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Comparative analysis of the use of various solid-state laser media for the self-starting of four-wave PCW generation in a loop laser resonator

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Abstract. **A generalised theory has been used to carry out a comparative analysis of the use of various four-level and quasi-three-level media for the self-starting of degenerate four-wave mixing PCW generation directly in a laser medium placed in a loop resonator. It has been shown that quasi-three-level media can compete with four-level media at long upper laser level lifetimes and increased pump intensities. The most attractive solid-state laser media for four-wave PCW generation have been identified that have the highest deposited energy at a given pump intensity. In addition to neodymium-doped crystals, which are already widely used for four-wave PCW generation, promising materials are four-level chromium-doped media, e.g. alexandrite and Cr:LiCAF, and quasi-three-level media with the longest upper laser level lifetime, such as Yb:YAG and Tm,Ho:YAG, at high pump intensities.**

Keywords: *four-level and quasi-three-level media, four-wave PCW generation, gain grating.*

1. Introduction

There is currently increasing interest in the study of diffraction-coupled lasing, in which a solid-state laser medium is not only an amplifier of laser radiation but also an element of optical coupling in four-wave wavefront conjugation [1–21]. This allows one to develop adaptive laser systems employing self-pumped phase-conjugate mirrors (PCMs) [1–14] and ensures coherent beam combination for such lasers [15–21].

The self-starting of a laser with a self-pumped PCM due to a positive feedback through degenerate four-wave mixing (DFWM) was first discussed in Refs [1, 22, 23] and first demonstrated by Vanherzeele et al. [22]. Lasing in a loop resonator with a self-pumped four-wave PCM directly in a gain medium was first demonstrated in a copper vapour laser [24]. For such a phase-conjugate laser oscillator, Bel'dyugin et al. [2] used for the first time rare-earth-doped solid-state laser media: lamp-pumped Nd:YAG, Nd:KGW and Nd, Cr:GSGG laser crystals. Minassian et al. [5] were the first to use a solid-state laser medium containing iron group ions: laser-pumped Ti:sapphire.

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The realisation of four-wave diffraction coupling directly in a laser medium requires an increase in its gain coefficient, which can be achieved by increasing the pump intensity, e.g. through narrow-band diode pumping focused into an active laser medium [6, 10, 11, 13]. Sillard et al. [6] excited a Nd:YAG laser crystal of length $L = 0.5$ cm in a phase-conjugate laser configuration using longitudinal pumping by a 2D laser diode array with a power in the order of one kilowatt. The pump beam was focused to a small spot 1.5×1.5 mm in dimensions, which ensured an increase in pump intensity to 35 kW cm^{-2}. The pump intensity was, however, found to decrease with increasing penetration depth in the gain element (GE) because of the pump beam absorption and divergence, and the small-signal gain after two passes through the GE was within 20 [6], so three such GEs were used in a laser configuration with a self-pumped phase-conjugate loop resonator. Thompson et al. [10] and Smith and Damzen [13] used side pumping in a bounce geometry [25] by 50- or 100-W diode stacks, whose output was focused by a cylindrical lens to a narrow line (0.1×15 mm) along a GE of length $L = 1.5$ cm. This increased the pump intensity to 4–7 kW cm^{-2} over the entire GE length, ensuring an ultrahigh small-signal gain (above 10^4) per pass through the Nd:YVO_4 GE [10, 13] for total internal reflection of the laser beam from the pump face. In the case of such guided propagation, the laser radiation retains the shape of its wavefront despite the distortion of the laser medium, as was first demonstrated by Mikaelyan and D'yachenko [26]. The choice of Nd:YVO_4 crystals as active media was prompted by their better performance (effective gain cross section $\sigma_{em}(\omega_L) = 15.6\times10^{-19}$ cm^2 [27]) compared to the other neodymium-doped gain media. This enabled the number of laser crystals in the phase-conjugate laser configuration to be reduced to one.

Antipov et al. [11] used side diode pumping in a bounce geometry for a Nd:YAG phase-conjugate laser. Because of the lower gain performance of Nd:YAG ($\sigma_{em}(\omega_L) = 2.8\times10^{-19}$ cm^2 [27]), they utilised a diode stack with a higher power, up to 300 W. The pump beam was focused to a spot 0.1×10 mm in dimensions, which corresponded to a pump intensity $I_p \approx 30$ kW cm^{-2} over the entire GE length ($L = 1$ cm) and ensured ultrahigh gain per pass through the Nd:YAG GE, equal to that in Nd:YVO_4 crystals [10, 13], with the possibility of using only one laser crystal in the phase-conjugate laser configuration. Basiev et al. [14] and Pogoda et al. [28] employed high-power side pumping of a long ($L = 10$ cm) Nd:YAG crystal in a phase-conjugate laser by six 2D diode arrays with a total power of 12.6 kW, without focusing. The pump radiation was distributed over the entire length of the laser crystal, which ensured a large gain coefficient and made it possible to use only one laser crystal in the phase-conjugate laser configuration.

High-intensity diode pumping extends the range of laser media suitable for the self-starting of four-wave phase-conjugate wavefront (PCW) generation and allows one to use not only four-level but also quasi-three-level laser media, which requires a detailed analysis.

This paper presents a comparative analysis of the use of various four-level and quasi-three-level laser media for the self-starting of DFWM PCW generation at increased pump intensities directly in a laser medium placed in a loop resonator.

2. Theoretical description of gain saturation gratings in quasi-three-level and four-level solid-state laser media

As shown earlier [29, 30], the main mechanism underlying the formation of a PCM in laser media is the inscription of holographic gain saturation gratings. Such gratings can be written by spatial hole burning in local population inversion through interaction between two or more interfering coherent beams.

In a number of instances, appreciable contributions can be made by other mechanisms, e.g. refractive index grating inscription. Temperature variations of the refractive index of the laser medium [31–33] are less significant, but electronically induced index changes can be noticeable when the laser frequency is detuned from the centre of the gain band. In the case of lasing near the centre of the band, one can also observe electronically induced index changes, which are caused by the difference in polarisability between laser levels and increase with increasing population inversion. However, according to Antipov et al. [32], at a gain coefficient under 0.4 cm^{-1} the imaginary part of the nonlinear susceptibility of Nd:YAG exceeds its real part by a factor of 2 or more, so electronically induced changes in refractive index do not play a decisive role in the intracavity DFWM process. As shown by Antipov et al. [33], the electronic changes induced in the refractive index of the Yb:YAG quasi-three-level medium by high-power diode pumping can be comparable to those induced by gain saturation grating inscription. Note however that, in the initial stage of lasing, when pumping has not yet increased the gain coefficient, we can restrict our consideration to gain grating formation. This facilitates comparative analysis of the use of various laser media for PCW generation through self-pumped DFWM.

To carry out such analysis, one should develop a generalised theory applicable to both quasi-three-level and four-level laser media. The gain coefficient of a laser medium can be represented in the form [34]

$$\alpha = \sigma_{\rm em}(\lambda_{\rm L})N_2 - \sigma_{\rm abs}(\lambda_{\rm L})N_1, \qquad (1)$$

where $\sigma_{\rm em}(\lambda_{\rm L})$ is the effective gain cross section at the laser wavelength, $\lambda_{\rm L}$; $\sigma_{\rm abs}(\lambda_{\rm L})$ is the effective absorption cross section at this wavelength; and $N_{1,2}$ are the populations of the upper and lower laser manifolds, respectively. Note that $\sigma_{\rm em}(\lambda_{\rm L})$ and $\sigma_{\rm abs}(\lambda_{\rm L})$ differ from the spectroscopic cross sections for transitions between particular energy levels: they describe transitions at arbitrary wavelengths between two manifolds and depend on the population distribution in each manifold. As pointed out by Contag et al. [35], this approach has the advantage that effective cross sections can be evaluated directly from absorption spectra measured at a preset temperature, without any calculations that would take into account the Boltzmann statistics of the population distribution, and this model is completely equivalent to a description that considers only transitions between particular energy levels.

The population of the upper laser manifold, N_2, can be found using the rate equation [34]

$$\frac{\partial N_2}{\partial t} = [\sigma_{\rm abs}(\lambda_{\rm p})N_1 - \sigma_{\rm em}(\lambda_{\rm p})N_2]\frac{I_{\rm p}}{\hbar\omega_{\rm p}} - [\sigma_{\rm em}(\lambda_{\rm L})N_2 - \sigma_{\rm abs}(\lambda_{\rm L})N_1]\frac{I_{\rm L}}{\hbar\omega_{\rm L}} - \frac{N_2}{\tau}, \qquad (2)$$

where $I_{\rm p}$, $I_{\rm L}$, $\hbar\omega_{\rm p}$ and $\hbar\omega_{\rm L}$ are the pump and laser intensities and photon energies, respectively; τ is the upper laser level lifetime; and $\sigma_{\rm em,abs}(\lambda_{\rm p})$ are the effective stimulated emission (em) and absorption (abs) cross sections at the pump wavelength, $\lambda_{\rm p}$. The population of the lower laser manifold in a quasi-three-level medium can be found as $N_1 = N_{\rm ions} - N_2$, where $N_{\rm ions}$ is the active-ion concentration in the solid-state laser medium.

Equations (1) and (2) can be used as well for four-level laser media as a particular case at $\sigma_{\rm abs}(\lambda_{\rm L}) = \sigma_{\rm em}(\lambda_{\rm p}) = 0$. In spectroscopic terms, a four-level laser medium has nonoverlapping absorption and luminescence lines, so $\sigma_{\rm abs}(\lambda_{\rm L}) = \sigma_{\rm em}(\lambda_{\rm p}) = 0$. In a quasi-three-level medium, these lines partially overlap, and both $\sigma_{\rm abs}(\lambda_{\rm L})$ and $\sigma_{\rm em}(\lambda_{\rm p})$ are greater than zero. For convenience of comparison, we introduce the following parameters of a quasi-three-level medium:

$$f_{\rm p} = \frac{\sigma_{\rm em}(\lambda_{\rm p})}{\sigma_{\rm abs}(\lambda_{\rm p})}, \quad f_{\rm L} = \frac{\sigma_{\rm abs}(\lambda_{\rm L})}{\sigma_{\rm em}(\lambda_{\rm L})}, \qquad (3)$$

which have the meaning of relative efficiencies of stimulated emission at the pump wavelength ($f_{\rm p}$) and absorption at the laser wavelength ($f_{\rm L}$). We have $f_{\rm p,L} = 0$ in four-level media and $f_{\rm p,L} > 0$ in quasi-three-level media.

For pump light with a finite bandwidth, the pump intensity in (2) is given by

$$I_{\rm p} = I_{\rm p}^{\rm int}\int_{-\infty}^{+\infty} g_{\rm abs}(\omega_{\rm p})g_{\rm p}(\omega_{\rm p}){\rm d}\omega_{\rm p} \approx I_{\rm p}^{\rm int}\frac{\Delta\omega_{\rm abs}}{\Delta\omega_{\rm abs}+\Delta\omega_{\rm p}}, \qquad (4)$$

where $I_{\rm p}^{\rm int}$ is the spectrum-integrated pump intensity, which is multiplied in (4) by the overlap integral of the absorption line of the laser medium and the pump line with form factors $g_{\rm abs}(\omega_{\rm p})$ and $g_{\rm p}(\omega_{\rm p})$, respectively [$g_{\rm abs}(\omega_{\rm p})$ is normalised to unit height, and $g_{\rm p}(\omega_{\rm p})$, to unit area]. The approximate equality in (4) corresponds to Lorentzian shapes of overlapping absorption and pump lines, where $\Delta\omega_{\rm abs}$ and $\Delta\omega_{\rm p}$ are the full widths at half maximum of these lines.

When there is continuous pumping ($I_{\rm p}$ = const) and no lasing ($I_{\rm L} = 0$) for time t, Eqn (2) subject to $N_2(0) = 0$ has the solution

$$N_2(t) = N_2^{\infty}\Big\{1 - \exp\Big[-\frac{t}{\tau}(1 + i_p(1 + f_p)\Big]\Big\}, \quad (5)$$

where $i_p = I_p/I_p^{sat}$ is the relative pump intensity;

$$I_p^{sat} = \frac{\hbar\omega_p}{\sigma_{abs}(\lambda_p)\tau} \quad (6)$$

is the pump absorption saturation intensity; and

$$N_2^{\infty} = N_{ions}\frac{i_p}{1 + i_p(1 + f_p)} \quad (7)$$

is the maximum upper laser manifold population for $t \to \infty$. From (1), we obtain an expression for the unsaturated gain coefficient of the laser medium:

$$\alpha_0(t) = \sigma_{em}(\lambda_L)N_{ions}\Big[(1 + f_L)\frac{i_p}{1 + i_p(1 + f_p)} \times\Big\{1 - \exp\Big[-\frac{t}{\tau}(1 + i_p(1 + f_p))\Big]\Big\} - f_L\Big]. \quad (8)$$

For a pump time $t \to \infty$, the term in braces tends to unity, and the limiting unsaturated gain coefficient is

$$\alpha_0^{\infty} = \sigma_{em}(\lambda_L)N_{ions}\Big[(1 + f_L)\frac{i_p}{1 + i_p(1 + f_p)} - f_L\Big]. \quad (9)$$

Note that, in a quasi-three-level medium, because of the absorption at the laser wavelength ($f_L > 0$) the gain coefficient $\alpha_0(t)$ in (8) is positive only for $t > t_0$, where

$$t_0 = \frac{-\tau}{1 + i_p(1 + f_p)}\ln\frac{1 - f_L f_p - f_L i_p^{-1}}{1 + f_L} \quad (10)$$

is the minimum necessary pump duration.

In addition, we need to find the pump duration t_p sufficient for increasing the unsaturated gain coefficient to a given level, p ($0 < p < 1$), relative to the maximum value α_0^{∞} in (9), i.e. $\alpha_0(t_p) = p\alpha_0^{\infty}$:

$$t_p = \frac{-\tau}{1 + i_p(1 + f_p)}\ln\Big[(1 - p)\frac{1 - f_L f_p - f_L i_p^{-1}}{1 + f_L}\Big]. \quad (11)$$

Note that, for four-level media ($f_{p,L} = 0$), we have $t_0 = 0$ from (10) and, at a given i_p, t_p in (11) is smaller than that for quasi-three-level media because the magnitude of the logarithm in (11) increases with $f_{p,L}$. It is also worth noting that, in the case of quasi-three-level media, there is a minimum relative cw pump intensity (for $t \to \infty$),

$$i_p^{min} = \frac{f_L}{1 - f_L f_p}, \quad (12)$$

that ensures $\alpha_0^{\infty} = 0$ and increases with increasing f_L and f_p.

To theoretically describe gain grating inscription in a laser medium, we must take into account gain saturation due to laser radiation ($I_L \neq 0$). We are interested in the case of seed gain grating inscription by intrinsic weak seed laser radiation from a self-pumped phase-conjugate oscillator above the lasing self-starting threshold. Seed radiation can be considered continuous (I_L = const).

Equation (2) subject to $I_{p,L}$ = const and $N_2(0) = 0$ has the solution

$$N_2(t) = N_{ions}\frac{i_p + f_L i_L}{1 + i_p(1 + f_p) + i_L(1 + f_L)} \times\Big\{1 - \exp\Big[-\frac{t}{\tau}(1 + i_p(1 + f_p) + i_L(1 + f_L))\Big]\Big\}, \quad (13)$$

where $i_L = I_L/I_L^{sat}$ is the relative laser intensity and

$$I_L^{sat} = \frac{\hbar\omega_L}{\sigma_{em}(\lambda_L)\tau} \quad (14)$$

is the gain saturation intensity. From (1), we obtain an expression for the gain coefficient:

$$\alpha = \sigma_{em}(\lambda_L)N_{ions}\Big[(1 + f_L)\frac{i_p + f_L i_L}{1 + i_p(1 + f_p) + i_L(1 + f_L)} \times\Big\{1 - \exp\Big[-\frac{t}{\tau}(1 + i_p(1 + f_p) + i_L(1 + f_L))\Big]\Big\} - f_L\Big]. \quad (15)$$

Taylor expanding Eqn (15) and retaining only the first two terms in the expansion, we obtain

$$\alpha \approx \alpha_0 - \alpha_0\frac{1 + f_L}{1 + i_p(1 + f_p)}\frac{I_L}{I_L^{sat}}, \quad (16)$$

where α_0 is given by (8).

The seed laser radiation intensity in DFWM can be found as the result of the interference of four laser beams [34]:

$$I_L = I_0 + I_1 + I_2 + I_3 + 2\sqrt{I_1 I_3}\,\gamma\cos\Big(\frac{2\pi z}{\Lambda_{13}} + \varphi_{13}\Big) + 2\sqrt{I_2 I_3}\,\gamma\cos\Big(\frac{2\pi x}{\Lambda_{23}} + \varphi_{23}\Big), \quad (17)$$

where Λ_{13} and Λ_{23} are the interference grating periods for two waves with intensities I_1 and I_3 and two waves with intensities I_2 and I_3, respectively (the gratings are oriented along the orthogonal axes z and x); φ_{13} and φ_{23} are the initial phases of the corresponding interference gratings; and γ is the degree of coherence of the laser beams, which will be taken to be unity because, in the case of coherent seed radiation, the phase matching condition is fulfilled for DFWM on gain gratings [8, 36]. In (17), gratings written by counterpropagating waves are left out of consideration because they make no contribution to the diffraction efficiency of DFWM in the case of a local response of the medium [37]. Also, Eqn (17) leaves out of account weak gratings written with the participation of the weakest wave, of intensity I_0, which is a phase-conjugate wave, because as a rule it is not deliberately directed to the input of the phase-conjugate medium but results from DFWM [38] and its intensity at the input of the medium corresponds to the spontaneous noise level. Substitution of (17) into (16) yields an expression for the gain coefficient with harmonic modulation:

$$\alpha \approx \alpha_0 - \beta_{13}\cos\left(\frac{2\pi z}{\Lambda_{13}} + \varphi_{13}\right) - \beta_{23}\cos\left(\frac{2\pi x}{\Lambda_{23}} + \varphi_{23}\right), \quad (18)$$

where the gain modulation amplitudes are given by

$$\beta_{ij} = \alpha_0 \frac{1 + f_L}{1 + i_p(1 + f_p)} \frac{2\sqrt{I_i I_j}}{I_L^{sat}}. \quad (19)$$

The diffraction efficiency of a gain grating is given by [9, 25]

$$\eta \approx G_0(bL)^2, \quad (20)$$

where $G_0 = \exp(\alpha_0 L)$ is the small-signal gain per pass through the laser medium; L is the length of the laser medium; and b is the DFWM coupling coefficient, which can be found as $b = \beta_{ij}/4$ [36, 39]. The diffraction efficiency of the ijth seed gain grating is then given by

$$\eta_{ij} \approx \frac{1}{4} G_0(\alpha_0 L)^2 \left[\frac{1 + f_L}{1 + i_p(1 + f_p)}\right]^2 \frac{I_i I_j}{(I_L^{sat})^2}, \quad (21)$$

where the writing wave intensities I_i and I_j depend on the seed radiation intensity I_0 and the self-pumped phase-conjugate loop resonator.

3. Self-starting threshold for four-wave PCW generation in a loop laser resonator with a self-pumped gain-grating PCM

Figure 1 shows the optical schemes of self-pumped phase-conjugate loop resonators with one (scheme 1) and two (schemes 2 and 3) GEs, where the extra GE is used as a single-pass (scheme 2) or double-pass (scheme 3) intracavity amplifier.

In the loop configuration of the self-pumped DFWM resonator, all waves originate from one seed wave of intensity I_0. For scheme 1, we indicate the designations of the intensities of the waves. In what follows, the waves in all the schemes presented in Fig. 1 are numbered in this manner.

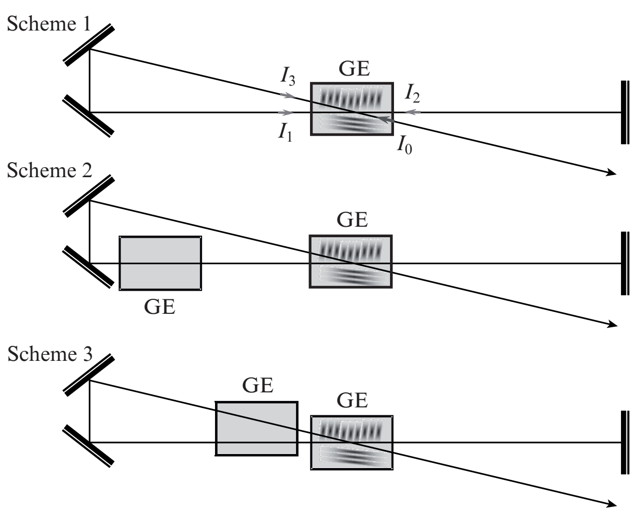

Figure 1. Optical schemes of loop laser resonators with one (scheme 1) and two (schemes 2 and 3) GEs, where the extra GE is used as a single-pass (scheme 2) or double-pass (scheme 3) intracavity amplifier.

The intensities of all the waves in scheme 1 are related to I_0 by $I_1 = I_0 G_0$, $I_2 = I_0 G_0^2$ and $I_3 = I_0 G_0^3$. Note that the waves of intensities I_1 and I_3 in Fig. 1 inscribe a transmission gain grating (the radiation diffracted by it passes through the DFWM medium), and the waves of intensities I_2 and I_3 inscribe a reflection gain grating (the radiation diffracted by it is reflected from the DFWM medium).

Substituting the intensities of the waves into (21), we obtain expressions for the diffraction efficiency of the transmission and reflection gain gratings:

$$\eta_{13} \approx \frac{1}{4} G_0^5(\alpha_0 L)^2 \left[\frac{1 + f_L}{1 + i_p(1 + f_p)}\right]^2 \left(\frac{I_0}{I_L^{sat}}\right)^2, \quad (22)$$

$$\eta_{23} \approx \frac{1}{4} G_0^6(\alpha_0 L)^2 \left[\frac{1 + f_L}{1 + i_p(1 + f_p)}\right]^2 \left(\frac{I_0}{I_L^{sat}}\right)^2. \quad (23)$$

We can now write a lasing threshold condition for transmission and reflection gain gratings in scheme 1: the intracavity radiation intensity I_1 returns to its original level after a resonator round trip, i.e. $I_1 G_0 \eta_{13} + I_1 \eta_{23} = I_1$, or

$$G_0 \eta_{13} + \eta_{23} = 1. \quad (24)$$

Substituting (22) and (23) into (24), we obtain identical terms on the left-hand side, which means that transmission and reflection gratings make identical contributions to initial lasing (during subsequent development of lasing, the contribution of a small-period reflection grating may decrease because it is effaced by the intracavity flow).

Similar considerations apply to the other schemes in Fig. 1. As a result, we obtain a general lasing threshold condition for all the schemes in Fig. 1:

$$\frac{1}{\sqrt{2}} \frac{1 + f_L}{1 + i_p(1 + f_p)} (\alpha_0 L)_{th} \times \exp[(\alpha_0 L)_{th}(2n + 1)] = \frac{1}{\kappa}, \quad (25)$$

where $n = 1-3$ is the number of the scheme and $\kappa = I_0/I_L^{sat}$ is the relative seed radiation intensity.

Condition (25) is an implicit expression for the threshold gain increment $(\alpha_0 L)_{th}$ at a given relative seed radiation intensity κ. It is worth pointing out that all the factors of the exponential on the left-hand side of (25) are in the order of unity, so they can be neglected in estimating $(\alpha_0 L)_{th}$, and we obtain

$$(\alpha_0 L)_{th} \approx \frac{1}{2n + 1} \ln\frac{1}{\kappa}. \quad (26)$$

According to theory [34], the amplified spontaneous emission intensity at the amplifier output for $G_0 \gg 1$ can be evaluated as

$$I_{ASE} \approx \frac{\Omega}{4\pi} \frac{G_0}{\sqrt{\ln G_0}} I_L^{sat}, \quad (27)$$

where Ω is the solid angle subtended by one GE end face seen from the centre of the other end face. The seed radiation intensity at the GE input can then be estimated as

$$I_0 \approx k\frac{I_{\rm ASE}}{G_0} = k\frac{\Omega/4\pi}{\sqrt{\ln G_0}} I_{\rm L}^{\rm sat} = \kappa I_{\rm L}^{\rm sat},$$

where k is the fraction of amplified spontaneous emission that meets the conditions for the angular and spectral selectivities of gain-grating DFWM [8, 36] and the relative seed radiation intensity is $\kappa = k(\Omega/4\pi)/\sqrt{\ln G_0}$. Note that I_0 is a rather weak function of the gain coefficient of the active medium, G_0. Because of this, when substituting the last expression into (25) the factor $\sqrt{\ln G_0}$ can be neglected compared to the factor $\exp[(\alpha_0 L)(2n+1)]$ in (25); i.e. in estimating the PCW generation threshold, κ can be treated as independent of the gain coefficient. Moreover, κ can be treated as independent of the luminescence linewidth of the laser medium because the spectral selectivity of four-wave PCMs is lower than the luminescence linewidth even for Nd:YAG, the narrowest band solid-state laser medium [8].

The value of κ can be estimated from the measured self-starting threshold for four-wave PCW generation. Antipov et al. [11] experimentally studied threshold conditions for PCW generation in a Nd:YAG laser diode-side-pumped in a bounce geometry for scheme 1 (Fig. 1). For a 1-cm-long pump region of the GE and 300-μs pump duration, the threshold power was 75 W, which corresponded to a pump intensity $I_{\rm p}$ = 7.5 kW cm^{-2} (in a spot 100 μm×1 cm in dimensions). According to (8), the gain coefficient at the GE input was $\alpha_0(x=0) \approx 10.5$ cm^{-1}. The pump absorption coefficient was $\mu \approx 3$ cm^{-1} [11], and the GE thickness in the pump propagation direction, d, and the corresponding transverse size of the generated laser beam were 0.4 cm. When the pump beam experienced total internal reflection from the GE face, the average gain coefficient was

$$\alpha_0 = \frac{1}{d}\int_0^d \alpha_0(x){\rm d}x \approx \alpha_0(x=0)\frac{1-\exp(-\mu d)}{\mu d},$$

where we take $\alpha_0(x) \approx \alpha_0(x=0)\exp(-\mu z)$. The threshold gain increment is then $(\alpha_0 L)_{\rm th} \approx 6.1$. Substituting this value into (25) or (26), we find the relative seed radiation intensity: $\kappa \approx 10^{-8}$.

Thus, at $\kappa \approx 10^{-8}$ we obtain from (26) $(\alpha_0 L)_{\rm th} \approx 6.1$ for scheme 1, ~3.7 for scheme 2 and ~2.6 for scheme 3. These estimates agree with the measured PCW generation thresholds of loop Nd:YAG lasers in schemes 2 [7] and 3 [18, 20]. It is worth pointing out that the threshold gain increment $(\alpha_0 L)_{\rm th}$ is independent of laser medium parameters and of how the required gain increment is ensured, but depends on the phase-conjugate laser configuration (Fig. 1). The use of an extra intracavity laser amplifier reduces the threshold gain increment.

The threshold gain increments obtained are large enough for the operation of the phase-conjugate laser to be influenced by amplified spontaneous emission, whose intensity is given by (27). Amplified spontaneous emission reduces the gain coefficient of the laser medium, so measures should be taken to reduce it. The main approach for reducing amplified spontaneous emission is to decrease the solid angle Ω [see Eqn (27)]. To this end, one usually increases the length of the loop resonator and the distance between GEs. For example, the resonator length in the Nd:YAG laser described in Refs [7, 8] exceeded 8 m. Use is also made of selective intracavity elements, such as an aperture, Faraday isolator [3, 13, 15] or saturable absorber [7]. Diode pumping in a bounce geometry ensures angular selection of amplified spontaneous emission by narrowing the pump region. Smith and Damzen [40] studied the spatial selection of amplified spontaneous emission in an ultrahigh-gain ($G_0 \sim 10^4$) Nd:YVO_4 GE in a bounce geometry, with a pump region 100×330 μm in cross section and 1.5 cm in length. This allowed Smith and Damzen in a subsequent study [13] to prevent the adverse effect of amplified spontaneous emission and achieve self-Q-switched gain-grating PCW generation in a compact loop Nd:YVO_4 laser resonator configuration using a Faraday isolator and an output coupler with a reflectivity under 1%.

Consider specific features of the development of lasing based on gain gratings when there is no adverse effect of amplified spontaneous emission. It should be noted that the self-starting of a laser with a self-pumped PCM due to a positive feedback through DFWM in the laser medium occurs in the self-Q-switching regime in the phase-conjugate resonator [3, 9, 12, 13]. In the initial stage (before lasing), the phase-conjugate resonator has a low Q because no gain-grating PCM is yet formed on account of the small gain coefficient of the laser medium. During this stage, pumping increases the gain coefficient of the laser medium, $\alpha_0(t)$, according to Eqn (8), and at time

$$t_{\rm th} = \frac{-\tau}{1+i_{\rm p}(1+f_{\rm p})}$$

$$\times \ln\left\{\left[1 - \frac{(\alpha_0 L)_{\rm th}}{\alpha_0^{\infty} L}\right]\frac{1 - f_{\rm L} f_{\rm p} - f_{\rm L} i_{\rm p}^{-1}}{1+f_{\rm L}}\right\}, \quad (28)$$

corresponding to the threshold gain level, $p_{\rm th} = (\alpha_0 L)_{\rm th}/(\alpha_0^{\infty} L)$, the lasing threshold is surpassed. Note that, for $\alpha_0^{\infty} L < (\alpha_0 L)_{\rm th}$, lasing based on gain gratings is impossible, so the length of the active medium, L, and/or the limiting gain coefficient, α_0^{∞}, should be increased. According to (9), α_0^{∞} can be increased by using laser media with a large gain cross section, $\sigma_{\rm em}(\lambda_{\rm L})$, and small $f_{\rm p}$ and $f_{\rm L}$ of the quasi-three-level medium. The use of four-level media ($f_{\rm p,L} = 0$) is preferable. It is also advantageous to utilise high relative pump intensity $i_{\rm p}$, which can be ensured not only by high pump intensity $I_{\rm p}$ but also by low pump absorption saturation intensity $I_{\rm p}^{\rm sat}$ (6), which decreases with increasing absorption cross section $\sigma_{\rm abs}(\lambda_{\rm p})$ and upper laser level lifetime τ.

Figure 2 shows the dependences, obtained using Eqn (28), of the pump duration $t_{\rm th}$ needed for four-wave PCW generation self-starting on threshold gain level $p_{\rm th}$ for four-level (Nd:YAG) and quasi-three-level (Yb:YAG) laser media at different pump intensities, $I_{\rm p}$. The graphs were obtained using data for Nd:YAG (τ = 230 μs, $f_{\rm p,L} = 0$ [34]) and Yb:YAG (τ = 950 μs, $f_{\rm L} = 0.057$, $f_{\rm p} = 0.195$ [41]), under the assumption that $p_{\rm th}$ was a function of L.

It is seen in Fig. 2 that, as the threshold gain level $p_{\rm th}$ increases to unity, the pump duration $t_{\rm th}$ increases and tends to infinity. In the Nd:YAG four-level system, $t_{\rm th}$ increases starting from zero, with a relatively low slope, whereas in the Yb:YAG quasi-three-level system it increases starting at $t_0 > 0$ (10) and with a steeper slope, which is due to the lower population inversion because $f_{\rm p,L} > 0$. Increasing the pump intensity $I_{\rm p}$ reduces the pump duration $t_{\rm th}$ needed for the onset of PCW generation [Fig. 2, curves (*1′*), (*2′*)].

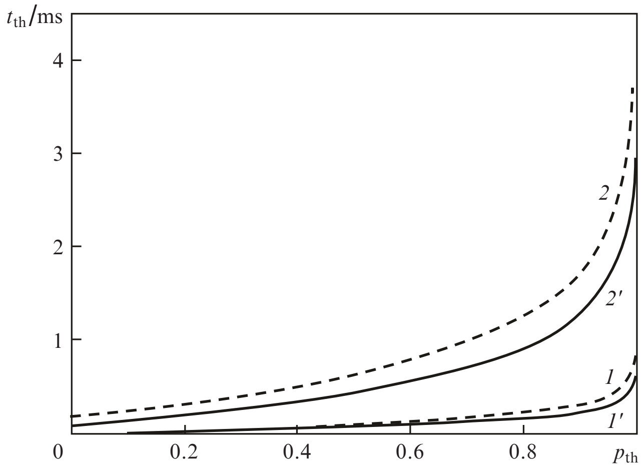

Figure 2. Pump duration t_{th} needed for the onset of PCW generation as a function of threshold gain level $p_{th} = (\alpha_0 L)_{th}/(\alpha_0^{\infty} L)$ for (*1, 1′*) a Nd:YAG four-level laser medium and (*2, 2′*) Yb:YAG quasi-three-level laser medium at pump intensities I_p = (*1, 2*) 10 and (*1′, 2′*) 20 kW cm^{-2}.

Thus, the pump duration t_{th} can be reduced by increasing $\alpha_0^{\infty} L$ relative to $(\alpha_0 L)_{th}$, which can be achieved, for a given laser medium, by increasing the pump intensity I_p or the length L of the medium.

Low values of the required pump duration t_{th} can be ensured for laser media with a large gain cross section $\sigma_{em}(\lambda_L)$ and small $f_{p,L}$ parameters, i.e. for near-four-level media and for media with a large pump absorption cross section $\sigma_{abs}(\lambda_p)$ and long upper laser level lifetime τ, which is due to the fact that the relative pump intensity $i_p = \sigma_{abs}(\lambda_p)\tau I_p/(\hbar\omega_p)$ increases with increasing $\sigma_{abs}(\lambda_p)$ and τ.

4. Optimal concentration of active ions for four-wave PCW generation self-starting in a quasi-three-level laser medium

We should take into account the spatial nonuniformity of the gain coefficient, due to the absorption-limited penetration of pump radiation into the laser medium. We will consider single-pass narrow-band pumping, exemplified by side pumping with 2D laser diode arrays, which was demonstrated experimentally for a high-power Nd:YAG laser with a self-pumped phase-conjugate loop resonator based on gain gratings [14].

The penetration of pump radiation into a laser medium follows the Bouguer–Lambert law:

$$i_p(x) = i_{p0}\exp(-\mu x), \qquad (29)$$

where i_{p0} is the relative pump intensity incident on the input face of the laser medium; $\mu = \sigma_{abs}(\lambda_p)N_{ions}$ is the pump absorption coefficient (with the absorption saturation neglected); and the x coordinate is measured along the pump direction from the input face. Because of the pump absorption according to (29), the population is inverted only in a local region of the laser medium that is the nearest to the pump source. Since the lower laser level is partially populated, the population inversion in quasi-three-level media is considerably more localised than that in four-level media. As a result, the pump energy deposited in quasi-three-level media depends on the concentration of active laser ions, N_{ions}, because the lower laser level population increases with N_{ions}.

Figure 3 shows spatial profiles of the unsaturated gain coefficient, $\alpha_0(x)$, normalised to the maximum value $\alpha_0(0)$ and calculated using Eqn (8) with allowance for (29) for four-level (Nd:YAG) and quasi-three-level (Yb:YAG) media with a pump absorption coefficient μ = 9.25 cm^{-1}, corresponding to 1 at % active ions in Nd:YAG ($\sigma_{abs}(\lambda_p)$ = 6.7×10^{-20} cm^2, $\sigma_{em}(\lambda_L)$ = 28×10^{-20} cm^2, τ = 230 μs, $f_{p,L}$ = 0 [34]) and 8.7 at% active ions in Yb:YAG ($\sigma_{abs}(\lambda_p)$ = 0.77×10^{-20} cm^2, $\sigma_{em}(\lambda_L)$ = 2.1×10^{-20} cm^2, τ = 950 μs, f_L = 0.057, f_p = 0.195 [41]) at a pump intensity of 10 kW cm^{-2} and different pump durations.

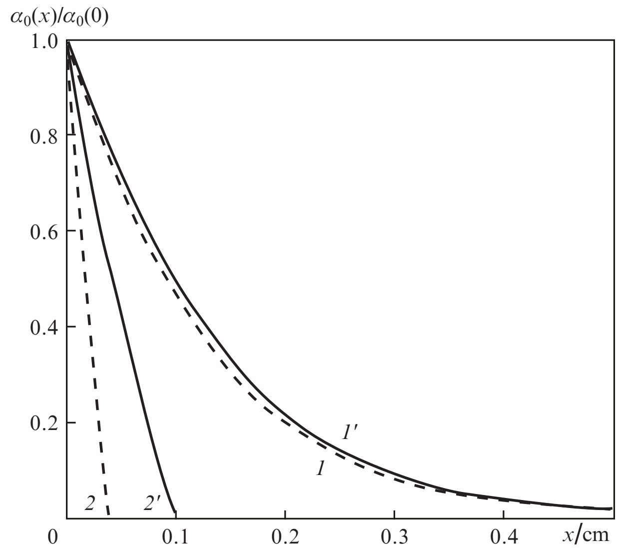

Figure 3. Unsaturated gain coefficient profiles, $\alpha_0(x)$, normalised to the maximum value $\alpha_0(0)$ and calculated using Eqn (8) with allowance for (29) for (*1, 1′*) Nd:YAG and (*2, 2′*) Yb:YAG media with a pump absorption coefficient μ = 9.25 cm^{-1}, corresponding to 1 at % active ions in Nd:YAG and 8.7 at % active ions in Yb:YAG at a pump intensity of 10 kW cm^{-2} and pump durations of (*1, 2*) 0.25 and (*1′, 2′*) 0.5 ms.

The gain coefficient is seen to decrease with increasing penetration depth in the active medium. In the case of the four-level medium Nd:YAG, the decrease is relatively slow, is a weak function of pump duration, and approximately follows the Bouguer–Lambert law (29): the gain coefficient approaches zero when the x coordinate tends to infinity.

A different situation occurs for the quasi-three-level medium Yb:YAG. The gain coefficient drops sharply, almost linearly, to zero as the x coordinate increases to the maximum value x_p, and the pump duration has a significant effect on the gain coefficient profile. With increasing pump duration, the pump radiation penetrates deeper into the medium, meaning that the pump depth x_p increases. For example, $x_p \approx 0.04$ cm at a pump duration of 0.25 ms and reaches $x_p \approx 0.1$ cm when the pump duration increases to 0.5 ms. The pump depth x_p can be determined for any pump duration t_p by substituting (29) into (8) and equating it to zero. For $t_p \to \infty$ (continuous pumping), the limiting relation $i_p^{min} = i_{p0}\exp(-\mu x_p^{max})$ is valid, where i_p^{min} is given by (12). The maximum pump depth in a quasi-three-level medium is then

$$x_{\rm p}^{\max} = \frac{\ln[I_{\rm p}(1 - f_{\rm L} f_{\rm p})/(I_{\rm p}^{\rm sat} f_{\rm L})]}{\sigma_{\rm abs}(\lambda_{\rm p}) N_{\rm ions}}. \quad (30)$$

A small pump depth $x_{\rm p}$ leads to a low pump energy density stored over time t:

$$U_{\rm stor}(t) = U_{\rm L}^{\rm sat} \int_0^h \alpha_0(t, x) {\rm d}x. \quad (31)$$

Here the upper integration limit is $h = x_{\rm p}$ for $x_{\rm p} < d$ and $h = d$ for $x_{\rm p} \geqslant d$, where d is the thickness of the laser medium in the pump propagation direction.

Return now to the self-starting of four-wave PCW generation. As mentioned above, a phase-conjugate laser operates in the self-Q-switching regime based on gain gratings and accumulates energy during the pump duration $t_{\rm th}$ (28). After time $t_{\rm th}$, the stored energy $U_{\rm stor}(t_{\rm th})$ is emitted in the form of a giant laser pulse, and one can then determine the energy efficiency of lasing as

$$\eta_{\rm L} = \frac{U_{\rm stor}(t_{\rm th})}{I_{\rm p} t_{\rm th}}. \quad (32)$$

Figure 4 shows the lasing efficiency $\eta_{\rm L}$ (32) as a function of relative pump intensity $i_{\rm p0}$ at different $N_{\rm ions}^{\rm Yb}$ values (Fig. 4a) and as a function of ytterbium concentration $N_{\rm ions}^{\rm Yb}$ at different $i_{\rm p0}$ values (Fig. 4b) for scheme 1 (Fig. 1) in an Yb:YAG quasi-three-level laser medium of thickness $d = 0.5$ cm.

It is seen in Fig. 4a that, at a high concentration of active ions, $N_{\rm ions}^{\rm Yb} = 8.7$ at % [curve (*1*)], the lasing threshold is low, but the limiting lasing efficiency is not high: below 17%. The reason for the low lasing efficiency is the small pump depth ($x_{\rm p} \approx 0.088$ cm), which means that pump energy is deposited in a very small volume. As the concentration of active ions decreases to 3.2 at % [curve (*2*)], the lasing threshold increases, but the limiting lasing efficiency also increases: to 38% and above. The value $N_{\rm ions}^{\rm Yb} = 3.2$ at % ensures an increase in pump depth to the thickness of the medium: $x_{\rm p} = d = 0.5$ cm. Further reducing the concentration of active ions to 1.5 at % [curve (*3*)] increases the pump depth ($x_{\rm p} > d$). However, too large a fraction of the pump radiation is then lost [passes through the medium according to the Bouguer–Lambert law (29)], leading to a decrease in limiting lasing efficiency, down to 30% or below.

Figure 4b demonstrates that there is an optimal concentration of active ions, $N_{\rm ions\,opt}^{\rm Yb} = 3.2$ at %, which is independent of relative pump intensity $i_{\rm p0}$ and corresponds to the maximum lasing efficiency ($\eta_{\max} = 34\%$ at $i_{\rm p0} = 1.0$ and $\eta_{\max} = 38\%$ at $i_{\rm p0} = 1.5$) in an Yb:YAG quasi-three-level medium of thickness $d = 0.5$ cm.

Thus, a quasi-three-level medium has an optimal concentration of active ions, $(N_{\rm ions}^{\rm Yb})_{\rm opt}$, for a particular phase-conjugate laser oscillator configuration. At this concentration, the pump depth $x_{\rm p}$ increases to the thickness of the medium, d, at time $t_{\rm th}$. The optimal concentration of active ions in an Yb:YAG quasi-three-level medium of thickness $d = 0.5$ cm for scheme 1 (Fig. 1) was found to be 3.2 at %. Using analogous calculations, we evaluated the optimal concentration of active ions in Yb:YAG for schemes 2 and 3: $N_{\rm ions\,opt}^{\rm Yb} = 2.7$ at % for scheme 2 and 2.3 at % for scheme 3.

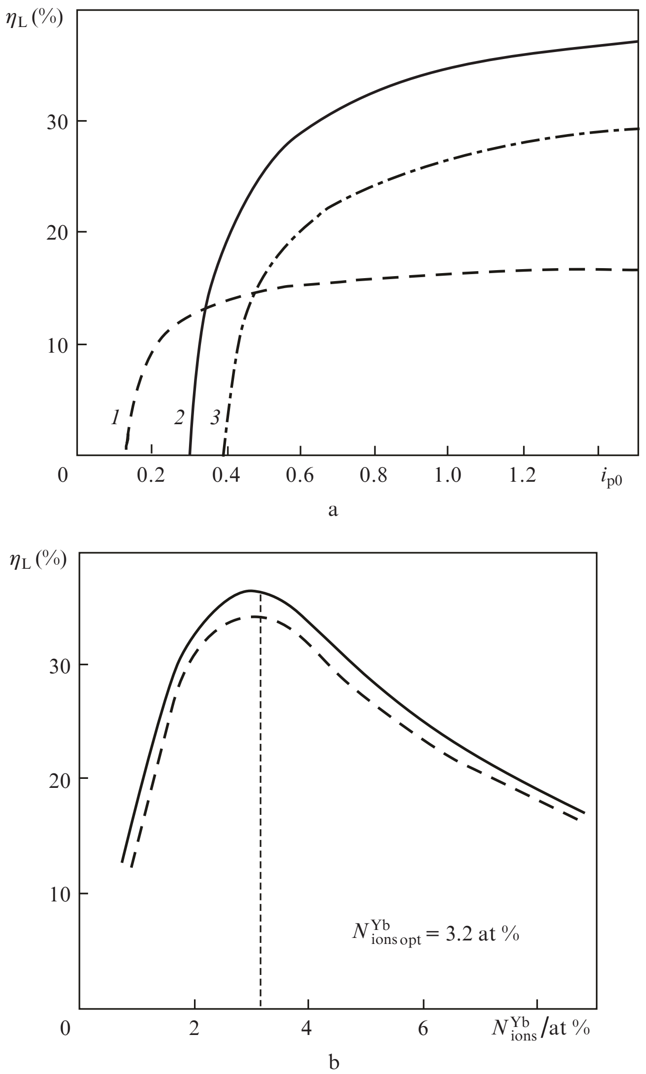

Figure 4. Four-wave lasing efficiency $\eta_{\rm L}$ (a) as a function of relative pump intensity $i_{\rm p0}$ at $N_{\rm ions}^{\rm Yb} =$ (*1*) 8.7 at %, (*2*) 3.2 at % and (*3*) 1.5 at % and (b) as a function of ytterbium concentration $N_{\rm ions}^{\rm Yb}$ at $i_{\rm p0} = 1.0$ (dashed line) and 1.5 (solid line) for scheme 1 (Fig. 1) in an Yb:YAG crystal of thickness $d = 0.5$ cm in the pump direction.

5. Analysis of the use of various laser media for the self-starting of four-wave PCW generation in a loop laser resonator

Let a laser medium be spatially infinite in the pump direction and the pump duration be infinite in time. The energy deposited in the medium is then also given by (31), but in the case of a four-level medium the upper integration limit h can be replaced by infinity. For a quasi-three-level medium, we can take $h = x_{\rm p}^{\max}$, where $x_{\rm p}^{\max}$ is given by (30). In this way, we find the limiting gain coefficient of the laser medium, α_0^{∞} (9), at a relative pump intensity $i_{\rm p}$ dependent on the x coordinate according to Eqn (29). Substituting (9) and (29) into (31), we obtain for a four-level medium ($f_{\rm p,L} = 0$, $h \to \infty$)

$$U_{\rm stor} = \frac{\omega_{\rm L}}{\omega_{\rm p}} \tau I_{\rm p}^{\rm sat} \ln\left(\frac{I_{\rm p}}{I_{\rm p}^{\rm sat}} + 1\right) \approx \frac{\omega_{\rm L}}{\omega_{\rm p}} \tau I_{\rm p}. \quad (33)$$

Substitution of (9) and (29) into (31) gives for a quasi-three-level medium ($f_{\rm p,L} > 0$, $h = x_{\rm p}^{\max}$)

$$U_{\rm stor} = \frac{\omega_{\rm L}}{\omega_{\rm p}}\tau I_{\rm p}^{\rm sat}\frac{1+f_{\rm L}}{1+f_{\rm p}}$$

$$\times\left\{\ln\left(\frac{I_{\rm p}}{I_{\rm p}^{\rm sat}}\frac{1-f_{\rm L}f_{\rm p}}{f_{\rm L}}\right)+\ln\left[\frac{I_{\rm p}(1+f_{\rm p})}{I_{\rm p}^{\rm sat}}+1\right]\right.$$

$$\left.-\ln\left[\frac{I_{\rm p}}{I_{\rm p}^{\rm sat}}\frac{1-f_{\rm L}f_{\rm p}}{f_{\rm L}}+\frac{I_{\rm p}(1+f_{\rm p})}{I_{\rm p}^{\rm sat}}\right]\right\}-\hbar\omega_{\rm L}N_{\rm ions}f_{\rm L}$$

$$\approx\frac{\omega_{\rm L}}{\omega_{\rm p}}\tau I_{\rm p}(1+f_{\rm L})-\frac{\hbar\omega_{\rm L}}{\sigma_{\rm abs}(\lambda_{\rm p})}\frac{1+f_{\rm L}}{1+f_{\rm p}}$$

$$\times\left(\ln\frac{1+f_{\rm L}}{f_{\rm L}}-\ln\frac{1-f_{\rm L}f_{\rm p}}{f_{\rm L}}\right)-\hbar\omega_{\rm L}N_{\rm ions}f_{\rm L}. \quad (34)$$

The approximate equality in (33) and (34) is valid when $I_{\rm p} < I_{\rm p}^{\rm sat}$, which is often the case in practice.

It follows from (33) and (34) that the energy deposited in the laser medium increases with increasing pump intensity $I_{\rm p}$ and upper laser level lifetime τ and also when the pump frequency $\omega_{\rm p}$ approaches the laser frequency $\omega_{\rm L}$. Therefore, laser media with the longest upper laser level lifetime are advantageous for four-wave PCW generation self-starting in a loop laser resonator.

The energy deposited in a four-level medium (33) is independent of the concentration of active ions, $N_{\rm ions}$. For $I_{\rm p} < I_{\rm p}^{\rm sat}$, it is independent as well of the absorption saturation intensity, $I_{\rm p}^{\rm sat}$. At the same time, the energy deposited in a quasi-three-level medium (34) decreases with increasing active-ion concentration $N_{\rm ions}$ and decreasing effective absorption cross section $\sigma_{\rm abs}(\lambda_{\rm p})$, which is due to partial filling of the lower laser level.

It also follows from (33) and (34) that, for $I_{\rm p} > I_{\rm p}^{\rm sat}$, the deposited energy increases with pump absorption saturation intensity, $I_{\rm p}^{\rm sat} = \hbar\omega_{\rm p}[\sigma_{\rm abs}(\lambda_{\rm p})\tau]^{-1}$, which means that a decrease in deposited energy upon partial filling of the lower laser level because of the small absorption cross section $\sigma_{\rm abs}(\lambda_{\rm p})$ of the quasi-three-level medium can be compensated for by raising the pump intensity.

Thus, in addition to neodymium-doped crystals, which are already widely used for four-wave PCW generation, attractive four-level laser media are chromium-doped materials, which also have a long upper laser level lifetime, e.g. alexandrite (τ = 260 μs [34]) and Cr : LiCAF (τ = 170 μs [42]). The worst medium in this respect is Ti : sapphire (τ = 3.2 μs [34]): the use of Ti : sapphire crystals for four-wave PCW generation self-starting requires a substantial increase in pump intensity $I_{\rm p}$, which was achieved by Minassian et al. [5] by pumping with a frequency-doubled high-power nanosecond Nd : YAG laser.

Promising quasi-three-level media are those with a long upper laser level lifetime, such as Yb : YAG (τ = 950 μs, $\lambda_{\rm L}$ = 1.03 μm [41]) and Tm, Ho : YAG (τ = 8500 μs, $\lambda_{\rm L}$ = 2.08 μm [34]), but, because of the partial filling of the lower laser level, one has to increase the pump intensity and reduce the concentration of active ions.

What is of interest for us is at what values of the $I_{\rm p}$ and $N_{\rm ions}$ variable parameters a quasi-three-level medium has the same stored energy as four-level media, i.e. compares well to it in energy efficiency in Q switching of a phase-conjugate laser. Equating the energy (33) deposited in a four-level medium, e.g. in Nd : YAG, to the energy (34) deposited in a quasi-three-level medium, e.g. in Yb : YAG, we obtain

$$N_{\rm ions}^{\rm Yb}(I_{\rm p}) = \frac{1+f_{\rm L}}{\sigma_{\rm abs}^{\rm Yb}(\lambda_{\rm p})f_{\rm L}(1+f_{\rm p})}$$

$$\times\left\{\ln\left[I_{\rm p}\frac{\sigma_{\rm abs}^{\rm Yb}(\lambda_{\rm p})\tau_{\rm Yb}(1-f_{\rm L}f_{\rm p})}{\hbar\omega_{\rm p}^{\rm Yb}f_{\rm L}}\right]\right.$$

$$+\ln\left[I_{\rm p}\frac{\sigma_{\rm abs}^{\rm Yb}(\lambda_{\rm p})\tau_{\rm Yb}(1+f_{\rm p})}{\hbar\omega_{\rm p}^{\rm Yb}}+1\right]$$

$$\left.-\ln\left[I_{\rm p}\frac{\sigma_{\rm abs}^{\rm Yb}(\lambda_{\rm p})\tau_{\rm Yb}(1-f_{\rm L}f_{\rm p})}{\hbar\omega_{\rm p}^{\rm Yb}f_{\rm L}}+I_{\rm p}\frac{\sigma_{\rm abs}^{\rm Yb}(\lambda_{\rm p})\tau_{\rm Yb}(1+f_{\rm p})}{\hbar\omega_{\rm p}^{\rm Yb}}\right]\right\}$$

$$-\frac{\omega_{\rm L}^{\rm Nd}/\omega_{\rm L}^{\rm Yb}}{\sigma_{\rm abs}^{\rm Nd}(\lambda_{\rm p})f_{\rm L}}\ln\left(I_{\rm p}\frac{\sigma_{\rm abs}^{\rm Nd}(\lambda_{\rm p})\tau_{\rm Nd}}{\hbar\omega_{\rm p}^{\rm Nd}}+1\right), \quad (35)$$

where the superscript Nd or Yb refers to a four-level (Nd : YAG) or quasi-three-level (Yb : YAG) medium.

Figure 5 shows the ytterbium ion concentration $N_{\rm ions}^{\rm Yb}$ in Yb : YAG evaluated as a function of pump intensity $I_{\rm p}$ using Eqn (35) and corresponding to the condition of equal energies deposited in Yb : YAG ($\lambda_{\rm L}$ = 1.03 μm) and Nd : YAG (1.064 μm). It is seen that increasing the pump intensity leads to an increase in the active ion concentration in Yb : YAG at which equal energies are deposited in Yb : YAG and Nd : YAG. In the grey region (under the curve), the energy deposited in the Yb : YAG quasi-three-level medium exceeds that in the Nd : YAG four-level medium. For this reason, at a given pump intensity, it is energetically favourable to use an active ion concentration below the $N_{\rm ions}^{\rm Yb}$ value on the curve, i.e., at a given pump intensity, the working range of $N_{\rm ions}^{\rm Yb}$ concentrations extends from zero to the value lying on the curve.

There is a minimum pump intensity, $I_{\rm p}^{\rm min}$ = 2.1 kW cm^{-2} (vertical dashed line), at which the use of the Yb : YAG quasi-three-level medium can be energetically favourable. The active ion concentration $N_{\rm ions}^{\rm Yb}$ should then be almost zero.

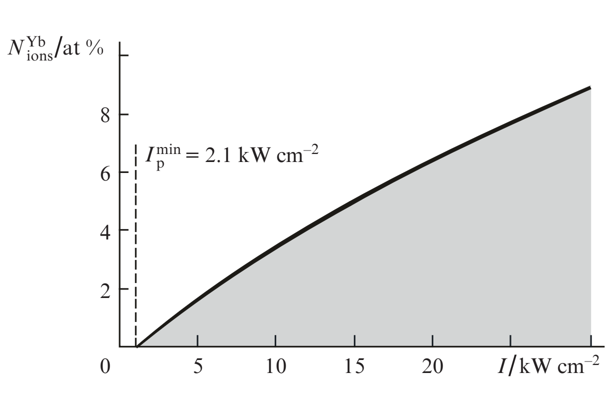

Figure 5. Ytterbium ion concentration $N_{\rm ions}^{\rm Yb}$ in Yb : YAG evaluated as a function of pump intensity $I_{\rm p}$ using Eqn (35) and corresponding to equal energies deposited in Yb : YAG ($\lambda_{\rm L}$ = 1.03 μm) and Nd : YAG (1.064 μm) crystals at the same pump intensity. In the grey region, the energy deposited in the Yb : YAG quasi-three-level medium exceeds that in the Nd : YAG four-level medium.

Note that the pump intensity of 2.1 kW cm^{-2} can be ensured by 2D SLM-3-2 laser diode arrays used in an earlier study [14], even without focusing the pump radiation. Pump intensities above $N_{\mathrm{ions}}^{\mathrm{Yb}}$ (e.g. with focused pump radiation) extend the working range of active ion concentrations to higher $N_{\mathrm{ions}}^{\mathrm{Yb}}$ values.

6. Conclusions

A generalised theory was used to carry out a comparative analysis of the use of various four-level and quasi-three-level doped laser media for the self-starting of DFWM PCW laser generation based on gain gratings in a loop resonator.

Expressions were derived for the threshold gain increment of a laser medium in various loop resonator configurations at the pulse duration necessary for four-wave PCW generation self-starting. The results demonstrate that, in contrast to four-level media, quasi-three-level media are characterised by a longer pump duration, which is due to the slow increase in gain coefficient under pumping because of the partial filling of the lower laser level. Quasi-three-level media are shown to have an optimal concentration of active ions corresponding to the maximum four-wave PCW generation efficiency. At this concentration, the pump depth of the laser medium increases at the instant of self-starting to the thickness of the medium in the pump direction. Quasi-three-level media can compete with four-level media in terms of deposited energy at long upper laser level lifetimes and increased pump intensities. We have identified the most attractive solid-state laser media for four-wave PCW generation that have the highest deposited energy at a given pump intensity. In addition to neodymium-doped crystals, which are already widely used for four-wave PCW generation, attractive four-level media are chromium-doped materials, e.g. alexandrite and Cr : LiCAF. Promising quasi-three-level media are those with the longest upper laser level lifetime, such as Yb : YAG and Tm, Ho : YAG, at high pump intensities.

References

1. Bel'dyugin I.M., Galushkin M.G., Zemskov E.M. *Kvantovaya Elektron.*, **11**, 887 (1984) [*Sov. J. Quantum Electron.*, **14**, 602 (1984)].
2. Bel'dyugin I.M., Berenberg V.A., Vasil'ev A.E., et al. *Kvantovaya Elektron.*, **16**, 1142 (1989) [*Sov. J. Quantum Electron.*, **19**, 740 (1989)].
3. Damzen M.J., Green R.P.M., Syed K.S. *Opt. Lett.*, **20**, 1704 (1995).
4. Antipov O.L., Belyaev S.I., Kuzhelev A.S. *Opt. Commun.*, **117**, 290 (1995).
5. Minassian A., Grofts G.J., Damzen M.J. *Opt. Lett.*, **22**, 697 (1997).
6. Sillard P., Brignon A., Huignard J.-P., Pocholle J.-P. *Opt. Lett.*, **23**, 1093 (1998).
7. Fedin A.V., Gavrilov A.V., Basiev T.T., Antipov O.L., Kuzhelev A.S., Smetanin S.N. *Laser Phys.*, **9**, 433 (1999).
8. Antipov O.L., Chausov D.V., Kuzhelev A.S., Vorob'ev V.A., Zinoviev A.P. *IEEE J. Quantum Electron.*, **37**, 716 (2001).
9. Fedin A.V., Gavrilov A.V., Kyalbieva S.A., Smetanin S.N. *Proc. SPIE Int. Soc. Opt. Eng.*, **4644**, 312 (2002).
10. Thompson B.A., Minassian A., Damzen M.J. *J. Opt. Soc. Am.*, **20**, 857 (2003).
11. Antipov O.L., Eremeykin O.N., Ievlev A.V., Savikin A.P. *Opt. Express*, **12**, 4313 (2004).
12. Basiev T.T., Garnov S.V., Klimentov S.M., Pivovarov P.A., Gavrilov A.V., Smetanin S.N., Solokhin S.A., Fedin A.V. *Kvantovaya Elektron.*, **37**, 956 (2007) [*Quantum Electron.*, **37**, 956 (2007)].
13. Smith G., Damzen M.J. *Opt. Express*, **15**, 6458 (2007).
14. Basiev T.T., Gavrilov A.V., Ershkov M.N., Smetanin S.N., Fedin A.V., Bel'kov K.A., Boreisho A.S., Lebedev V.F. *Kvantovaya Elektron.*, **41**, 207 (2011) [*Quantum Electron.*, **41**, 207 (2011)].
15. Basiev T.T., Gavrilov A.V., Osiko V.V., Smetanin S.N., Fedin A.V. *Kvantovaya Elektron.*, **33**, 659 (2003) [*Quantum Electron.*, **33**, 659 (2003)].
16. Basiev T.T., Fedin A.V., Gavrilov A.V., Smetanin S.N. *Laser Phys.*, **16**, 1610 (2006).
17. Fedin A.V., Gavrilov A.V., Smetanin S.N. *Laser Phys.*, **19**, 1117 (2009)
18. Basiev T.T., Gavrilov A.V., Smetanin S.N., Fedin A.V. *Dokl. Akad. Nauk*, **430**, 321 (2010).
19. Shardlow P.C., Damzen M.J. *Opt. Lett.*, **35**, 1082 (2010).
20. Basiev T.T., Gavrilov A.V., Smetanin S.N., Fedin A.V. *Kvantovaya Elektron.*, **41**, 202 (2011) [*Quantum Electron.*, **41**, 202 (2011)].
21. Fedin A.V., Gavrilov A.V., Ershkov M.N., Smetanin S.N., Solokhin S.A. *Izv. Akad. Nauk. Ser. Fiz.*, **76**, 713 (2012) [*Bull. Russ. Acad. Sci. Phys.*, **76**, 637 (2012)].
22. Vanherzeele H., Van Eck J.L., Siegman A.E. *Opt. Lett.*, **6**, 467 (1981).
23. Karr T. *J. Opt. Soc. Am.*, **73**, 600 (1983).
24. Bel'dyugin I.M., Zolotarev M.V., Kireev S.E., Odintsov A.I. *Kvantovaya Elektron.*, **13**, 825 (1986) [*Sov. J. Quantum Electron.*, **16**, 535 (1986)].
25. Bernard J.E., Alcock A.J. *Opt. Lett.*, **18**, 968 (1993).
26. Mikaelyan A.L., D'yachenko V.V. *Pis'ma Zh. Eksp. Teor. Fiz.*, **16**, 25 (1972) [*JETP Lett.*, **16**, 17 (1972)].
27. Zagumennyi A.I., Mikhailov V.A., Shcherbakov I.A., in *Handbook of Laser Technology and Applications* (Bristol – Philadelphia: Institute of Physics Publishing, 2004).
28. Pogoda A.P., Lebedeva T.B., Yusupov M.R., Liventsov R.A., Lebedev V.F., Boreysho A.S., Gavrilov A.V., Smetanin S.N., Fedin A.V. *LO-2012 Technical Program* (St. Petersburg, 2012) p. 43.
29. Syed K., Green R.P.M., Crofts G.J., Damzen M.J. *Opt. Commun.*, **112**, 175 (1994).
30. Damzen M.J., Matsumoto Y., Crofts G.J., Green R.P.M. *Opt. Commun.*, **123**, 182 (1994).
31. Galushkin M.G., Mitin K.V., Sviridov K.A. *Kvantovaya Elektron.*, **21**, 1157 (1994) [*Quantum Electron.*, **24**, 1073 (1994)].
32. Antipov O.L., Kuzhelev A.S., Luk'yanov A.Yu., Zinov'ev A.P. *Kvantovaya Elektron.*, **25**, 891 (1998) [*Quantum Electron.*, **28**, 867 (1998)].
33. Antipov O.L., Bredikhin D.V., Eremeikin O.N., Ivakin E.V., Savikin A.P., Sukhodolov A.V., Fedorova K.A. *Kvantovaya Elektron.*, **36**, 418 (2006) [*Quantum Electron.*, **36**, 418 (2006)].
34. Svelto O. *Principles of Lasers* (New York: Plenum, 1998; St. Petersburg: Lan', 2008).
35. Contag K., Karszewski M., Stewen C., Giesen A., Hugel H. *Kvantovaya Elektron.*, **28**, 139 (1999) [*Quantum Electron.*, **29**, 697 (1999)].
36. Damzen M.J., Matsumoto Y., Crofts G.J., Green R.P.M. *Opt. Commun.*, **123**, 182 (1996).
37. Odoulov S., Soskin M., Khyzhniak A. *Optical Oscillators with Degenerate Four-Wave Mixing* (*Dynamic Grating Lasers*) (Chur: Harwood Academic, 1991; Moscow: Nauka, 1990).

38. Zeldovich B.Ya., Pilipetsky N.F., Shkunov V.U. *Principles of Phase Conjugation* (Berlin: Springer, 1985; Moscow: Nauka, 1985).
39. Kogelnik H. *Bell Syst. Tech. J.*, **48**, 2909 (1969).
40. Smith G., Damzen M.J. *Opt. Express*, **14**, 3318 (2006).
41. DeLoach L.D., Payne S.A., Chase L.L., Smith L.K., Kway W.L., Krupke W.F. *IEEE J. Quantum Electron.*, **29**, 1179 (1993).
42. Payne S.A., Chase L.L., Newkirk H.W., Smith L.K., Krupke W.F. *IEEE J. Quantum Electron.*, **24**, 2243 (1988).