

PACS numbers: 42.40.-i; 42.50.-p; 42.50.Ct; 42.79.Hp; 42.50.Gy
DOI: 10.1070/QE2013v043n07ABEH014950

Information coding of exciting laser pulses in an optical echo-processor

I.A. Rusanova

Abstract. **We report the possibility of controlling the distribution of quantum bits within an inhomogeneously broadened line of a resonant transition in recording and transforming information in optical echo-processors. We consider the efficiency of realisation of the elementary logic XOR gate based on a two-pulse excitation of a resonant medium with phase memory. The encoded information is incorporated into the temporal shape of laser pulses in the form of amplitude modulation of an 'echelon' of present ('1') and absent ('0') pulse-codes for obtaining more efficient logic elements that reduce the noise in a quantum communication channel.**

Keywords: *photon echo, optical echo-processing, echo-holography, quantum information, qubit, optical information processing, logic gate, XOR.*

1. Introduction

Currently, much attention is paid to the development of optical storage devices (OSDs) and optical echo-processors, which are used to solve technically important operations of compression of information signals and to implement different versions of their convolutions and Fourier transforms [1, 2]. In designing such devices, of interest is echo-holographic information processing, given the existence of effective methods for recording, storing and reproducing information. The main advantages of the photon echo-based OSDs are high-density information recording, operation speed and direct access to memory cells, possibility of multiple usage of the resonant medium for recording and ability to read and write information in real time, including information in analogue form [3, 4]. When the resonant medium interacts with a train of laser pulses, atoms can act as quantum gates that perform logical operations. Realisation of a set of elementary quantum XOR and NOT gates allows, in principle, any unitary quantum qubit (q-bit) operations. The possibilities of quantum systems of information transmission and transformation depend on the density of encoding quantum information and the existence of quantum algorithms that make it possible to solve any problems more effectively. Physical systems that implement the q-bits can be any objects with two quantum states, such as the polarisation states of photons, spin states of nuclei, etc. Another urgent problem today is the organisation of the control of individual q-bits and the interaction between them, while ensuring a sufficiently long time of decoherence [5, 6].

In this connection, of interest is to study the optimisation of information recording and transformation processes in optical echo-processors for the development of logic elements of quantum computers. Data carriers are transient dynamic population and polarisation gratings of a resonant medium, which can be represented as a space – frequency distribution of q-bits within an inhomogeneously broadened line of a resonant transition. In this paper we used an information-theoretic method for studying quantum information processes in resonant media with phase memory, developed on the basis of the ideas of Shannon and Kolmogorov's algorithmic information theory [7, 8]. The use of the von Neumann entropy in the study of quantum information processes revealed its low suitability for description of optical transient processes [9]. We report the possibility of controlling the distribution of quantum bits within an inhomogeneously broadened line in recording and transforming the information based on the two-pulse excitation of a resonant medium with phase memory by implementing elementary logic XOR and NOT gates as well as by selecting the coding and laser pulse duration to produce more efficient logic elements that reduce noise in a quantum communication channel.

2. Processing and transformation of quantum information

The information received and processed in quantum systems is significantly different from classical information. One of the qualitative differences of quantum information is the impossibility to copy it. The amount of transmitted classical information can be increased when use is made of a quantum communication channel, accurately transmitting any quantum state. The quantum error correcting code was constructed independently by Shor and Steane [10, 11]. Shannon's theorem implies the possibility of encoding information, which reduces errors, at lower bandwidth transmission rates [12]. In the case of a code alphabet, consisting of two symbols – '0' and '1', in transmitting information over a communication channel with noise, the most direct way to prevent mistakes is to repeat the message. To reduce the error probability, the cascade structure is used at the output, the depth of this structure being determined by the logarithm of the length of the program algorithm of the scheme, which needs to be implemented [6, 7].

One of the elementary logic gates for an arbitrary state $|q\rangle = \alpha|0\rangle + \beta|1\rangle$ is NOT. This gate is capable of performing the functions of two logic operations – XOR (OR) and FANOUT

I.A. Rusanova Institute of Physics, Kazan (Volga region) Federal University, ul. Kremlevskaya 18, 420008 Kazan, Russia;
e-mail: irusanova@yandex.ru

Received 8 August 2012; revision received 18 February 2013
Kvantovaya Elektronika **43** (7) 651 – 655 (2013)
Translated by I.A. Ulitkin

('unfolding', a nonstandard gate, which duplicates the input data). The FANOUT gate does not lose information and is logically reversible due to re-action. Consider the implementation of the logic element in the phenomenon of the primary photon echo (PPE), which involves generation of an echo-signal by a two-level resonant medium with phase memory after two-pulse exposure (Fig. 1a). The response of the photon echo is formed in two main stages: the dephasing of the oscillating dipole moments of optical centres and their subsequent phasing, leading to the emergence of a macroscopic polarisation of the medium observed in the form of a coherent response. Physically, the logical XOR operation corresponds to the reorientation of dephased oscillating dipole moments by 180° under the action of the second resonant laser pulse (Figs 1b and 1 c). At the end of its action the operation of the XOR-gate is performed and the information convolution of the two laser pulses is realised. This scheme is similar to the elementary command of a quantum computer, if $|a\rangle = |1\rangle$, then $|b\rangle \rightarrow \mathrm{NOT}(|a\rangle$.

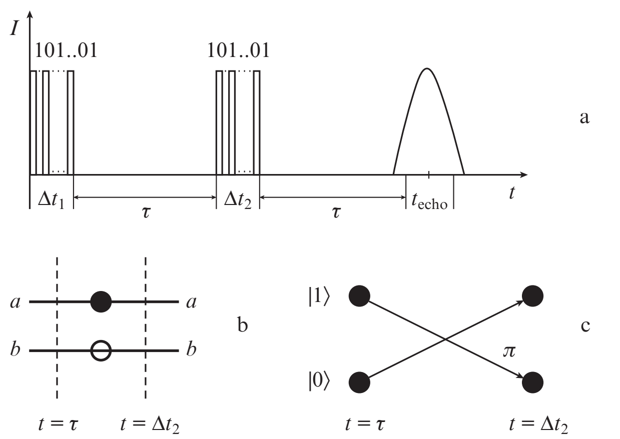

Figure 1. (a) Information writing using the scheme of a two-pulse photon echo ($t_{\mathrm{echo}} = 2\tau$ is the time of the coherent response of the medium, Δt_1 and Δt_2 are the duration of the first and second excitation pulses) and (b) scheme of the elementary logic XOR gate, implemented by applying two excitation laser pulses; (c) time evolution of a quantum system under the action of 'trigger' pulse (π is the phase shift).

The encoded information may be incorporated both into the temporary shape or polarisation of the exciting pulse and into the wave front. The simplest way to specify the temporary shape of the exciting laser pulse is the amplitude modulation in the form of two or more pulses delayed relative to each other in time. By changing the number of such pulse-codes and their intensity, we can set the complex temporal shape. Let the exciting pulse be set by a train (echelon) of present ('1') and absent ('0') n 'different' pulse-codes provided that $n\partial t \ll T_1, T_2$, where T_1 and T_2 are the longitudinal and transverse irreversible relaxation times of the system, and ∂t is the length of a message element. Consider the optical transient processes on time scales that are close to the irreversible relaxation of the medium, subject to relatively large inhomogeneous broadening of the optical transitions $\sigma = 5$ ns^{-1}, which make it possible to realise a high speed writing and reading of information ($T_2^* = 0.2$ ns, $T_2 = 2$ ns, $T_1 = 10$ ns) [2, 7].

Consider now the transformation of the classical information stored in the coded object laser pulse upon its impact on the system of two-level atoms. The equation for the single-particle density matrix can be written as

$$\mathrm{i}\hbar \frac{\partial \rho}{\partial t} = [H, \rho], \tag{1}$$

where $H = H_0 + H_{\mathrm{m}} + U + V$; H_0 and H_{m} are the Hamiltonians of the atom and the medium; U is the operator of their interaction; and V is the operator of the interaction of the atom with the radiation field.

Transforming expression (1) in the rotating coordinate system, we obtain

$$\mathrm{i}\hbar \frac{\partial \tilde{\rho}}{\partial t} = [B, \tilde{\rho}], \tag{2}$$

where $B = \tilde{H}_0 - \hbar A + \tilde{V}$; $H_0 = \hbar(\Omega - \Omega')P_{22}$; $A = (\omega - \omega')P_{22}$; and A is the transition matrix in the rotating coordinate system. For a two-layer system

$$\exp(\pm\,\mathrm{i}At) = P_{11} + P_{22}\exp[\pm\,\mathrm{i}(\omega - \omega')t],$$

$$H_0 = \hbar(\Omega - \Omega')P_{22},$$

$$\tilde{V} = -\frac{1}{2}d\left[E^*(\omega')P_{12} + E(\omega')P_{21}\right],$$

where P_{ij} is the projective matrix operator (with elements ij = 1 and all other elements equal to zero); d is the resonant transition dipole moment; and E is the electric field strength of an exciting laser pulse.

Neglecting the field broadening and spectral diffusion during the action of the object pulse, we find the density matrix when the atom interacts with a single Fourier component of the pulse field followed by averaging over all frequencies. The solution to (2) can be found as

$$\tilde{\rho}(n\partial t) = \exp[-\,\mathrm{i}\hbar^{-1}B(n\partial t)]\rho(0)\exp[\mathrm{i}\hbar^{-1}B(n\partial t)], \tag{3}$$

where

$$B = \begin{pmatrix} 0 & -\frac{1}{2}dE^*(\omega') \\ -\frac{1}{2}dE(\omega') & \hbar(\omega' - \Omega') \end{pmatrix}.$$

Assuming that before the exposure to the object pulse the atom was in its ground state, for the matrix elements of the density matrix we obtain

$$\begin{aligned} \tilde{\rho}(n\partial t) \approx{}& P_{11}\left(\cos^2\frac{\Theta}{2} + \frac{\Delta^2}{\Theta'^2}\sin^2\frac{\Theta}{2}\right) \\ &+ P_{12}\left(-\mathrm{i}\frac{a^*}{2\Theta'}\sin\Theta + \frac{a^*\Delta}{\Theta'^2}\sin^2\frac{\Theta}{2}\right) \\ &+ P_{21}\left(\mathrm{i}\frac{a}{2\Theta'}\sin\Theta + \frac{a\Delta}{\Theta'^2}\sin^2\frac{\Theta}{2}\right) + P_{22}\frac{|a|^2}{\Theta'^2}\sin^2\frac{\Theta}{2}, \end{aligned} \tag{4}$$

where

$$\Delta = \omega' - \Omega';\ \Theta = \Theta' t;\ \Theta = \sqrt{\Delta^2 + d^2E_0^2\hbar^{-2}|\tilde{\varepsilon}|^2};$$

$$a = dE_0\hbar^{-1}\tilde{\varepsilon}\exp(\mathrm{i}\boldsymbol{kr});\ a^* = dE_0\hbar^{-1}\tilde{\varepsilon}^*\exp(\mathrm{i}\boldsymbol{kr}).$$

After the exposure to the exciting pulse ($B = P_{22}\hbar\Delta$)

$$\exp[\pm\mathrm{i}\hbar^{-1}B(t' - n\partial t)] = P_{11} + P_{22}\exp[\pm\mathrm{i}\Delta(t' - n\partial t)], \quad (5)$$

$$\tilde{\rho}(t' - n\partial t) = \{P_{11} + P_{22}\exp[-\mathrm{i}\Delta(t' - n\partial t)]\}\tilde{\rho}(n\partial t)$$

$$\times\{P_{11} + P_{22}\exp[\mathrm{i}\Delta(t' - n\partial t)]\}. \quad (6)$$

The information incorporated in a structure becomes potential (structural). In a resonant medium the potential information carriers are transient dynamic gratings described by the density matrix ρ, i.e., the amplitude and phase structure of the density matrix contains structural information. We associate with such a matrix a weighted graph. Then the measure of structural information is determined by the measure of the uncertainty of such a graph with the elements $\in V(G)$, where V is the final set consisting of N vertices (marked), corresponding to the diagonal elements of the density matrix, and q edges corresponding to the off-diagonal elements. Kolmogorov in his algorithmic information theory [13] defines the relative complexity of the object K as the minimum length $l(p)$ of the program p of deriving G_0 from G, thereby removing the uncertainty of the system. The operator D represents a set of rules for processing (removing) by any means the elements of the active part of the object G into the object $G_0 = D(G)$. The amount of structural information in G with respect to G_0 is defined by the expression

$$J = K(G, G_0) - K(G_0). \quad (7)$$

We assume that the object with the elements belonging to the zero set $\varnothing$ has the structural information that is equal to zero. In view of the time evolution the weight function is defined as the sum [corresponding to the diagram $G^{(k)}$] of the values of the elements of the active part of the object G at a time t, referred to the sum of the values of the elements of the active part of the object G at an initial time t_0. Implementation of procedure (2) leads to an ensemble of sets $Q^{(k)}$.

Since the density matrix

$$\rho = \sum_{i,j=1}^{N}\rho_{ij}P_{ij}$$

corresponds to the weighted graph G, at an initial time the sum of the values of the elements of the active part, $S(t_0)$, will have the form:

$$S(t_0) = \mathrm{abs}\left(\sum_{i\neq j}\rho_{ij}(t_0)\right). \quad (8)$$

By calculating the corresponding sum $S'(t) = \sum_k S^{(k)}(t)$ at a time t for an ensemble of sets $Q^{(k)}$, we finally obtain an expression for the amount of quantum information in q-bits:

$$J_{\mathrm{q}} = \log_2\left(\frac{S'(t)}{S(t_0)}\right). \quad (9)$$

Since ρ is a Hermitian operator, its matrix elements $\rho_{ij} = \rho_{ji}^*$. The foregoing selection of the operator D leads to the values of $S'(t)$ and $S(t_0)$, consisting of the sum of matrix elements $\rho_{ij} + \rho_{ji}$. In the presence of only two quantum states in the system, $|1\rangle$ and $|2\rangle$, the general state is a linear superposition $|\psi\rangle = \alpha|1\rangle + \beta|2\rangle$ (α and β are the complex numbers). The corresponding density matrix operator has the form

$$\rho|\psi\rangle\langle\psi| = |\alpha|^2|1\rangle\langle 1| + |\beta|^2|2\rangle\langle 2| + \alpha\beta^*|1\rangle\langle 2| + \alpha^*\beta|2\rangle\langle 1|, \quad (10)$$

where the active part is given by the last two terms. The action of the operator D in this case yields $J_{\mathrm{q}} = 1$. Thus, expression (9) specifies the amount of quantum information of the system in the q-bits.

Using the solutions for the density matrix when the atom interacts with a single Fourier component of the pulse field followed by averaging over all frequencies, the amount of structural information $J_{\mathrm{q}}(\omega', \Omega')$ over a separate isochromate of an inhomogeneously broadened line of a resonant transition of two-level atoms is defined as

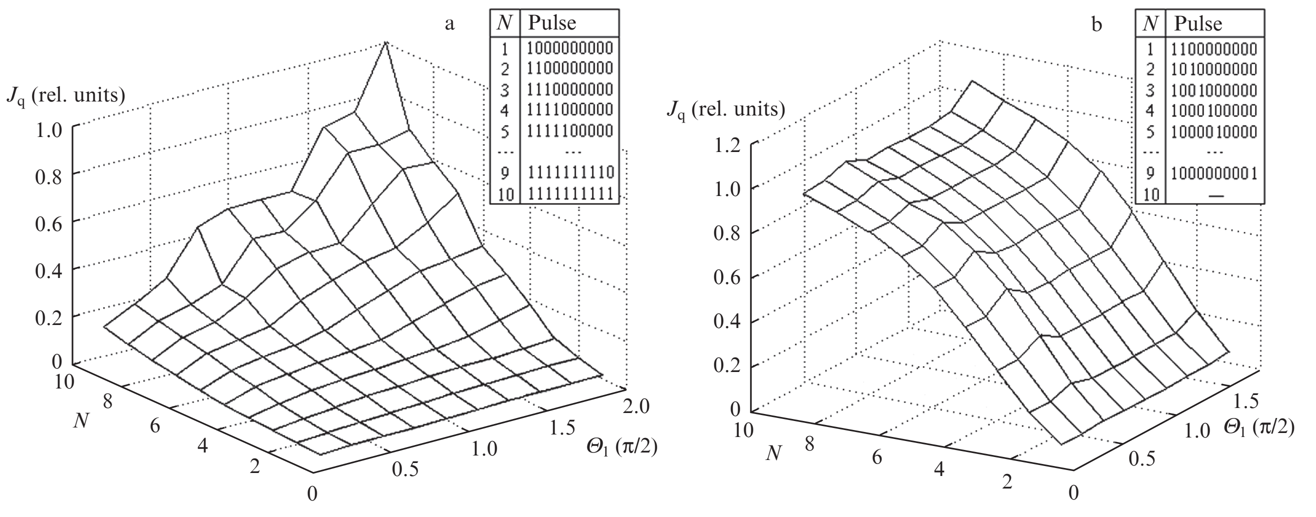

Figure 2. Dependence of the amount of quantum information J_{q} on the encoding scheme of the first excitation pulse N and its normalised power Θ_1.

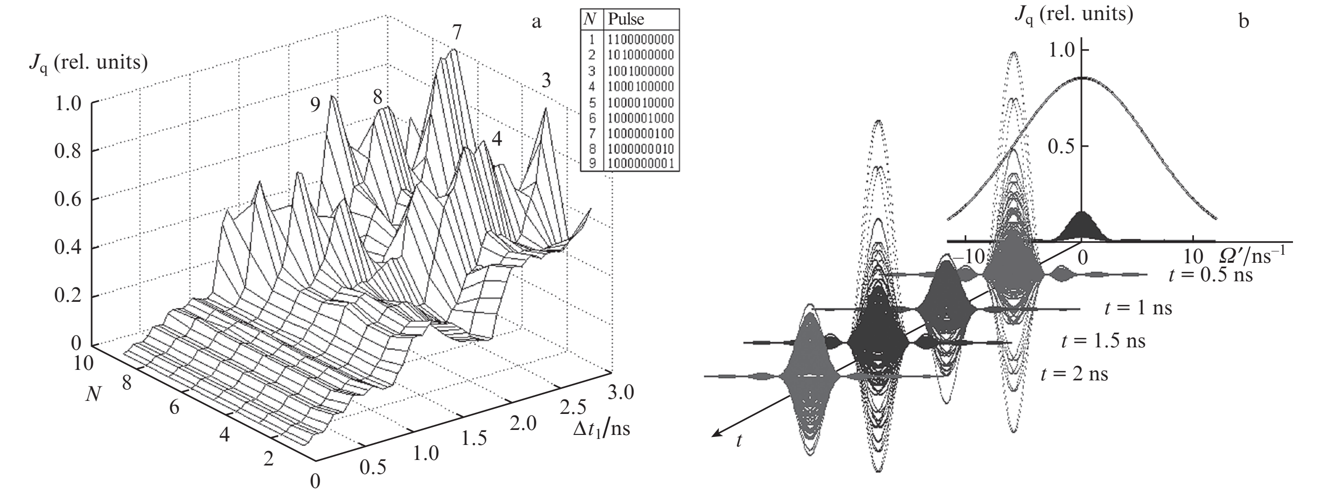

Figure 3. Efficiency of writing quantum information J_q at times close to the irreversible relaxation time of the medium: (a) dependence on the duration Δt_1 and encoding scheme N of the first exciting laser pulse ($\sigma = 5$ ns^{-1}, $0.05T_2 \leqslant \Delta t_1 \leqslant 1.5T_2$); (b) time evolution of the distribution of quantum bits within the inhomogeneously broadened line ($t = 0$ is the instant of time after exposure to the first laser pulse encoded as 1100000000).

$$J_q = \int_{-\infty}^{\infty} g_1(\omega')\,d\omega' \int_{-\infty}^{\infty} g_2(\Omega')\log_2\left(\frac{S'(t)}{S(t)}\right)(\omega',\Omega')\,d\Omega', \quad (11)$$

where $g_1(\omega')$ is the frequency distribution function of the Fourier spectrum of the object pulse, normalised per unit area per pulse duration $n\partial t$; $g_2(\Omega')$ is the frequency distribution function of the inhomogeneously broadened line of a resonant transition [8].

Figure 2 shows the result of the transformation of encoded information (word, phrase), incorporated in the first excitation laser pulse [in the form of a train (echelon) of pulse-codes '1' and '0'] as a function of the pulse power $J_q(\Theta_1)$ and type of encoding. More efficient data recording is due to the algorithm of amplitude modulation of the laser pulse temporal shape, corresponding to the scheme in Fig. 2b, which allows one to encode information in time slots (channels) by the absent pulse-codes '0' between a pair of pulse-codes '1' moving apart in time. The transformation of quantum information of a resonant medium strongly depends not only on the encoding scheme of the exciting pulse, but also on its duration, has a significant impact on the distribution of quantum bits within the inhomogeneously broadened line of the resonant transition (Fig. 3a) through the formation of entangled states and the emerging 'information and phase grating', allowing one to control the number of quantum bits in a medium with time-addressable phase memory within the inhomogeneous broadening of the optical transition. The efficiency of the distribution of quantum bits within an inhomogeneously broadened line (Fig. 3b) has an effect on the reversible transverse relaxation time T_2^*, which is associated with the recovery of coherence in the system. Depending on the encoding of the exciting pulses one can control the efficiency of the distribution of quantum bits on an inhomogeneously broadened line and the transformation of the amount of quantum information in a resonant medium; there is also the possibility of reading and writing data in the frequency domain [3–5]. Limiting the existence of the phase memory depends on the irreversible transverse relaxation time T_2, which is explained by the irreversible loss of coherence of the system [phase memory loss $\exp(-t/T_2 < 1)$]. Irreversible relaxation leads to the destruction of superposition states (entanglement of the corresponding states with the states of the reservoir). The appearance of negative values of quantum information is connected, like quantum teleportation, with the presence of entangled states and is observed at times close to the irreversible relaxation time of the system [14, 15].

In studying the impact of two pulses on the resonant medium (the intensity maxima of the pulses correspond to the times $t = 0$ and $t = \tau$ (see Fig. 1a), the information is written in the temporary shape of both excitation pulses in the form of an echelon of pulse-codes: 1000000001. Figure 4 shows the quantum information J_q of the convolution of two laser pulses versus their duration ($\Delta t = \Delta t_1 = \Delta t_2$) and the time interval τ between them at a point in time after the second pulse. It is shown that the optimum writing of quantum information is strongly influenced by the duration and the time interval between code-pulses, which is associated with the formation of various transient dynamic population and polarisa-

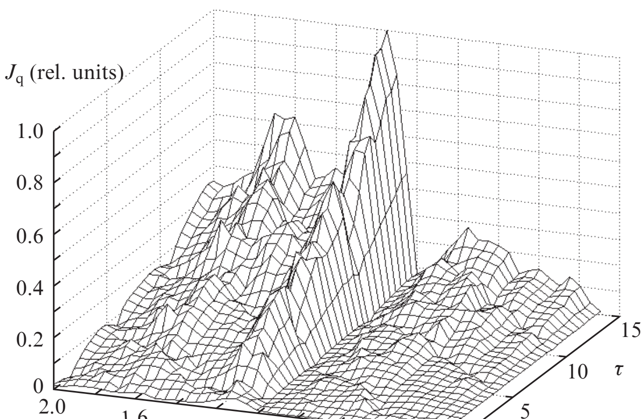

Figure 4. Quantum information J_q during two-pulse excitation of a resonant medium by laser pulses with the same encoded information stored in the temporal shape in the form of the echelon of pulse-codes 1000000001 as a function of pulse duration Δt and time interval τ between them.

tion gratings of the medium within the inhomogeneously broadened line [16]. At a pulse duration 1 ns < Δt <2 ns one can observe an intense spatial modulation of quantum information with a pronounced maximum at Δt = 1.15 ns and at Δt = 1.6 and 1.7 ns within the inhomogeneously broadened line of the optical transition $\sigma = 5\ \text{ns}^{-1}$, the inverse value of which determines the minimum pulse duration of the echelon of pulse-codes (information array).

3. Conclusions

Thus, we report the possibility of controlling the number of quantum bits in a medium with time-addressable phase memory within the inhomogeneously broadened lines of the resonant transition in order to ensure optimal processes of recording and transforming information in optical echo-processors. This allows one to develop logical elements, resulting in noise reduction in the quantum communication channel by selecting efficient encoding schemes of excitation laser pulses, which incorporate information in their temporary shape in the form of an echelon of pulse-codes '1' and '0'.

References

1. Nefed'ev L.A., Samartsev V.V. *Zh. Prikl. Spektrosk.*, **47** (4), 640 (1987).
2. Kalachev A.A., Samartsev V.V. *Kogerentnye yavleniya v optike* (Coherent Phenomena in Optics) (Kazan: Kazan State University Publishing House, 2003).
3. Mitsunaga M., Yano R., Uesugi N. *Opt. Lett.*, **16**, 1890 (1991).
4. Kroll S., Tidlund P. *Appl. Opt.*, **32**, 7233 (1993).
5. Mossberg T.W. *Opt. Lett.*, **17**, 535 (1992).
6. Kitaev A.Yu. *Usp. Mat. Nauk*, **52** (6), 53 (1997).
7. Nefed'ev L.A., Rusanova I.A. *Opt. Spektrosk.*, **90**, 1000 (2001).
8. Nefed'ev L.A., Rusanova I.A. *Laser Phys.*, **12** (3), 571 (2002).
9. Rusanova I.A. *Vestnik ChelGU*, **13** (14), 88 (2012).
10. Shor P.W. *Phys. Rev. Ser. A.*, **52**, (4), 2493 (1995).
11. Steane A.M. *Proc. Roy. Soc. London Ser. A.*, **1954**, 2551 (1996).
12. Shannon C. *Raboty po teorii informatsii i kibernetike* (Works on Information Theory and Cybernetics) (Moscow: IL, 1963).
13. Kolmogorov A.N. *Teoriya informatsii i teoriya algoritmov* (Information Theory and The Theory of Algorithms) (Moscow: Nauka, 1987).
14. Kilin S.Ya. *Usp. Fiz. Nauk*, **169** (5), 507 (1999).
15. Cerf N.J., Adami C. *Phys. Rev. Lett.*, **79** (26), 5194 (1997).
16. Rusanova I.A., in *Sb. Dokl. VII Vseross. Konf. 'Neobratimye protsessy v prirode i tekhnike'* (Abstracts of the VII All-Russian Conf. 'Irreversible Processes in Nature and Technology') (Moscow: Izd. Bauman MSTU, 2013) p. 92.