z) - 1)}, \quad (20)$$

$$\tau_s = \tau_0 [1 + \delta(\exp(G_R z) - 1)]^{1/3},$$

where  $\delta = 6\alpha_0 D_0 / G_R$ .

In a low-optical-loss ( $\gamma \rightarrow 0$ ) passive fibre, the GVD that satisfies the condition for the existence of similaritons should have a hyperbolic profile, i.e.  $D(z) = D_0(1 + 6\alpha_0 D_0 z)^{-1}$ . As shown below, such ‘passive’ fibres can be used for further modulation of similaritons (to increase their duration) and, more importantly, as adapters between amplifying (active) fibre segments of a cascade amplifier.

It follows from (9) that, when the condition  $\gamma/3\alpha_0 > D_0$  is satisfied, efficient amplification of FM pulses can be ensured

by fibres that have an increasing dispersion. For  $\gamma/3\alpha_0 < D_0$ , fibres with a decreasing dispersion should be used for efficient amplification. At  $\gamma/3\alpha_0 \approx D_0$ , longitudinally homogeneous fibres should be used. The design features of modern laser pulse sources and amplifiers provide conclusive evidence in favour of the first scenario of FM pulse amplification, i.e. the use of active fibres with an increasing dispersion, less than 10 m in length and with a relatively high gain ( $\gamma > 0.2 \text{ m}^{-1}$ ). This conclusion differs fundamentally from what was inferred in previous studies [1–6], where fibres with a gradually decreasing GVD were proposed for similariton amplification. Of special note is that, in the case of the proposed rapid (over a length less than 10 m) amplification of chirped pulses, the influence of higher order dispersion effects, beginning with the third order, can be neglected.

Note also that, to ensure stable amplification, it is desirable that, in some frequency range, the gain should be constant (a weak function of frequency), i.e.  $\partial^n G_R / \partial \omega^n = 0$ . A flat gain profile can be effectively produced using multiwavelength pump sources – an approach that is employed rather widely in optical fibre communication systems [17–19].

### 3. Compression of frequency-modulated pulses

It follows from the above analysis that the amplification of a parabolic pulse in a normal-dispersion medium is accompanied by an increase in pulse duration, whereas the frequency modulation rate remains unchanged. It is desirable that further increase in pulse peak power through temporal compression should occur in a passive dispersive medium, which minimises the influence of nonlinear effects (in order to minimise the development of various instabilities [9, 10]). This can be done beyond the amplifying fibre: either in an anomalous-dispersion passive fibre with low cubic nonlinearity (e.g. in a hollow-core microstructured fibre) or using a diffraction grating pair, acting as an efficient dispersive component. At present, it is the latter approach which is better developed for high-energy laser pulse generation [21, 22].

If a transform-limited pulse has parameters  $\tau_s(L)$  and  $\alpha_0$  at the compressor input (after passing a fibre amplifier of length  $L$ ), its duration at the compressor output is [9, 23]

$$\tau_{\text{com}} = \frac{\tau_s(L)}{(1 + \alpha_0^2 \tau_s^4(L))^{1/2}}. \quad (21)$$

If  $\alpha_0 \tau_s^2(L) \gg 1$ , we obtain  $\tau_{\text{com}} \sim 1/\alpha_0 \tau_s(L)$ . The peak power of the compressed pulse is then

$$P_{\text{max}} \cong P_0 \alpha_0 \tau_0 \tau_s(L) \exp\left(2 \int_0^L \gamma(z) dz\right). \quad (22)$$

It follows from the above relations that the more we ‘stretch’ a pulse with a constant, nonzero frequency modulation rate, the more the pulse can be compressed and the higher pulse peak power can be reached at the compressor output.

### 4. Cascade amplification

As shown above, a significant increase in pulse energy is rather difficult to achieve through the use of long lengths of low-gain active fibres. It is therefore more advantageous to use high-gain fibres with a steep rise in GVD along their length. This, however, presents problems related to the marked increase in GVD due to the reduction in fibre diam-

eter because a decrease in diameter must lead to an increase in optical loss. At the same time, in W-profile fibres the required longitudinal GVD profile can be produced at relatively small variations in outer fibre diameter. The fibre drawing technology in current use enables considerable changes in dispersion even at very small variations in outer fibre diameter, typically, no more than 3  $\mu\text{m}$  over a 100-m length. At an average fibre diameter of 100–125  $\mu\text{m}$ , this is a very low value. The effective mode area and nonlinearity coefficient in such fibres are almost constant throughout their length [13, 14].

It is also worth noting that, in the case of single-mode propagation of a wave packet along a fibre, the GVD  $D(z)$  most likely cannot exceed  $5 \times 10^{-26} \text{ s}^2 \text{ m}^{-1}$ . Since the energy of a soliton-like pulse and GVD in a single-mode amplifying fibre satisfy the relation

$$\frac{D(z)}{D_0} = \frac{W(z)}{W_0} = \exp\left(2 \int_0^z \gamma(z') dz'\right), \quad (23)$$

with high accuracy, the pulse energy at the amplifier output cannot be increased by more than one order of magnitude. The problem can be resolved by using a cascade configuration where high-gain elements with a sharp increase in GVD alternate with passive elements having a gradually decreasing dispersion. In this configuration, an amplifier segment with a decreasing cladding diameter grades into a modulator segment, where pulses are stretched in time without amplification and the fibre diameter increases to its original level. Such a two-element cascade has identical input and output sizes and supports single-mode pulse propagation.

To illustrate the proposed amplification scheme, consider a two-element cascade. The GVD and pulse energy are taken to reach the highest levels,  $D_{\text{max}}$  and  $W_{\text{max}}$ , at the output of the amplifier element. Because of the small length of this element, the duration of a pulse propagating through it changes insignificantly. In the passive element behind the amplifier, the decrease in GVD and increase in pulse duration are

$$D = D_{\text{max}}(1 + 6\alpha_0 D_{\text{max}} z)^{-1}, \quad \tau_s(z) = \tau_0(1 + 6\alpha_0 D_{\text{max}} z)^{1/3}, \quad (24)$$

and the pulse energy varies little (the loss is typically well below 1 dB  $\text{km}^{-1}$ ). The total length of the two-element cascade is  $L = L_1 + L_2$ , where the length of the amplifier element,  $L_1$ , can be found using (23):

$$\ln\left(\frac{D_{\text{max}}}{D_0}\right) = 2 \int_0^{L_1} \gamma(z) dz. \quad (25)$$

If the gain does not vary along the fibre, we have

$$L_1 = \frac{1}{2\gamma} \ln\left(\frac{D_{\text{max}}}{D_0}\right).$$

The length of the modulator element in the cascade is then

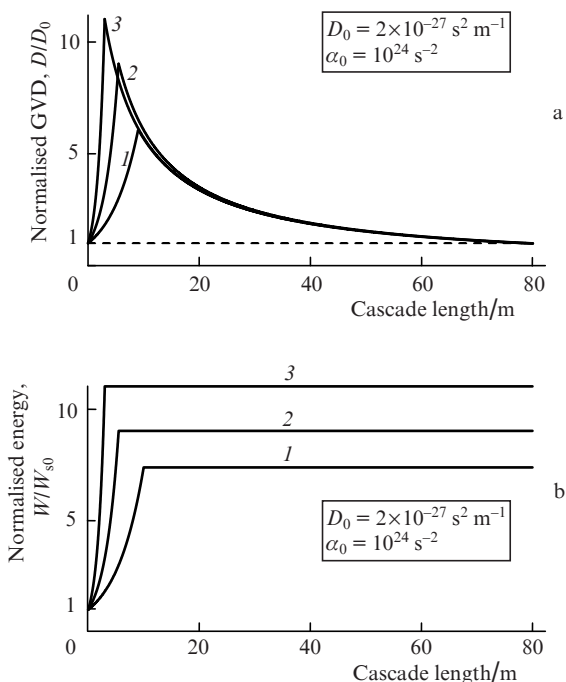
$$L_2 = \left(\frac{W_s}{W_0} - 1\right) \frac{1}{6\alpha_0 D_{\text{max}}}. \quad (26)$$

Figure 3 illustrates the variation of normalised dispersion  $D(z)$  and pulse energy  $W(z)$  along a cascade for W-profile active fibres. The curves were obtained for  $D_0 = 2 \times 10^{-27} \text{ s}^2 \text{ m}^{-1}$ ;  $\alpha_0 = 10^{24} \text{ s}^{-2}$ ;  $\gamma = 0.1, 0.2$  and  $0.3 \text{ m}^{-1}$ ; and  $L_1 = 10, 5.5$  and  $3 \text{ m}$ , respectively. It follows from these data that the pulse energy increases in the amplifier by approximately a factor of  $D_{\text{max}}/D_0$ . Next, the pulse must pass through a pas-



sive modulator element of length  $L_2 = L - L_1$ . At its output, the GVD decreases to  $D_0$ . After chirp suppression (e.g. using diffraction gratings), the net increase in the peak power of a pulse that has passed through such a cascade, with allowance for possible pulse compression to  $\tau_{\text{com}} = \tau_0(D_{\text{max}}/D_0)^{-1/3}$ , is

$$P_{\text{com}}(L) \simeq P_0(D_{\text{max}}/D_0)^{4/3}. \quad (27)$$



**Figure 3.** Normalised (a) GVD and (b) pulse energy as functions of distance travelled in a two-element cascade: (1)  $\gamma = 0.1 \text{ m}^{-1}$ ,  $L_1 = 10 \text{ m}$ ; (2)  $\gamma = 0.2 \text{ m}^{-1}$ ,  $L_1 = 5.5 \text{ m}$ ; (3)  $\gamma = 0.3 \text{ m}^{-1}$ ,  $L_1 = 3 \text{ m}$ .

The above estimates suggest that, after chirp suppression, the peak power of the wave packet may increase by 15–25 times, depending on fibre parameters.

In one two-element cascade (fabricated from standard optical fibres), the pulse energy will most likely increase by no more than one order of magnitude. Therefore, to ensure a higher gain several double (amplifier + modulator) elements should be connected in series. In the case of a complex cascade composed of  $N$  identical double elements, the pulse peak power at the output of the cascade, of length  $L = N(L_1 + L_2)$ , after chirp suppression (i.e. after pulse compression) is

$$P_{\text{com}}(L) \simeq P_0(D_{\text{max}}/D_0)^{4N/3}. \quad (28)$$

Note that a key role in determining the amplification result is played by the GVD profile, which should correspond to the gain profile. As compact amplifiers capable of ensuring ultrarapid amplification of FM pulses, one can use either active fibres with variable-period Bragg gratings (chirped gratings) or active photonic crystal structures. Such structures can ensure an increase in GVD by two to three orders of magnitude over a very small length, within several centimetres, and the corresponding increase in the energy of the wave packet.

## 5. Conclusions

We have identified conditions for the formation of stable parabolic pulses in optical fibres with a W-shaped radial refractive index profile in the normal GVD region. An expression has been derived for the GVD profile that ensures an optimal, ‘rapid’ amplification of parabolic FM pulses. It has been shown that preferable amplifiers of FM similariton-like pulses are fibres with an increasing dispersion and a relatively high gain per unit length (over  $0.2 \text{ m}^{-1}$ ). The proposed scheme of the amplification and subsequent compression of FM pulses is at present actively used in solid-state laser systems to produce high-power pulses. Such a scheme, with the similariton amplification regime and a grating compressor, may enable the generation of pulses of duration down to 100 fs with an energy of  $\sim 100 \text{ nJ}$  and, as a consequence, with a record-breaking power (among all-fibre laser systems) of over 100 kW.

**Acknowledgements.** This work was supported by the RF Ministry of Education and Science (federal targeted programmes ‘The Scientists and Science Educators of Innovative Russia, 2009–2013’ and ‘Research and Development in the Priority Areas of the Science and Technology Sector of Russia in 2007–2012’).

## References

1. Fermann M.E., Kruglov V.I., Thomsen B.C., Dudley J.M., Harvey J.D. *Phys. Rev. Lett.*, **84**, 6010 (2000).
2. Chang G., Winful H.G., Galvanauskas A., Norris T.B. *Phys. Rev. E*, **72**, 016609 (2005).
3. Dudley J.M., Finot C., Richardson D.J., Millot G. *Nature*, **3**, 597 (2007).
4. Hirooka T., Nakazawa M. *Opt. Lett.*, **29**, 498 (2004).
5. Anderson D., Desaix M., Karlson M., et al. *J. Opt. Soc. Am. B*, **10**, 1185 (1993).
6. Latkin A., Turitsyn S.K., Sysolyatin A. *Opt. Lett.*, **32**, 331 (2007).
7. Plotskii A.Yu., Sysolyatin A.A., Latkin A.I., et al. *Pis'ma Zh. Eksp. Teor. Fiz.*, **85**, 397 (2007).
8. Andrianov A.V., Muraviov S.V., Kim A.V., Sysolyatin A.A. *JETP Lett.*, **85**, 364 (2007).
9. Zolotovskii I.O., Sementsov D.I., Senatorov A.K., et al. *Kvantovaya Elektron.*, **40**, 229 (2010) [*Quantum Electron.*, **40**, 229 (2010)].
10. Zeytunyan A.S., Palandjan K.A., Esayan G.L., Muradyan L.Kh. *Kvantovaya Elektron.*, **40**, 327 (2010) [*Quantum Electron.*, **40**, 327 (2010)].
11. Abdullaev F.Kh., Navotnyi D.V. *Pis'ma Zh. Tekh. Fiz.*, **28**, 39 (2002).
12. Abdullaev F.Kh., Abdumalikov A.A., Baizakov B.B. *Kvantovaya Elektron.*, **24**, 176 (1997) [*Quantum Electron.*, **27**, 171 (1997)].
13. Akhmetshin U.G., Bogatyrev V.A., Senatorov A.K., et al. *Kvantovaya Elektron.*, **33**, 265 (2003) [*Quantum Electron.*, **33**, 265 (2003)].
14. Sysolyatin A.A., Nolan D.A. *J. Nonlinear Opt. Phys. Mater.*, **16**, 171 (2007).
15. Likhachev M.E., Bubnov M.M., Zotov K.V., et al. *Kvantovaya Elektron.*, **40**, 633 (2010) [*Quantum Electron.*, **40**, 633 (2010)].
16. Kamke E. *Gewöhnliche Differentialgleichungen* (Leipzig: Akademie, 1959; Moscow: Fizmatlit, 1976).
17. Grant A.R. *IEEE J. Quantum Electron.*, **38**, 1503 (2002).
18. Yoshihiro Emori, Soko Kado, Shu Namiki. *Opt. Fiber Technol.*, **8** (2), 107 (2002).
19. Kobtsev S.M., Pustovskikh A.A. *Kvantovaya Elektron.*, **34**, 1054 (2004) [*Quantum Electron.*, **34**, 1054 (2004)].

20. Dianov E.M., Abramov A.A., Bubnov M.M., Prokhorov A.M., Shipulin A.V., Gur'yanov A.N., Devyatykh G.G., Khopin V.F. *Kvantovaya Elektron.*, **22** (7), 643 (1995) [*Quantum Electron.*, **25** (7), 615 (1995)].
21. Mourou G., Tajima T., Bulanov S.V. *Rev. Mod. Phys.*, **78**, 309 (2006).
22. Lozhkarev V.V., Garanin S.G., Gerke S.G., et al. *Pis'ma Zh. Eksp. Teor. Fiz.*, **82**, 196 (2005).
23. Akhmanov S.A., Vysloukh V.A., Chirkin A.S. *Optics of Femtosecond Laser Pulses* (New York: Am. Inst. of Physics, 1992; Moscow: Nauka, 1988).