Quantum Electronics **42** (11) 1034–1038 (2012)

SOLITONS

PACS numbers: 42.65.Tg; 42.65.Wi; 42.81.Dp
DOI: 10.1070/QE2012v042n11ABEH014947

Gap soliton formation in a nonlinear anti-directional coupler

M.S. Ryzhov, A.I. Maimistov

Abstract. **We consider propagation of electromagnetic solitary waves in two tunnel-coupled waveguides. It is assumed that one of the waveguides is made of a positive-index dielectric, having a Kerr nonlinearity. The other waveguide is made of a linear optical metamaterial characterised by the so-called negative refraction. The gap soliton formation in such a system, which, as shown, has a threshold character, is studied numerically.**

Keywords: *optical solitons, tunnel-coupled waveguides, forward and backward waves, metamaterials.*

1. Introduction

Recently, much attention has been paid to the study of a new class of artificial materials – metamaterials, which are produced due to advances in the fabrication technology of nanocomposites and nanostructured media. Among metamaterials, a special place belongs to materials with the so-called negative refraction of electromagnetic waves. Negative refraction at an interface between two media, as noted in [1], is due to the fact that in one of the media the wave vector and Poynting vector are oppositely directed, while in another medium, they have the same direction. Moreover, this is a general property of the waves of any nature with a negative group velocity and positive phase velocity (or vice versa). Such waves are called backward waves, and they have been known for a long time. To use Snell's law to describe negative refraction, it is necessary to formally introduce a negative refractive index. After the pioneering work of Mandelstam [1], the waves in negative-index media were studied theoretically in [2–5]. The first experiments demonstrating the negative-index manifestation were performed in the microwave frequency range [6, 7]. The authors of papers [8, 9] reported the creation of a 'bulk' (a multilayer structure with a thickness on the order of the wavelength) negatively refracting material. Although modern materials with optical negative refraction have large losses, there are reasons to expect in the future either new materials with low losses, or ways to compensate for the loss [10]. The reviews of the properties of metamaterials are presented in [11–16].

M.S. Ryzhov National Research Nuclear University 'Moscow Engineering Physics Institute', Kashirskoe sh. 31, 115409 Moscow, Russia;
A.I. Maimistov Moscow Institute of Physics and Technology (State University), Institutskii per. 9, 141700 Dolgoprudnyi, Moscow region, Russia; National Research Nuclear University 'Moscow Engineering Physics Institute', Kashirskoe sh. 31, 115409 Moscow, Russia;
e-mail: aimaimistov@gmail.com

Received 10 July 2012; revision received 6 August 2012
Kvantovaya Elektronika **42** (11) 1034–1038 (2012)
Translated by I.A. Ulitkin

The unusual properties of negative-index metamaterials appear when an electromagnetic wave is refracted or localised near the interface between an ordinary medium and a negative-index medium [17, 18]. One interesting example of the interaction of forward and backward waves is the relation between the waves propagating in closely spaced waveguides, one of which is made of a positive-index nonlinear material, and the other is made of a negative-index linear or nonlinear material [16, 19–22]. Such a device in integrated optics is called a directional coupler, if the directions of propagation of the wave energy fluxes coincide [23–25]. In the case we are discussing the energy fluxes have opposite directions (forward and backward waves are coupled). Therefore, this device can be called an anti-directional coupler (Fig. 1). The spectrum of linear waves in the anti-directional coupler has a band gap – a gap similar to the gap in a Bragg waveguide. However, in this coupler there is no periodic change in the refractive index, and the forward and backward waves are spatially separated. In addition, in a Bragg waveguide both waves propagating in opposite directions are forward waves.

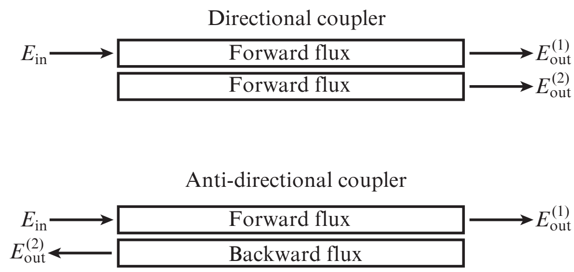

Figure 1. Scheme illustrating the differences between the directional and anti-directional couplers.

The authors of papers [20, 21] considered an extended nonlinear anti-directional coupler (NADC) and found the solutions that meet a stationary pulse of the electromagnetic field, which propagates along both tunnel-coupled waveguides as a whole. Based on the analogy between the properties of these pulses for a nonlinear Bragg waveguide and a NADC, a stationary solitary wave in the latter case can be called a gap soliton.

In this paper, we study the formation of a stationary solitary wave in an extended nonsymmetric (i.e., only an ordinary waveguide has nonlinear optical properties) nonlinear anti-directional coupler [21]. By solving numerically the equations describing the coupler under study, it is shown that a small-amplitude electromagnetic pulse, coupled into one of the NADC channels, is emitted in the opposite direction from the other channel of the coupler. When the amplitude of the input pulse exceeds a certain threshold, a pair of coupled pulses propagating in both waveguides in one, general direction is formed. Thus, the formation of a gap soliton in the NADC has a threshold character. The dependence of the threshold amplitude of the input pulse on a single parameter of the model is obtained numerically.

2. Basic equations of the NADC model

Following [21], we consider a pair of tunnel-coupled waveguides, one of which is made of a conventional optically nonlinear dielectric, the other is made of a negative-index linear material. The linear properties of the first waveguide are determined by the dielectric permittivity $\varepsilon_1(\omega_0)$ at the carrier wave frequency ω_0, and its magnetic permeability is equal to unity. It is assumed that the medium is transparent at the frequency ω_0. The nonlinear properties of the first waveguide are characterised by the effective third-order nonlinear susceptibility $\chi^{(3)}_{\rm eff}$. The waveguides are assumed short enough for the second-order group velocity dispersion effects to be ignored, and the condition of the wave matching is fulfilled.

The system of equations for the slowly varying envelopes of the electric field, E_1, in the ordinary waveguide and, E_2, in the negative-index waveguide has the form [21]:

$$\begin{gathered}
\mathrm{i}\frac{\partial E_1}{\partial z} + \frac{\mathrm{i}}{v_{\rm g1}}\frac{\partial E_1}{\partial t} + K_{12}E_2 + \frac{2\pi\omega_0}{c\sqrt{\varepsilon_1(\omega_0)}}\chi^{(3)}_{\rm eff}|E_1|^2E_1 = 0, \\
-\mathrm{i}\frac{\partial E_2}{\partial z} + \frac{\mathrm{i}}{v_{\rm g2}}\frac{\partial E_2}{\partial t} + K_{21}E_1 = 0,
\end{gathered} \tag{1}$$

where $v_{{\rm g}j}$ is the group velocity in the jth waveguide, and K_{12} and K_{21} are the tunnelling coupling constants of the waves.

It is convenient to pass to dimensionless variables by selecting them as follows:

$$\zeta = z/L_c, \quad \tau = t_0^{-1}(t - z/V_0), \quad L_{\rm c} = (K_{12}K_{21})^{-1/2},$$

$$t_0 = L_{\rm c}(v_{\rm g1} + v_{\rm g2})/2v_{\rm g1}v_{\rm g2}, \quad V_0^{-1} = (v_{\rm g2} - v_{\rm g1})/2v_{\rm g1}v_{\rm g2}.$$

The electric fields $E_j(z,t)$ in the jth waveguide ($j = 1, 2$) can be written as

$$E_1(z,t) = A_0Q_1(z,t), \quad E_2(z,t) = \sqrt{\frac{K_{21}}{K_{12}}}A_0Q_2(z,t).$$

In these new variables, the system of equations (1) has the form

$$\begin{gathered}
\mathrm{i}\Big(\frac{\partial}{\partial\zeta} + \frac{\partial}{\partial\tau}\Big)Q_1 + Q_2 + r|Q_1|^2Q_1 = 0, \\
\mathrm{i}\Big(\frac{\partial}{\partial\zeta} - \frac{\partial}{\partial\tau}\Big)Q_2 - Q_1 = 0,
\end{gathered} \tag{2}$$

where

$$r = \frac{2\pi\omega_0A_0^2\chi^{(3)}_{eff}}{c\sqrt{\varepsilon_1(\omega_0)}K_{12}K_{21}}$$

is a dimensionless parameter that characterises the nonlinearity of the waveguide. We will call this system of equations the NADC equations.

Stationary solitary waves – gap solitons – correspond to the solutions of equations (2), which are written as $Q_{1,2}(\zeta,\tau) = a_{1,2}(\eta)\mathrm{e}^{\mathrm{i}\phi_{1,2}(\eta)}$, where

$$\eta = \frac{\zeta + \beta\tau}{\sqrt{1-\beta^2}}$$

and β is a free parameter. Real amplitudes (envelopes of a gap soliton) $a_{1,2}(\eta)$ and phases $\phi_{1,2}(\eta)$ are defined by the expressions [21]:

$$a_1^2(\eta) = \frac{4}{\theta(1+\beta)\cosh 2(\eta - \eta_{\rm c})}, \tag{3}$$

$$a_2^2(\eta) = \frac{4}{\theta(1-\beta)\cosh 2(\eta - \eta_{\rm c})}, \tag{4}$$

$$\phi_1(\eta) = \arctan(\exp 2(\eta - \eta_{\rm c})), \tag{5}$$

$$\phi_2(\eta) = \arctan(\exp 2(\eta - \eta_{\rm c})) - \pi/2, \tag{6}$$

where

$$\theta = \frac{r}{1+\beta}\sqrt{\frac{1-\beta}{1+\beta}}$$

and the parameter $\eta_{\rm c}$ is a constant of integration, i.e., position of the maximum of the gap soliton envelope. If we go back to the original variables (z, t), then for the group velocity $v_{\rm s}$ of the gap soliton we can obtain the expression

$$\frac{1}{v_{\rm s}} = \frac{1}{2v_{\rm g1}v_{\rm g2}}\Big[v_{\rm g2}\Big(1 - \frac{1}{\beta}\Big) - v_{\rm g1}\Big(1 + \frac{1}{\beta}\Big)\Big].$$

Thus, the parameter β ($|\beta| < 1$, $\beta \neq 0$) determines the group velocity of the gap soliton and the direction of its motion in the NADC.

3. Results of the numerical solution of the NADC equations

It is assumed that a pulse of Gaussian shape is coupled into the input of the waveguide made of an ordinary (in this case, nonlinear) dielectric, and no radiation is coupled into the output of the second negative-index waveguide. Hence, the boundary conditions for system (2) can be written in the form

$$Q_1(z = 0, t) = a_{\rm in}\mathrm{e}^{-\tau^2}, \; Q_2(z = L, t) = 0. \tag{7}$$

The waveguide length L is chosen finite but sufficiently large (much larger than the coupling length $L_{\rm c}$) to ensure maximum total reflection in the linear limit. Along the time axis, use was made of an ordinary condition for solitary waves

$$\lim_{|t|\to\infty} Q_{1,2} = 0.$$

To control the accuracy of numerical solutions of equations (2), we used the integral of motion, i.e., the Manley–Rowe relation, which takes into account the relationship of forward and backward waves:

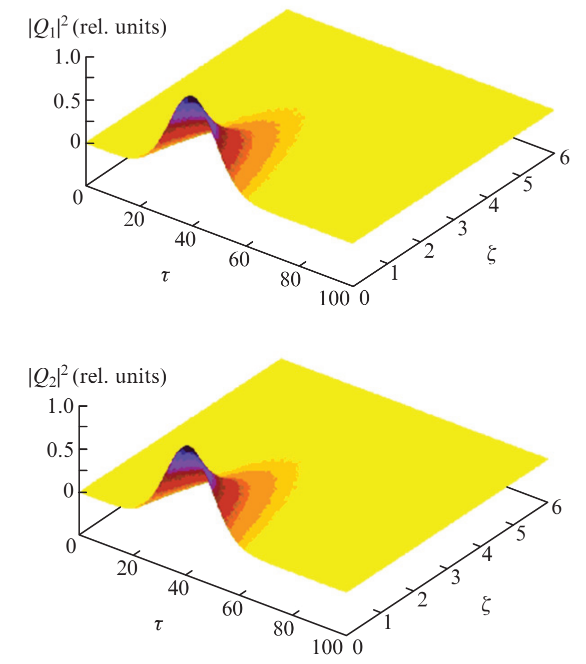

Figure 2. Reflection of a weak signal $(r = 1, a_{\mathrm{in}} = 1.0)$.

$$\int_{-\infty}^{+\infty}(a_2^2 - a_1^2)\mathrm{d}\tau = \mathrm{const}. \tag{8}$$

To verify the accuracy of the numerical code, we performed the calculations using boundary conditions corresponding to the gap soliton in [21]; the numerical solution coincided with the analytical solution (3)–(6).

Figure 2 shows the reflection of a weak signal coupled into the positive-index waveguide. The Gaussian pulse from the negative-index waveguide due to frustrated total internal reflection penetrates into the positive-index waveguide, where the energy flux is directed in the opposite direction, and the initial Gaussian pulse appears at the NADC input, but from another waveguide.

Gradually increasing the amplitude of the input pulse, a_{in}, it is possible to find a value when the input pulse reflection becomes negligible and a stationary pulse, localised in both waveguides, is formed (Figs 3 and 4). A further increase in the amplitude a_{in} slightly changes the velocity of the stationary pulse, which corresponds to gap solitons. The value of a_{in}, which yields a gap soliton, will be considered a threshold amplitude $a_{1\mathrm{th}}$. Figure 3 illustrates (in gray scale) how the input pulse with the amplitude smaller than the threshold one broadens and excites the broadening pulse in the second waveguide. But when its amplitude exceeds the threshold, two coupled pulses localised in both waveguides and retaining their shape are formed. Figure 4 shows that when the amplitude of the input pulse exceeds the threshold, a stationary pulse (gap soliton), the propagation velocity of which is less than the pulse velocity in the linear regime, is produced in both waveguides.

By changing the value of the nonlinearity parameter r in (2), we can numerically find the dependence of the amplitude threshold $a_{1\mathrm{th}}$ on this parameter (Fig. 5). From these graphs, it follows that

$$a_{1\mathrm{th}} \sim r^{-1/2}$$

[taking into account the error in determining $a_{1\mathrm{th}}$ from the results of numerical solutions of system (2).]

Such a dependence of the threshold amplitude of the input pulse on the nonlinearity parameter is typical of the gap soliton formation in the Bragg waveguide [26]. Qualitatively, it can be explained as follows. The frequency spectrum of linear waves has a gap of width (taken here in dimensionless variables) $\Delta\nu = 2$ [20, 21]. The soliton is formed near the beginning of the waveguide, which includes the pulse $Q_1 = a_1 \mathrm{e}^{\mathrm{i}\phi_1}$ and where we can neglect the spatial variations of amplitude and phase. It follows from (2) that the time derivative of the phase ϕ_1 determined by the nonlinearity of the waveguide

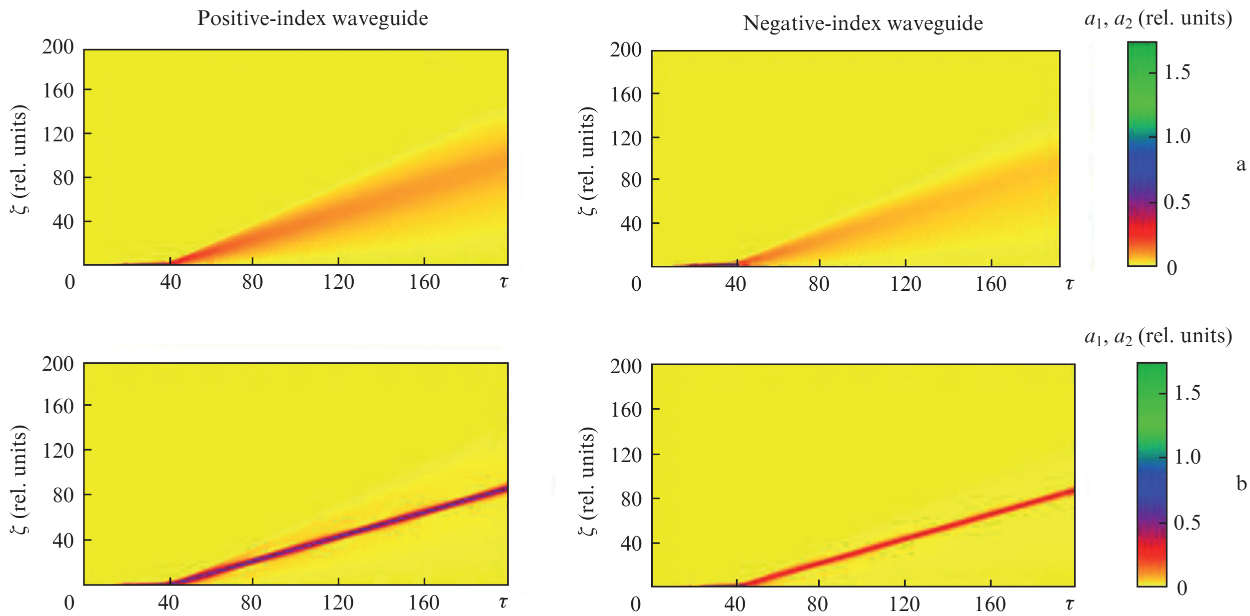

Figure 3. Gap soliton formation at $r = 1$ and different amplitudes of the input pulse: $a_{\mathrm{in}} = 2.09 < a_{1\mathrm{th}} = 2.10$ (a) and $a_{\mathrm{in}} = 2.13 > a_{1\mathrm{th}} = 2.10$ (b).

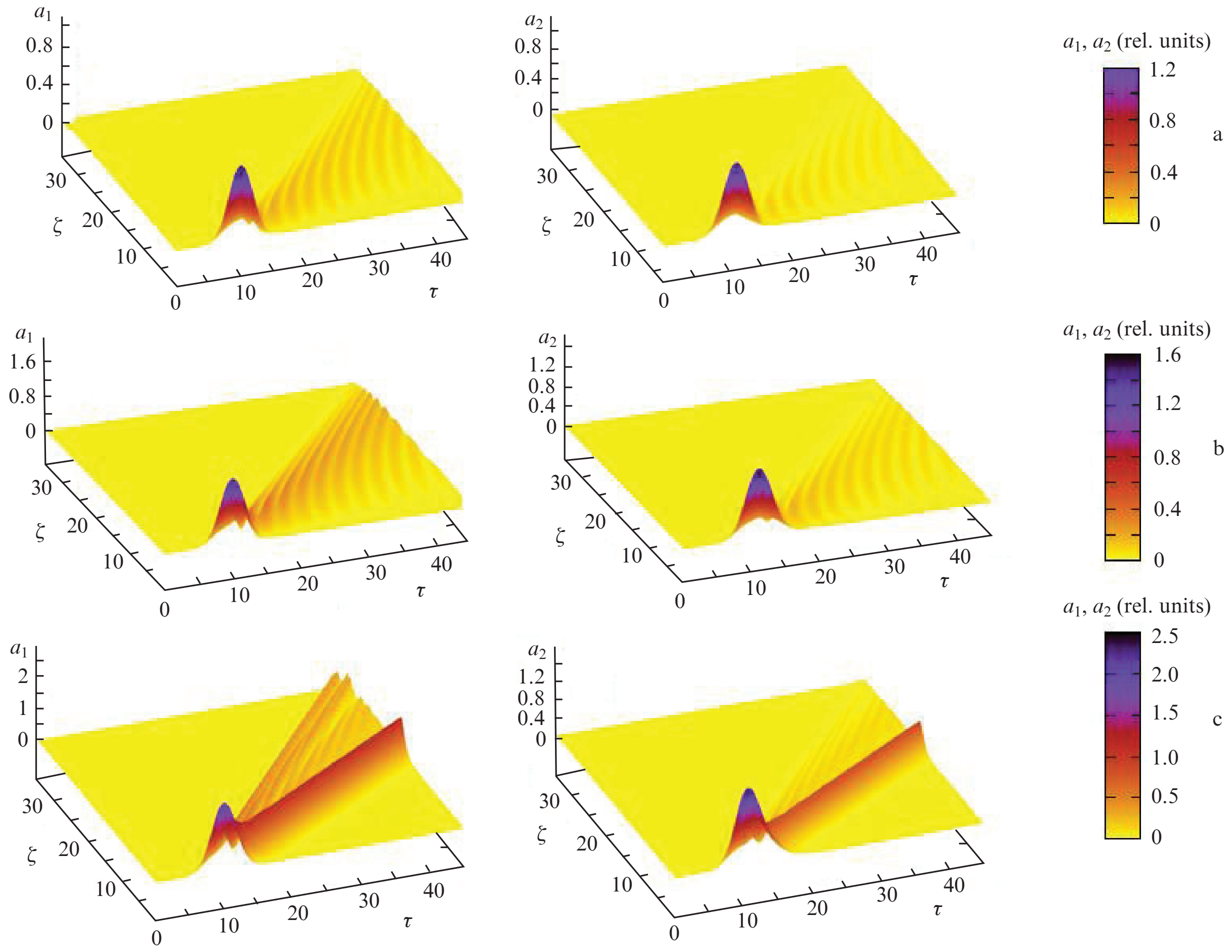

Figure 4. Evolution of pulses in NADC waveguides at $r = 1$ and different amplitudes of the input pulse: $a_{in} = 1.0$ (a), 1.5 (b) and 2.11 (c).

material is proportional $ra_1^2 \approx ra_{in}^2$. Therefore, the carrier frequency of the pulse is shifted by $\sim ra_{in\,max}$. If this shift is small compared to the $\Delta\nu/2$, the pulse cannot propagate in the waveguide (both in the Bragg waveguide and in the NADC) and is reflected. Otherwise, propagation takes place. Sometimes it is said that a sufficiently strong pulse locally changes the properties of the medium to such an extent that its spectrum turns out to be in the allowed band. Such a rough estimate of the threshold yields the formula $a_{1th} \sim r^{-1/2}$

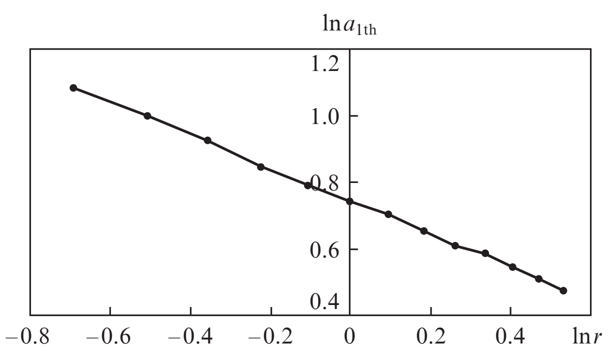

Figure 5. Logarithmic dependence of the threshold amplitude a_{1th} on the nonlinearity parameter r.

4. Conclusions

The formation of a stationary pulse (gap soliton) in a nonlinear anti-directional coupler, where one waveguide maintaining backward wave propagation is optically linear and the other waveguide made of an ordinary dielectric has a Kerr nonlinearity, is considered numerically. This configuration of (linear and nonlinear) waveguides is sufficient for the gap soliton formation. The resulting dependence of the threshold value of the input pulse amplitude on the nonlinearity parameter has the same form as in the case of a nonlinear Bragg waveguide. This is an expected result, because in the linear regime the dispersion law is the same for the NADC and Bragg waveguide (in particular, the spectrum has a band gap – an energy gap). Although structurally the NADC and Bragg waveguide differ, in both cases the existing stationary pulse can be attributed to the same type of nonlinear waves – a gap soliton.

In the calculations the duration of the input pulse was assumed fixed and equal to unity (in terms of normalised units). To analyse the effect of the input pulse duration on the threshold value of the gap soliton formation, one can, as above, consider the region near the NADC, where this process occurs. If, using the pulse duration t_p to determine the initial pulse amplitude as $a_1(t) = a_1(t/t_p)$, then at about $z = 0$, the equation for the input pulse phase ϕ_1 is approximately written as

$$\frac{\partial \phi_1}{\partial \tau} \approx r a_1^2 (\tau/\tau_{\rm in}),$$

where $\tau_{\rm in} = t_{\rm p}/t_0$ is the normalised pulse duration. When replacing the variable $\tau/\tau_{\rm in} = \tau'$, the equation takes the form

$$\frac{\partial \phi_1}{\partial \tau'} \approx r \tau_{\rm in} a_1^2 (\tau').$$

This equation is similar to the equation in the case of unit duration of the input pulse, the difference lies in the renormalisation of the nonlinearity coefficient. Therefore, the carrier frequency of the pulse is shifted by about $\tau_{\rm in} r a_{1\max}^2 = \tau_{\rm in} r a_{\rm in}^2$, which should exceed the value of the band gap in order to stop reflection. It follows that $a_{1\rm th} \sim (\tau_{\rm in} r)^{-1/2}$.

It should also be noted that the spectral width of the input pulse must not exceed the width of the band gap, which imposes a limit on the minimum pulse duration: $\tau_{\rm in} \Delta\nu \gg 1$. The maximum pulse duration may also be limited because of the development of the modulation instability [22].

Account for losses in the waveguides (or only one waveguide) prevents the gap soliton propagation at large distances. But the formation of such a soliton is possible if the loss coefficient is less than one-tenth of the reverse coupling length. At the value of the loss coefficient on the order of the inverse coupling length, the gap soliton formation was not found in the numerical calculations.

The fundamental point for the phenomenon considered in this paper is the coupling of the forward and backward waves. The NADC is a possible implementation of this kind of coupling. Some examples of the media where propagation of the backward waves is possible are discussed in [13, 17]. In recent years, surface plasmon waves and surface plasmon–polariton waves are actively studied, among which there are both forward and backward waves [27, 28]. Surface waves at the interface between a homogeneous medium and photonic crystal can act as backward waves in some frequency range [29]. The system of equations (2) describes in the coupled-mode approximation the linear interaction of the forward and backward waves in the cases mentioned. In this sense, system (2) is universal. Accounting for group-velocity dispersion, dissipation effects and nonlinear interaction (for example, due to parametric processes), of course, will require a further generalisation.

Acknowledgements. We are pleased to thank our colleagues S.O. Elyutin, I.R. Gabitov and E.V. Kazantseva. A.I. Maimistov is grateful to the Nonlinear Physics Centre, Research School of Physics and Engineering (The Australian National University) for their support and hospitality during the preparation of the manuscript. This work was supported by the Russian Foundation for Basic Research (Grant No. 12-02-00561).

References

1. Mandelstam L.I. *Zh. Eksp. Teor. Fiz.*, **15**, 475 (1945).
2. Sivukhin D.V. *Opt. Spektrosk.*, **3**, 308 (1957).
3. Pafomov V.E. *Zh. Eksp. Teor. Fiz.*, **30**, 761 (1956); **33**, 1074 (1957).
4. Pafomov V.E. *Zh. Eksp. Teor. Fiz.*, **36**, 1853 (1959).
5. Veselago V.G. *Usp. Fiz. Nauk.*, **92**, 517 (1967).
6. Smith D.R., Padilla W.J., Vier D.C., Nemat-Nasser S.C., Schultz S. *Phys. Rev. Lett.*, **84**, 4184 (2000).
7. Shelby R.A., Smith D.R., Schultz S. *Science*, **292**, 77 (2001).
8. Valentine J., Zhang S., Zentgraf T., Ulin-Avila E., Genov D.A., Bartal G., Zhang X. *Nature*, **455**, 376 (2008).
9. Yao J., Liu Z., Liu Y., Wang Y., Sun Ch., Bartal G., Stacy A., Zhang X. *Science*, **321**, 930 (2008).
10. Shumin Xiao, Drachev Vl.P., Kildishev Al.V., Xingjie Ni, Chettiar U.K., Hsiao-Kuan Yuan, Shalaev Vl.M. *Nature*, **466** (Lett.), 735 (2010).
11. Ramakrishna S.A. *Rep. Prog. Phys.*, **68**, 449 (2005).
12. Veselago V., Braginsky L., Shklover V., Hafner Ch. *J. Computational and Theoretical Nanoscience*, **3**, 189 (2006).
13. Agranovich V.M., Garshtein Yu.N. *Usp. Fiz. Nauk*, **176**, 1052 (2006).
14. Eleftheriades G.V., Balmain K.G. (Eds) *Negative-refraction Metamaterials: Fundamental Principles and Applications* (New York: Wiley, 2005).
15. Noginov M.A., Podolskiy V.A. (Eds) *Tutorials in Metamaterials* (Boca Raton – London – New York: Taylor and Francis Group, LLC/CRC Press, 2012).
16. Maimistov A.I., Gabitov I.R. *Eur. Phys. J. Spec. Top.*, **147**, 265 (2007).
17. Agranovich V.M., Shen Y.R., Baughman R.H., Zakhidov A.A. *Phys. Rev. B*, **69**, 165112 (2004).
18. Zharova N.A., Shadrivov I.V., Zharov A.A., Kivshar Yu.S. *Opt. Express*, **13**, 1291 (2005).
19. Litchinitser N.M., Gabitov I.R., Maimistov A.I. *Phys. Rev. Lett.*, **99**, 113902 (2007).
20. Maimistov A.I., Gabitov I.R., Litchinitser N.M. *Opt. Spektrosk.*, **104**, 292 (2008).
21. Kazantseva E.V., Maimistov A.I., Ozhenko S.S. *Phys. Rev. A*, **80**, 43833 (2009).
22. Xiang Yu., Wen Sh., Dai X., Fan D. *Phys. Rev. E*, **82**, 056605 (2010).
23. Marcatili E.A.J. *Bell. Syst. Techn. J.*, **48**, 2071 (1969).
24. Marcuse D. *Bell. Syst. Tech. J.*, **50**, 1791 (1971).
25. Tamir T. (Ed.) *Integrated Optics* (Berlin: Springer-Verlag, 1983; Moscow: Mir, 1978).
26. Kivshar Yu.S., Agrawal G.P. *Optical Solitons. From Fibers to Photonic Crystals* (San Diego: Acad. Press, 2003; Moscow: Fizmatlit, 2005).
27. Berini P. *Adv. Opt. Photon.*, **1**, 484 (2009).
28. Chern R.-L., Chang Ch.C., Chang C.Ch. *Phys. Rev. E*, **73**, 036605 (2006).
29. Vinogradov A.P., Dorofeenko A.V., Merzlikin A.M., Lisyansky A.A. *Usp. Fiz. Nauk*, **180**, 249 (2010).