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Dynamics of frequency-modulated soliton-like pulses in a longitudinally inhomogeneous, anomalous group velocity dispersion fibre amplifier

I.O. Zolotovskii, D.A. Korobko, O.G. Okhotnikov, A.A. Sysolyatin, A.A. Fotiadi

Abstract. **We examine conditions for the formation and amplification of frequency-modulated soliton-like pulses in longitudinally inhomogeneous, anomalous group velocity dispersion fibres. The group velocity dispersion profiles necessary for the existence and amplification of such pulses in active fibres are identified and the pulse duration and chirp are determined as functions of propagation distance.**

Keywords: *soliton-like pulses, fibre amplifier, anomalous group velocity dispersion.*

1. Introduction

The dynamics of optical solitons has recently attracted increased attention because of their fundamental importance and considerable potential for a variety of technological applications [1, 2]. One of the most important issues is the optimal amplification of soliton pulses, with no changes in their shape or elastic interaction. It is well known that, in the case of incoherent amplification in a longitudinally homogeneous nonlinear active medium, a soliton is not amplified as a whole. Independent of the amplification mechanism, increasing the soliton energy by approximately a factor of e leads to immediate significant distortions of the shape and spectrum of the soliton due to the increase in the nonsoliton component of the pulse. As a result, the nonlinear wave packet loses its structural stability and, as a consequence, its solitonic properties. This dynamic scenario of soliton amplification had long been thought to be the only possible one, until Serkin and Belyaeva [3, 4] and Melo Melchor et al. [5] demonstrated the possibility of amplifying an optical soliton as a whole provided the gain per unit length is a hyperbolic function of propagation distance and the phase of the soliton-like pulse at the fibre input is a parabolic function of time. Interaction between such frequency-modulated pulses is fully elastic when their phases and the gain coefficient of the medium are properly matched.

One of the main obstacles to the experimental demonstration of the proposed scenario of the 'ideal' amplification of shape-retaining wave packets is the necessity of producing an appropriate gain profile in a fibre. The problem can be solved by using longitudinally inhomogeneous fibres having not only hyperbolic but essentially any gain profiles if the group velocity dispersion (GVD) profile along the fibre is properly adjusted. In this paper, we examine the possibility of producing and amplifying a subpicosecond soliton in a fibre with longitudinally varying material parameters and establish functional relations between such parameters. The most attractive fibres for producing and amplifying soliton-like pulses with strong frequency modulation are those with an anomalous dispersion gradually decreasing in magnitude along the fibre and a W-shaped transverse refractive index profile. A required dispersion profile in such fibres can be produced by adjusting its transverse dimensions [6–8].

2. Model of amplification

Erbium-doped optical fibres are among the most widespread active fibres because they are widely used in fibre amplifiers for optical communication systems. The operating principle of the fibre amplifier – just as that of any other quantum amplifier – is that, if pumping produces population inversion in an active medium, a weak resonance electromagnetic field induces transitions from the upper, metastable level to the ground state, which lead to field amplification. An electron transition in erbium ions from the $^4I_{13/2}$ metastable level to the $^4I_{15/2}$ ground state corresponds to the spectral range 1.53–1.6 μm. Population inversion in this three-level medium is due to the long lifetime of the metastable level, 10–12 ms, whereas the decay from the pump level to the metastable level takes microseconds. The erbium-doped fibre amplifier operates as a three-level system when pumped at 0.98 μm and as a quasi-three-level system when pumped at 1.48 μm directly to the band corresponding to the metastable level, which ensures the highest efficiency. The possibility of 1.48-μm pumping is due to the significant Stark splitting in silica glass.

Assuming for simplicity that the dopant and field are uniformly distributed across the fibre core, we can represent the gain in this system as [9]

$$\frac{dN_2}{dt} = K_p(N - N_2) - K_s[(1+b)N_2 - bN] - \frac{N_2}{\tau_{21}},$$
$$2\gamma = \eta_s\sigma_s[(1+b)N_2 - bN], \tag{1}$$

I.O. Zolotovskii, D.A. Korobko Ulyanovsk State University, ul. L. Tolstogo 42, 432017 Ulyanovsk, Russia; e-mail: rafzol.14@mail.ru;
O.G. Okhotnikov Ulyanovsk State University, ul. L. Tolstogo 42, 432017 Ulyanovsk, Russia; present address: Optoelectronics Research Centre, Tampere University of Technology, Sähkötalo, K- and L-wings, Korkeakoulunkatu 3 (Hervanta), 33720 Tampere, Finland;
A.A. Sysolyatin Fiber Optics Research Center, Russian Academy of Sciences, ul. Vavilova 38, 119333 Moscow, Russia;
A.A. Fotiadi Ulyanovsk State University, ul. L. Tolstogo 42, 432700 Ulyanovsk, Russia; present address: University of Mons, B7000 Mons, Belgium

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Here, N_2 and N are the population of the metastable level and the total erbium concentration, respectively; τ_{21} is the lifetime of the metastable level; $b = \sigma_p/\sigma_s$ is the ratio of the pump absorption cross section σ_p to the emission cross section σ_s at the wavelength of the signal being amplified; the parameter $\gamma(z)$ has the meaning of a local gain per unit length; η_s is the fraction of the signal being amplified in the core; and K_p and K_s are the rates of transitions at the pump and signal wavelengths, respectively.

In a steady state, the population of the metastable level does not vary over time, i.e. $dN_2(z)/dt = 0$, and we have

$$N_2 = \frac{N(K_p + bK_s)}{K_p + (1+b)K_s + 1/\tau_{21}}. \tag{2}$$

Substituting this expression into the first equation in (1), we find the gain per unit length:

$$\gamma = \frac{1}{2}\frac{\sigma_s\eta_s N(K_p - b/\tau_{21})}{K_p + (1+b)K_s + 1/\tau_{21}}. \tag{3}$$

In the case of a copropagating pump and signal configuration, the equations for the variation of the pump intensity I_p and amplified signal intensity I_s along the fibre have the form

$$\frac{dI_p}{dz} = -I_p\sigma_p(N - N_2),$$
$$\frac{dI_s}{dz} = 2\gamma I_s. \tag{4}$$

In solving Eqns (4), one should take into account that K_p and K_s satisfy the relations [10]

$$K_p(z) = \frac{\sigma_p\eta_p I_p(z)}{h\nu_p},$$
$$K_s(z) = \frac{\sigma_s\eta_s[I_s(z) + I_f(z)]}{h\nu_s}, \tag{5}$$

where I_f is the spontaneous luminescence intensity and η_p is the fraction of the pump power in the core. If a source (e.g. an erbium-doped fibre laser) ensures the generation of picosecond pulses with a repetition rate v, the average intensity of a sequence of signals can be estimated as $I_s(z) = vW_s(z)$, where $W_s(z)$ is the energy of a separate signal pulse at an amplifier fibre length z.

Under the assumption that the erbium concentration is constant along the fibre, it can be shown that the net rate of stimulated transitions does not vary along the fibre, which is analogous to the conservation of the number of photons in Raman scattering [11]:

$$K_p(z) + (1+b)K_s(z) = K_0. \tag{6}$$

With (6), the equation for the pump intensity has the form

$$\frac{dI_p}{dz} = \frac{-\eta_p\sigma_p^2 N(I_0 - I_p)I_p}{(K_0 + 1/\tau_{21})(1+b)h\nu_p},$$

where

$$I_0 = \frac{[K_0 + (1+b)/\tau_{21}]h\nu_p}{\eta_p\sigma_p}.$$

Integrating the pump intensity equation and substituting initial conditions, we obtain

$$K_p = \frac{K_{p0} - (1+b)/\tau_{21}}{(I_0/I_{p0} - 1)\exp(\vartheta z) + 1}, \tag{7}$$

where

$$\vartheta = N\frac{K_0 + (1+b)/\tau_{21}}{K_0 + 1/\tau_{21}}\frac{\sigma_p}{1+b}; \tag{8}$$

I_{p0} is the pump intensity at $z = 0$; and $K_{p0} = \sigma_p\eta_p I_{p0}/(h\nu_p)$.

Using this solution, we obtain an equation for the signal intensity:

$$\frac{dI_s}{dz} = I_s\frac{\eta_s\sigma_s N(K_p - b/\tau_{21})}{K_0 + 1/\tau_{21}}.$$

Its solution is

$$\ln\frac{I_s}{I_{s0}} = \frac{\eta_s\sigma_s(1+b)}{\sigma_p}\ln\frac{I_0/I_{p0}}{(I_0/I_{p0} - 1) + \exp(-\vartheta z)} - \frac{N\eta_s\sigma_s b}{K_0\tau_{23} + 1}z.$$

Here, I_{s0} is the initial signal intensity. Thus, the effective gain, equal to the relative increase in signal pulse energy over length z,

$$G_{eff}(z) = \frac{W_s(z)}{W_0} = \exp\left(2\int_0^z \gamma(\xi)\,d\xi\right), \tag{9}$$

can be represented in the form

$$G_{eff}(z) = \left[\frac{I_0/I_{p0}}{(I_0/I_{p0} - 1) + \exp(-\vartheta z)}\right]^{\beta}$$
$$\times\exp\left[-\frac{\eta s\sigma s N}{(K0 + 1/\tau 21)\tau 21}\frac{bz}{\tau 21}\right], \tag{10}$$

where

$$\beta = \frac{\eta_s\sigma_s(1+b)}{\sigma_p}.$$

This relation was obtained with allowance for spontaneous luminescence, which results in an exponential signal decay. The effect of luminescence can be neglected for $\tau_{21} \to \infty$. In the case of high pump power ($I_{p0} \gg I_{s0}$), we can also neglect pump depletion in the initial stage of amplification for $z \ll \vartheta^{-1}\ln(I_0/I_{p0} - 1)$, which leads to a known expression for the relative pulse energy gain:

$$G_{eff}(z) = \exp(N\eta_s\sigma_s z).$$

However, in the case of FM soliton-like pulses propagating through anomalous-dispersion fibre amplifiers, the use of high pump powers leads to an undesirable influence of modulation instability and stimulated Raman scattering (SRS), which can break up a stable wave packet. For the same reasons, the energy of the FM soliton-like signals under consideration does not exceed several picojoules, which also indicates that the spontaneous luminescence intensity cannot be neglected. Therefore, Eqn (10) should be used in the case under consideration.

Figure 1 shows $G_{\rm eff}(z)$ curves for an amplifier with the following parameters: $N = 5 \times 10^{24}$ m^{-3}, $\tau_{21} = 10^{-2}$ s, $\eta_{\rm s} = 0.9$, $\eta_{\rm p} = 0.8$, $\sigma_{\rm s} = 10^{-25}$ m^2, $\sigma_{\rm p} = 1.5 \times 10^{-25}$ m^2, $\nu_{\rm p} = 2.03 \times 10^{14}$ Hz and $\nu_{\rm s} = 1.96 \times 10^{14}$ Hz. These parameters will be used below. The initial signal intensity is $I_{\rm s0} = 5 \times 10^9$ W m^{-2}. For pulses several picoseconds in duration, this corresponds to energies of the order of a picojoule. The curves in Fig.1 are for pump intensities $I_{\rm p0} = (2.5, 5$ and $10) \times 10^{10}$ W m^{-2}.

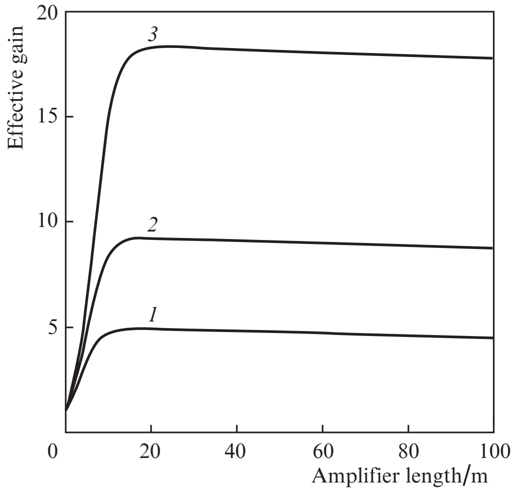

Figure 1. Relative signal intensity $G_{\rm eff}$ as a function of amplifier length at initial pump intensities $I_{\rm p0}$ = (*1*) 2.5 × 10^{10}, (*2*) 5 × 10^{10} and (*3*) 10^{11} W m^{-2}. The amplifier parameters are specified in text.

3. Dynamics of a soliton-like pulse in an inhomogeneous amplifier

Consider the dynamics of an optical wave packet propagating through an inhomogeneous amplifying fibre. The temporal envelope of the wave packet, $A(t,z)$, meets a nonlinear Schrödinger equation [1, 2] with coefficients dependent on z:

$$\frac{\partial A}{\partial z} - {\rm i}\frac{D(z)}{2}\frac{\partial^2 A}{\partial \tau^2} + {\rm i}R(z)|A|^2 A = g(z)A. \quad (11)$$

Here,

$$\tau = t - \int_0^z \frac{{\rm d}\xi}{u(\xi)}$$

is the time in a moving frame of reference; $u(z)$ is the group velocity; $D(z)$ is the group velocity dispersion; and $R(z)$ is the Kerr nonlinearity coefficient. The effective gain per unit length is given by

$$g(z) = \gamma(z) - \frac{\partial S_{\rm m}}{\partial z}\frac{1}{2S_{\rm m}}, \quad (12)$$

where $\gamma(z)$ is the gain per unit length of the fibre material. The second term on the right-hand side of (12) represents the contribution of possible changes in an effective mode area:

$$S_{\rm m}(z) = 2\pi\int_0^\infty |U(r,z)|^2 r{\rm d}r, \quad (13)$$

where $U(r,z)$ is the mode profile of the fibre. For convenience of further analysis, the dispersion and nonlinearity parameters can be represented in the form $D(z) = D_0 d(z)$ and $R(z) = R_0 r(z)$, where D_0 and R_0 are the corresponding parameters at the fibre input. With the new variable

$$\eta(z) = \int_0^z d(\xi){\rm d}\xi$$

and the pulse envelope

$$C(\tau,z) = \sqrt{r(z)/d(z)}\,A(\tau,z),$$

Eqn (11) transforms into an equation with constant GVD and nonlinearity parameters but a nonuniform effective gain:

$$\frac{\partial C}{\partial \eta} - {\rm i}\frac{D_0}{2}\frac{\partial^2 C}{\partial \tau^2} + {\rm i}R_0|C|^2 C = g_{\rm eff}(\eta)C. \quad (14)$$

Here,

$$g_{\rm eff}(\eta) = \frac{\gamma(\eta)}{d(\eta)} - \frac{1}{2}\frac{\partial}{\partial \eta}\ln\frac{\tilde{S}_m(\eta)d(\eta)}{r(\eta)} \quad (15)$$

is a nonuniform (with respect to the new longitudinal coordinate η) effective gain per unit length and $\tilde{S}_{\rm m} = S_{\rm m}(\eta)/S(0)$ is a normalised effective mode area. As a result of the above transformations, the problem of nonlinear pulse propagation through a fibre nonuniform in material parameters along its length reduces to the problem of pulse propagation through a fibre uniform in dispersion D_0 and nonlinearity R_0 but nonuniform in effective gain $g_{\rm eff}(\eta)$.

Equation (14) has a growing FM soliton solution if $D(\eta)R(\eta) < 0$ and the effective gain (15) can be represented in the form $g_{\rm eff}(\eta) = Q/(1 - 2Q\eta)$, where $Q = g_{\rm eff}(0)$. The solution to Eqn (14) can then be represented in the form

$$C(\tau,\eta) = \frac{C_0}{1 - 2Q\eta}{\rm sech}\frac{\tau}{\tau_{\rm s}}\exp\left({\rm i}\frac{\alpha_0\tau^2 - \Gamma_0\eta}{1 - 2Q\eta}\right), \quad (16)$$

where the soliton duration is $\tau_{\rm s} = \tau_0(1 - 2Q\eta)$. The parameters of this equation must satisfy the relations $2\Gamma_0 = |D_0|/\tau_0^2 = R_0|A_0|^2$ and $\alpha_0 = Q/(2\Gamma_0\tau_0^2)$. The formation energy of a soliton-like wave packet is $W_{\rm s} = \tau_0|A_0|^2 = |D_0|/(R_0\tau_0)$. The nonlinear wave packets represented by Eqn (16) are referred to in the literature as bright FM solitons and exhibit elastic interaction, a property important for practical application [1, 2].

Given that $Q = \alpha_0|D_0|$, the condition for the existence of an FM soliton in the case of the inhomogeneity and anomalous GVD under consideration can be represented in the form

$$g_{\rm eff}(z) = -\frac{\alpha_0 D_0}{1 + 2\alpha_0\eta} = \frac{Q}{1 - 2Q\eta}. \quad (17)$$

Substitution of (15) into (17) yields

$$\left(1 + 2\alpha_0\int_0^z D(\xi){\rm d}\xi\right)\exp\left(2\int_0^z \gamma(\xi){\rm d}\xi\right) = \frac{D(z)}{D_0}\frac{S_{\rm m}(z)}{S_{\rm m0}}\frac{R_0}{R(z)}. \quad (18)$$

The GVD profile needed for the formation of an FM soliton-like pulse is given by

$$D(z) = D_0 f(z) \exp\left(2\alpha_0 D_0 \int_0^z f(\xi)\,\mathrm{d}\xi\right), \tag{19}$$

where $f(z) = F(z)G_{\rm eff}(z)$ and $F(z) = R(z)S_{\rm m}(0)/(R_0 S_{\rm m}(z))$. If the conditions for the existence of FM pulses in an anomalous-dispersion medium are satisfied, the exact solution for the duration of a sech-shaped soliton-like pulse has the form

$$\tau_{\rm s}(z) = \frac{\tau_0}{F(z)}\frac{D(z)}{D_0}\exp\left(-2\int_0^z \gamma(\xi)\,\mathrm{d}\xi\right)$$

$$= \tau_0 \exp\left(2\alpha_0 D_0 \int_0^z f(\xi)\,\mathrm{d}\xi\right). \tag{20}$$

Moreover, a sech-shaped FM soliton can be taken to meet the relation $\tau_{\rm s}(z)\alpha(z) = \mathrm{const} = \tau_{\rm s0}\alpha_0$. It is easy to derive an expression for the pulse chirp from this relation:

$$\alpha(z) = \alpha_0 \exp\left(-2\alpha_0 D_0 \int_0^z f(\xi)\,\mathrm{d}\xi\right). \tag{21}$$

Note that, in studies of the dynamics of a subpicosecond pulse in an inhomogeneous fibre, one should in general take into account third-order dispersion (TOD), which has a significant effect on the pulse shape and, at the pulse durations in question, may cause the pulse to decay. In the fibres under consideration, with an anomalous GVD that gradually decreases in magnitude, the effective TOD coefficient increases with distance (because of the decrease in d). Even if the TOD is low at the fibre input, its influence becomes significant starting at a certain fibre length. Therefore, to amplify an FM soliton, it is desirable that, throughout the fibre length, the condition $|\beta_3(z)| < |D(z)|/\Delta\omega(z)$ should be satisfied, where $\Delta\omega(z)$ is the spectral width of the wave packet.

However, with decreasing wave packet duration (i.e. with increasing spectral width) and decreasing GVD modulus along the fibre, this condition becomes rather difficult to satisfy. As mentioned above, this condition can be satisfied in recently proposed fibres with a W-shaped transverse refractive index profile [6–8]. In such fibres, one can produce the required longitudinal GVD profile at very small third-order dispersion parameters. The technology currently used to draw such fibres ensures significant changes in dispersion even at a small diameter difference over the entire fibre length: typically no greater than 3 μm. At an average fibre diameter of about 100 μm, this is a very low value. The effective mode area and nonlinearity coefficient in such fibres are almost constant throughout their length, which allows the function $F(z)$ introduced in (19) to be set to equal unity with high accuracy. Given this, the dispersion profile necessary for the formation of an FM soliton in this type of active fibre can be represented in the form

$$D(z) = -|D_0|\,G_{\rm eff}(z)\exp\left(-2\alpha_0 |D_0| \int_0^z G_{\rm eff}(\xi)\,\mathrm{d}\xi\right). \tag{22}$$

Accordingly, the duration and chirp of a soliton-like pulse in a W-profile active fibre are given by

$$\tau(z) = \tau_0 \exp\left(-2\alpha_0 |D_0| \int_0^z G_{\rm eff}(\xi)\,\mathrm{d}\xi\right), \tag{23}$$

$$\alpha(z) = \alpha_0 \exp\left(2\alpha_0 |D_0| \int_0^z G_{\rm eff}(\xi)\,\mathrm{d}\xi\right). \tag{24}$$

The function $G_{\rm eff}(z)$ was found above [Eqn (10)] in the proposed model of amplification. The integral of this function can be expressed through a hypergeometric function [12]:

$$\int_0^z G_{\rm eff}(\xi)\,\mathrm{d}\xi = \left(\frac{I_0}{I_0 - I_{\rm p0}}\right)^{\beta}\frac{1}{\mu\vartheta}\left\{{}_2F_1\left(\beta,\mu,1+\mu;-\frac{I_{\rm p0}}{I_0 - I_{\rm p0}}\right)\right.$$

$$\left.-[\exp(-\vartheta z)]^{\mu}\,{}_2F_1\left(\beta,\mu,1+\mu;-\frac{I_{\rm p0}\exp(-\vartheta z)}{I_0 - I_{\rm p0}}\right)\right\}, \tag{25}$$

where

$$\mu = \frac{\eta_{\rm s}\sigma_{\rm s} b}{[1 + K_0\tau_{21}/(1+b)]\sigma_{\rm p}}.$$

Taking $\tau_{21} \to \infty$, i.e. neglecting the spontaneous luminescence noise, we have for passive fibres $G_{\rm eff}(z) = 1$. Therefore,

$$\int_0^z G_{\rm eff}(\xi)\,\mathrm{d}\xi = z.$$

The expressions for the dispersion profile in the fibre and the duration and chirp of a soliton-like pulse then transform to those derived earlier by Zolotovskii et al. [13]. Note also that, in the above model, absorption is left out of consideration because the absorption coefficient in real passive fibres with a W-shaped radial refractive index profile is under 0.5 dB km^{-1}.

Figure 2 shows $D(z)/D_0$ profiles (22) that ensure FM soliton propagation through a fibre amplifier at different pump levels, $D_0 = -2 \times 10^{-26}$ s^2 m^{-1} and $\alpha_0 = 2 \times 10^{-23}$ s^{-2} (the other parameters were specified above). The dashed line represents data for a passive fibre.

Figures 3 and 4 show relative pulse duration $\tau(z)/\tau_0$ (23) and relative frequency modulation rate $\alpha(z)/\alpha_0$ (24) curves at

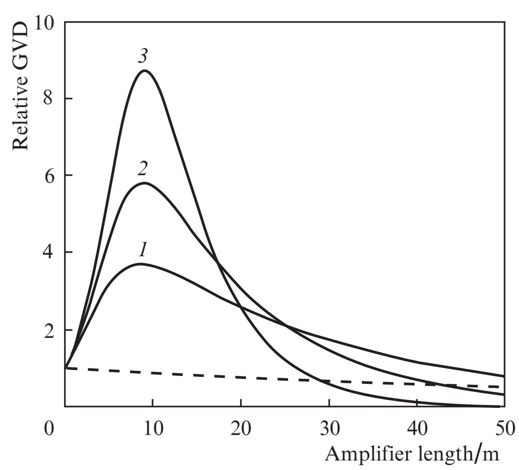

Figure 2. Relative GVD (D/D_0) profiles that ensure FM soliton propagation through an amplifier at the same pump levels [curves (1)–(3)] and amplifier parameters as in Fig. 1. The dashed line represents data for a passive fibre.

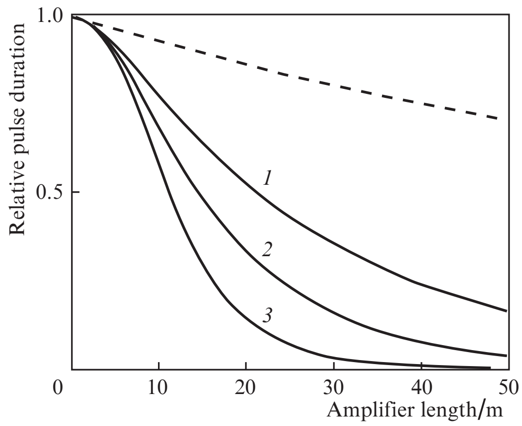

Figure 3. Relative soliton-like pulse duration τ/τ_0 as a function of amplifier length at the same pump levels [curves (*1*)–(*3*)] and amplifier parameters as in Fig. 1. The dashed line represents data for a passive fibre.

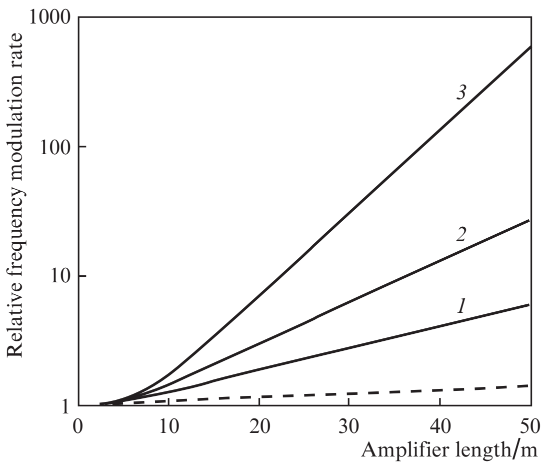

Figure 4. Relative soliton-like pulse frequency modulation rate α/α_0 as a function of amplifier length at the same pump levels [curves (*1*)–(*3*)] and amplifier parameters as in Fig. 1. The dashed line represents data for a passive fibre.

different pump levels and the parameters specified above. The dashed lines represent data for a passive fibre.

It can be seen from Figs 3 and 4 that the active fibres considered above can be used not only (not so much) to amplify FM solitons but also (as) to generate pulses with strong frequency modulation and an almost linear variation in instantaneous frequency. An FM pulse at the output of an inhomogeneous fibre can then be compressed in a normal-effective-dispersion medium. This can be done using a diffraction grating pair or a photonic crystal fibre with low Kerr nonlinearity [2, 14–16]. In the latter case, an all-fibre system can be made.

4. Conclusions

It follows from the above analysis that the conditions for the existence of FM solitons of the form (16) in active fibres can be satisfied when there is an appropriate fibre diameter profile, defined by (19), which is of most interest for the experimental demonstration of such nonlinear wave packets. Varying the fibre diameter is the most convenient approach for achieving 'soliton control' in the W-profile fibres under consideration, i.e. for tailoring the parameters of soliton-like pulses by producing appropriate gain, nonlinearity and dispersion profiles.

There may be great practical interest in the ability to produce FM pulses with a relatively low energy (about 10 pJ) but large chirp (over 10^{24} s^{-2}), short duration and, hence, high peak power, which can be amplified in normal-dispersion amplifier fibres. Thus, inhomogeneous anomalous dispersion fibres can be used as 'seed' elements generating FM soliton-like pulses with a relatively low energy, which are then amplified in multistage subpicosecond and femtosecond laser systems to peak powers above 100 kW [17, 18].

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