

# Compression of femtosecond pulses with a Gaussian temporal and spatial intensity distribution

S.Yu. Mironov, V.V. Lozhkarev, E.A. Khazanov, G. Mourou

**Abstract.** The possibility of using the cubic self-action effect of intense radiation for the additional time compression of Gaussian beams with a quasi-uniform cross section is investigated. The ability to recompress 30-fs Gaussian pulses down to 14 fs (16 fs) with the heterogeneity of less than 1.5 fs (2 fs) on the spatial scale, which corresponds to the energy level 63% (86%) of the beam, is theoretically demonstrated at the  $B$ -integral of  $\sim 3$ .

**Keywords:** compression of intense femtosecond pulses, self-phase modulation, cubic nonlinearity.

## 1. Introduction

Under the influence of intense laser radiation, the dependence of the medium polarisation on the external field strength is nonlinear. In an isotropic medium the cubic nonlinearity is most important. It leads to a number of negative effects such as large-scale and small-scale self-focusing and self-phase modulation [1–4]. The last effect can be used for an additional time compression of high-power laser pulses. The idea of the pulse compression is rather simple: an intense optical pulse propagating in a nonlinear medium broadens its spectrum, acquires the phase modulation and stops to be Fourier transform limited in the spectral-temporal domain; in this case, external chirped mirrors allows the phase to be corrected and the pulse duration to be reduced. This technique is widely used in fibre optics, where the pulse broadens its own spectrum upon propagation in an optical fibre [5, 6]. At present, temporal self-compression of femtosecond pulses has been experimentally studied during their propagation and filamentation in gases [7, 8] and capillaries [9–11]. However, it should be emphasised that these techniques due to the optical breakdown cannot be used for the high energy laser pulses. In this case, use is made of collimated beams without focusing and transparent dielectrics, i.e. crystals or glasses, as a spectrum

broadening optical medium. The authors of [12, 13] demonstrated theoretically the possibility of using self- and cross-action effects to shorten the optical pulse duration, resulting from the generation of the second harmonic of femtosecond radiation with a uniform intensity distribution in space. Experimental confirmation of the possible application of the self-phase modulation effect arising during the propagation of intense ( $1 \text{ TW cm}^{-2}$ ) pulses in glass for their temporal compression is presented in [14, 15].

In the case of nonuniform intensity distribution in space, different regions of a laser beam accumulate a different nonlinear phase ( $B$ -integral), which leads to nonuniform temporal compression across the beam cross section with help of chirped mirrors. In this paper we consider the possibility of accumulation of a quasi-uniform nonlinear phase across the beam by optical radiation with initial Gaussian intensity distribution in space and time. The main idea is to implement a glass defocusing lens as a nonlinear medium. Using numerical simulations we have demonstrated the possibility of quasi-uniform temporal recompression of an optical pulse in the cross section of the Gaussian beam.

## 2. Accumulation of a quasi-uniform nonlinear phase

Temporal compression of optical pulses with the Gaussian temporal and spatial intensity distribution is controlled by the nonlinear phase accumulation, commonly known as the  $B$ -integral

$$B(r, t) = \frac{2\pi}{\lambda_0} \int_0^L \gamma I(r, t, \xi) d\xi, \quad (1)$$

where  $\lambda_0$  is the centre wavelength;  $\gamma$  is the cubic nonlinearity;  $I(r, t, \xi)$  is the intensity;  $r$  is the transverse coordinate;  $L$  is the nonlinear medium thickness; and  $\xi$  is the longitudinal coordinate of the beam in a nonlinear medium. The typical value of the nonlinearity coefficient is  $\gamma = (3–15) \times 10^{-7} \text{ cm}^2 \text{ GW}^{-1}$ .

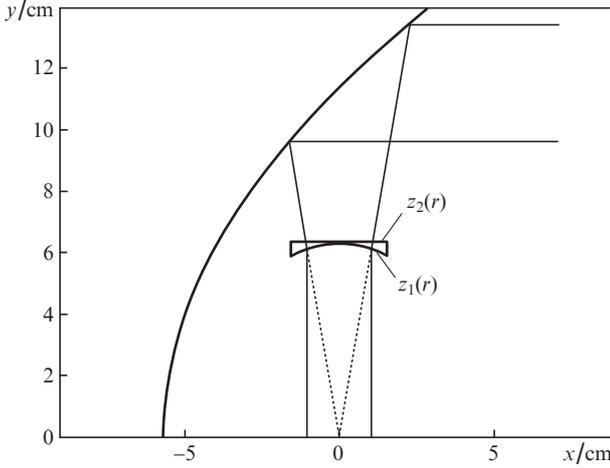
The propagation of the laser beam with the Gaussian intensity distribution in space through a plane-parallel plate leads to a spatially nonuniform accumulation of the nonlinear phase. The accumulation of a quasi-uniform nonlinear phase is possible, if the laser beam passes through a defocusing lens with parabolic or spherical surfaces. The lens parameters are chosen to maintain an almost constant product of intensity and optical path length in a nonlinear medium across each beam. The laser beam passed through such a lens can be collimated by an off-axis parabolic mirror. As a result, the beam will preserve the plane phase and the Gaussian intensity profile as well as will accumulate the quasi-uniform  $B$ -integral

**S.Yu. Mironov, V.V. Lozhkarev** Institute of Applied Physics, Russian Academy of Sciences, ul. Ul'yanova 46, 603950 Nizhnii Novgorod, Russia; e-mail: Sergey.Mironov@mail.ru;

**E.A. Khazanov** Institute of Applied Physics, Russian Academy of Sciences, ul. Ul'yanova 46, 603950 Nizhnii Novgorod, Russia; N.I. Lobachevsky State University of Nizhnii Novgorod, prosp. Gagarina 23, 603950 Nizhnii Novgorod, Russia;

**G. Mourou** Institut de la Lumière Extrême, ENSTA, Chemin de la Hunière, 91761 Palaiseau, France; N.I. Lobachevsky State University of Nizhnii Novgorod, prosp. Gagarina 23, 603950 Nizhnii Novgorod, Russia

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**Figure 1.** Principal scheme of accumulation of a quasi-uniform  $B$ -integral by the Gaussian beams.

across the beam cross section. Figure 1 illustrates the principal accumulation scheme.

The optimum thickness and the curvature of the input and output surfaces of the negative lens can be found using the geometric optics approximation and numerical simulation. Let us define the input and output surfaces of the lens by the functions  $z_1(r)$  and  $z_2(r)$  (Fig. 1). For spherical surfaces of the lens these functions have the form:

$$z_{1,2}(r) = \pm \sqrt{R_{1,2}^2 - r^2} \mp \left( \frac{d}{2} + R_{1,2} \right) + |F|, \quad (2)$$

where  $R_{1,2}$  are the radii of spherical surfaces;  $d$  is the lens thickness; and  $F$  is the focal distance of the lens.

We define the transverse intensity distribution as

$$I = I_0 \exp\left(-\frac{r^2}{R_{\perp}^2} - 4\frac{t^2}{T^2} \ln 2\right), \quad (3)$$

where  $I_0$  is the peak intensity;  $R_{\perp}$  is the Gaussian beam radius;  $t$  is the time; and  $T$  is the pulse duration FWHM. The minimum thickness of the lens should be chosen based on the desired  $B$ -integral accumulated by the central ray:

$$B_0 = \frac{2\pi}{\lambda} \gamma I_0 d. \quad (4)$$

Depending on the transverse coordinate, the rays accumulate the  $B$ -integral

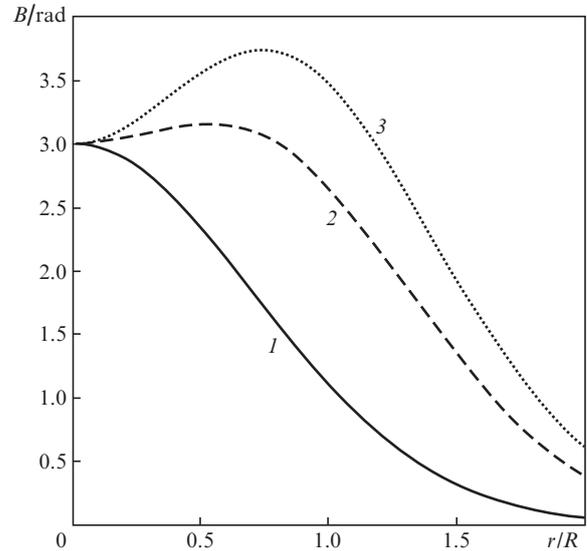
$$\begin{aligned} B(r_1, t) &= \frac{B_0}{d} \int_0^{L(r_1)} \exp\left\{-\frac{[r_1 + \Delta r(\xi)]^2}{[R_{\perp} + \Delta R(\xi)]^2} - 4\frac{t^2}{T^2} \ln 2\right\} d\xi \\ &\approx B_0 \exp\left(-\frac{r_1^2}{R_{\perp}^2} - 4\frac{t^2}{T^2} \ln 2\right) \frac{L(r_1)}{d}, \end{aligned} \quad (5)$$

where  $L(r_1) = \sqrt{(r_2 - r_1)^2 + [z_2(r_2) - z_1(r_1)]^2}$  and  $r_{1,2}$  are the input and output transverse coordinates. In (5) we assumed that the laser beam radius on the output surface of the lens,  $R$ , increases insignificantly, i.e.  $\Delta R/R \ll 1$ . The problem of the accumulation of a quasi-uniform nonlinear phase by the Gaussian beams is reduced to the problem of minimising the

difference  $|B(r_1) - B_0|$  at fixed values of the variable  $t$  and duration  $T$ . By varying the curvature radii of the input and output surfaces of the lens,  $R_1$  and  $R_2$ , it is possible to provide a quasi-uniform accumulation of the nonlinear phase for a given set of laser beam parameters.

Let us minimise the difference on two spatial scales  $|r_1| \leq R$  and  $|r_1| \leq w = R\sqrt{2}$ , corresponding to uniform nonlinear phase accumulation and spectrum broadening of 63% and 86% of the total beam energy. The choice between the spatial scales  $R$  or  $w$  (or other) should be determined by the requirements to the deviation of the pulse duration across the Gaussian beam cross section from the average value.

As an example, we consider a Gaussian beam with  $R_{\perp} = 0.74$  cm with the pulse duration  $T = 30$  fs, energy 100 mJ and centre wavelength 800 nm. For these parameters the peak intensity is  $1.8$  TW cm $^{-2}$ , and the lens thickness  $d = 0.63$  mm for the K8 glass ( $\gamma = 3.35 \times 10^{-7}$  cm $^2$  GW $^{-1}$ , the refractive index  $n = 1.5$ ) corresponds to the  $B$ -integral value of 3. The optimal curvature radii of spherical surfaces are  $R_1 = 3.17$  cm and  $R_2 = 739$  cm and  $R_1 = 2.12$  cm and  $R_2 = 740$  cm for  $R$  and  $w$  spatial scales. The focal distances are equal to  $-6.32$  cm and  $-4.24$  cm, respectively. In this case, the increase in the beam radius,  $\Delta R/R$ , does not exceed  $3 \times 10^{-2}$ . The lens, which is optimal for the  $R$  scale, and an off-axis parabolic mirror are shown in Fig. 1. Figure 2 presents the dependences of the  $B$ -integral accumulated in the lenses and in the plane-parallel glass plate of the same thickness  $d$  on the transversal coordinate normalised to the radius  $R$ . In the case of optimal defocusing lenses, the variation of the  $B$ -integral for the Gaussian beam does not exceed 10% and 27% of the value of  $B = 3$  for  $R$  and  $w$  spatial scales, respectively. At the same time for the plane-parallel glass plate, this value is 63%.



**Figure 2.**  $B$ -integral accumulated by a Gaussian beam in the plane-parallel glass plate (1) and in the defocusing lens, which are optimal for  $R$  (2) and  $w$  (3) scales vs. the normalised transverse coordinate.

It is also important to note that the cubic nonlinearity leads to the appearance of self-focusing. The self-focusing can be of two types: large-scale and small-scale. For the laser parameters mentioned above, the spatial scale of large-scale self-focusing is about 30 cm, which significantly exceeds the thickness of the lens. The large-scale self-focusing changes the

beam divergence, but is not crucial for the above beam parameters. The spatial scale of small-scale self-focusing (SSSF) is determined by the equality of the  $B$ -integral to unity. The SSSF leads to amplification of the spatial noise of the laser beam, thereby causing the damage of optical elements. For the observed case ( $B = 3$ ) there is a possibility to damage the defocusing lens, but the implementation of the SSSF suppression method [13, 16] makes the probability insignificant.

### 3. Temporal compression

The cubic nonlinearity makes an initially Fourier transform limited pulse broaden spectrally and become self-phase modulated. Once, the desired uniform spectral broadening is produced; a nonlinear chirp can be partially compensated by reflecting light from mirrors with anomalous dispersion. As shown in [12, 13], a correction (even of a quadratic spectral phase component) can significantly reduce the pulse duration. Mathematically, the operation of the compensation can be written as:

$$A_c(t) = F^{-1}(\exp(-i\alpha\omega^2/2)F(A_{\text{out}}(t, L))), \quad (6)$$

where  $A_{\text{out}}(t, L)$  is the complex field amplitude at the output of the lens;  $F$  and  $F^{-1}$  are the direct and inverse Fourier transform in the spectral-temporal domain; and  $\alpha$  is the quadratic dispersion coefficient.

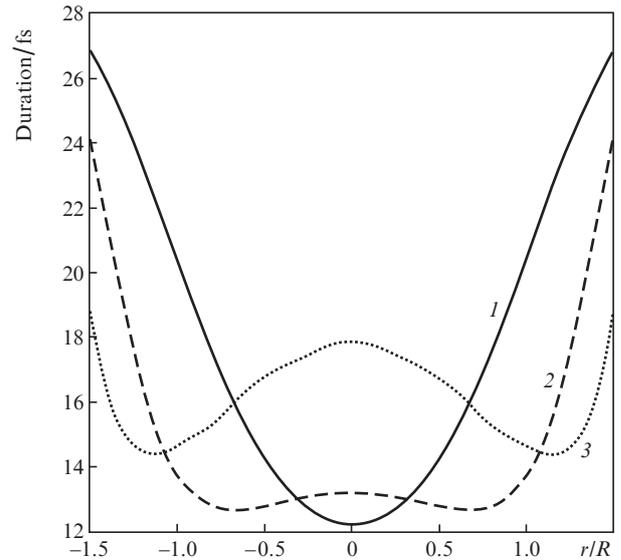
In general, to find the field distribution at the lens output, in addition to the laws of geometrical optics and the influence of cubic nonlinearity, the effect of group velocity dispersion is needed to be taken into account. The evolution of the field amplitude of the incidence pulse can be described by the equation [1]:

$$\frac{\partial A}{\partial z} + \frac{1}{u} \frac{\partial A}{\partial t} - i \frac{k_2}{2} \frac{\partial^2 A}{\partial t^2} + i\beta|A|^2 A = 0, \quad (7)$$

where  $A(t-z/u, z)$  is the complex amplitude of the electric field;  $\beta = (3\pi k\chi^{(3)})/(2n^2)$  is the cubic nonlinearity parameter;  $u$  is the group velocity;  $z$  is the longitudinal coordinate;  $k_2 = \partial^2 k/\partial\omega^2|_{\omega=\omega_0}$  is the linear material dispersion, which is responsible for pulse broadening in a medium with a refractive index dispersion;  $\omega_0 = 2\pi c/\lambda_0$ ; and  $k$  is the wave vector. For the glass in question, the parameter  $k_2 = 47.1 \text{ fs}^2 \text{ mm}^{-1}$  at 800 nm. Equation (7) can be solved numerically.

Figure 3 shows the distributions of the pulse durations across the Gaussian beam cross section after the quadratic spectral phase correction. For the Gaussian beam passed through the defocusing lens, the quadratic dispersion coefficient  $\alpha$  was chosen to minimise the pulse duration at the edge of the beam, i.e. at  $r = R$  (or  $w$ ), and for the laser parameters presented above it was  $-105 \text{ fs}^2$  ( $-169 \text{ fs}^2$ ). In considering the spectrum broadening of the Gaussian beam in the plane-parallel glass plate, the parameter  $\alpha$  was chosen to minimise the pulse duration in the beam centre and was equal to  $-65 \text{ fs}^2$ .

According to Fig. 3, implementation of the defocusing lens, which is optimal for  $R$  (or  $w$ ) scales, makes it possible to broaden the spectrum and compress the 30-fs Gaussian pulse down to 14 fs (16 fs) with the deviation of the duration 1.5 fs (2 fs). When using the plane-parallel glass plate with the same thickness, the variation of the duration across the beam cross section at the  $R$  spatial scale exceeds 7 fs.



**Figure 3.** Pulse durations across the Gaussian beam vs. the normalised transversal coordinate after the correction of the quadratic phase component of the spectrum for the cases when the  $B$ -integral is accumulated in the plane-parallel glass plate (1) and in the defocusing lens, which are optimal for  $R$  (2) and  $w$  (3) spatial scales.

The presented technique of quasi-uniform temporal compression of the Gaussian beams has one significant disadvantage, which restricts its implementation for pulses with shorter durations. The dispersion properties of glass are as important as values of cubic nonlinearity and refractive index, because the defocusing lens produces the radial group velocity delay. The difference between group and phase velocities does not permit compensating the effect completely. But the right choice of the lens material can minimise it. For the lens in question (optimal for the  $R$  scale) the group velocity delay of the beam centre and its edges ( $r = R$ ) after propagation through the defocusing lens and the off-axis parabolic mirror does not exceed the duration of the initial pulse under an assumption that the telescope preserves the plane wave front. Moreover, at present there has been developed the technique of almost complete correction of the radial group delay for ultrashort pulses [17].

### 4. Conclusions

In this paper, a method for accumulating a quasi-uniform nonlinear phase across the Gaussian beams with a peak intensity of a few  $\text{TW cm}^{-2}$  using defocusing lenses with spherical surfaces has been proposed and investigated. The accumulated  $B$ -integral leads to quasi-uniform spectrum broadening across the beam cross section. The beam with the broaden spectrum may be compressed by chirped mirrors introducing only a linear negative chirp. The possibility of quasi-uniform temporal compression of a Gaussian beam with a pulse duration of 30 fs down to 14 fs (16 fs) with the deviation 1.5 fs (2 fs) on the beam radius, which corresponds to 63% (86%) of the total beam energy, has been demonstrated with help of numerical methods. In the framework of the presented model, the observed techniques can be applied to any optical beams with the Gaussian spatial profile. For reliable experimental implementation of the technique it is important to operate with laser beams with a stable intensity distribution.

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