

# Stochastic resonance at a subharmonic of a periodic modulation signal in solid-state lasers

N.V. Kravtsov, E.G. Lariontsev, S.N. Chekina

**Abstract.** The stochastic excitation of a subharmonic of a periodic modulation signal in the intensity spectrum of a solid-state laser is experimentally studied upon modulation of the pump rate by the noise and periodic signal. The stochastic resonance (SR) is observed in the presence of bistability in the laser. The conditions for SR at a subharmonic of the periodic modulation signal are determined.

**Keywords:** ring laser, nonlinear dynamics, stochastic resonance, relaxation frequency, noise modulation, subharmonic of a periodic modulation signal.

## 1. Introduction

The stochastic resonance (SR) reveals a constructive role of noise, which means that the action of random noise oscillations enhances the response of a system to a periodic modulation signal [1, 2]. The SR was observed in many dynamic systems with bistable states (in particular, in a ring laser [3, 4]). In some works, it was shown that SR is also possible in the absence of bistability [4–7]. Previous studies showed that SR can occur both at the fundamental frequency of the modulation signal  $f_p$  and at its higher harmonics [6, 8, 9].

In [10], the excitation of relaxation oscillations in a solid-state laser was studied in the case of parametric resonance, when the  $f_p/2$  subharmonic of the periodic pump modulation signal is close to the frequency of relaxation oscillations  $f_r$ . In the experiments described in this paper, noise modulation was performed in addition to periodic modulation. As a result, we observed the stochastic excitation of a subharmonic of the periodic modulation signal in the intensity spectrum of a solid-state laser and observed SR at the  $f_p/2$  subharmonic of the modulation signal.

## 2. Experimental setup

As the object of study, we chose a monoblock Nd:YAG chip laser with a nonplanar cavity operating at a wavelength of 1.06  $\mu\text{m}$ . The geometric perimeter of the ring laser was 28 mm, and the nonplanarity angle was 85°. The laser was pumped by a semiconductor laser diode ( $\lambda = 0.810 \mu\text{m}$ ). The

diode supply circuit included a generator of periodic oscillations, which modulated the pump power of the studied laser, and a noise generator. The noise spectral width was 200 kHz.

In the process of experiments, the pump noise intensity was changed by changing the output voltage of the noise generator from zero to a maximum value determined by the maximum electric noise intensity at the exit of the noise generator, which reached  $10^{-6} \text{ W Hz}^{-1}$ . When processing the experimental data, the pump noise intensity  $D$  was measured in relative units (this unit corresponds to the electric noise intensity at the generator exit of  $10^{-7} \text{ W Hz}^{-1}$ ).

As was shown in our previous investigations [11], the lasing regimes of a ring chip laser can be efficiently controlled using an external magnetic field created by a constant magnet. The laser under study operated in the self-modulation regime of the first kind in the absence of an external field and in the unidirectional steady-state regime when an external field was applied. The switching from the self-modulation regime of the first kind to the unidirectional lasing occurred due to a significant amplitude nonreciprocity induced in the ring cavity by a magnetic field.

In experiments, we recorded the temporal and spectral characteristics of the laser radiation intensity in the unidirectional lasing regime at different noise powers. The signals were recorded using a 20-12-PCI analogue-to-digital converter and a Tektronix TDS 2014 broadband digital oscilloscope.

## 3. Experimental results

Upon periodic modulation, the excess of pumping over the threshold is

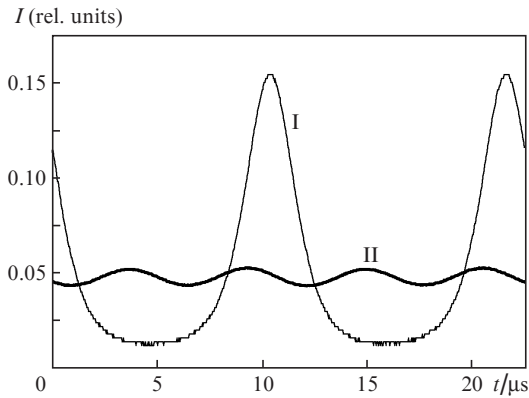
$$\eta(t) = \eta_0 + h \sin(2\pi f_p t), \quad (1)$$

where  $\eta_0$  is the excess of pumping over the threshold in the absence of pump modulation and  $h$  is the pump modulation depth.

The main part of experiments was performed at a constant excess of pumping over the threshold ( $\eta_0 = 0.1$ ) and a constant modulation depth ( $h = 0.05$ ); the frequency of relaxation oscillations was  $f_r = 89 \text{ kHz}$  and the periodic pump modulation frequency was  $f_p = 180 \text{ kHz}$ .

As was shown in [10], in the absence of external noise ( $D = 0$ ), the laser response to the periodic pump modulation is bistable: there are two branches of bistable states (branches I and II). At the modulation frequencies  $f_p < 182 \text{ kHz}$ , sinusoidal modulation of radiation intensity with the frequency  $f_p$  takes place on branch II, while the intensity modulation on

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**Figure 1.** Oscillograms of the laser radiation intensity for branches I and II at the pump modulation frequency  $f_p = 180$  kHz in the absence of external noise.

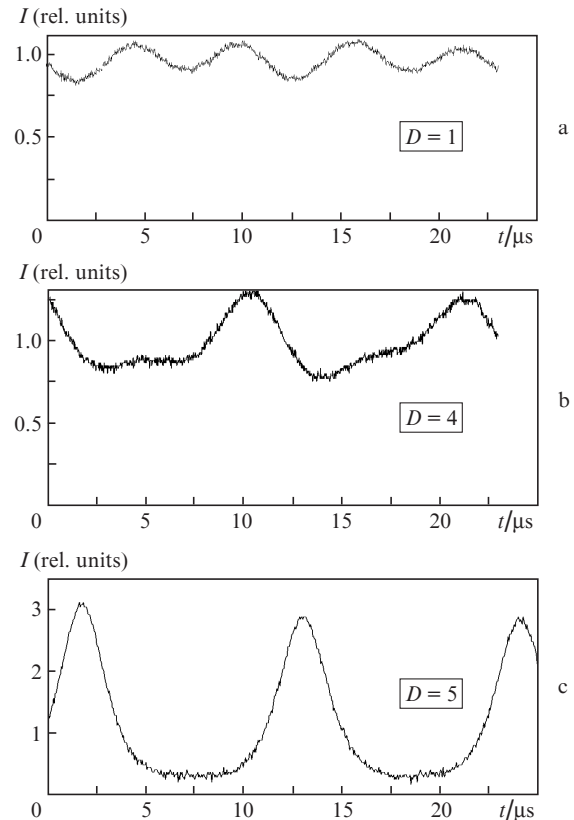
branch I (within the frequency range  $120 < f_p < 195$  kHz) occurs with a double period. Figure 1 presents the oscillograms of radiation intensity for branches I and II at  $f_p = 180$  kHz.

### 3.1. Stochastic excitation of a subharmonic of a periodic modulation signal

In the experiments performed in the present work in the absence of an external noise created by the generator, we chose the initial laser state corresponding to branch II. This state always appears in the absence of an external noise if the pump modulation frequency is smoothly increased from an initial (below 120 kHz) to a working point (180 kHz).

The investigations showed that, in the presence of an external noise, the laser response considerably changes with increasing external noise intensity  $D$ . Figure 2 shows the radiation intensity oscillograms measured at three typical values of  $D$ . At  $D < 1.5$  (Fig. 2a), the laser response is close to sinusoidal with a period equal to the modulation signal period  $T_p = 1/f_p$ , while the modulation at the noise intensities exceeding the threshold value ( $D > 1.5$ ) (Fig. 2b) occurs with a double period ( $T = 2T_p = 2/f_p$ ).

Figure 3 shows the radiation intensity spectra  $S(\omega)$  at different external noise intensities. The spectrum in Fig. 3a is obtained at  $D = 0$ . In this case, the observed response is caused by the intrinsic noise in the system. From Fig. 3a, one can see that the spectrum contains a discrete component at the modulation frequency  $f_p = 180$  kHz, while the component at the subharmonic frequency  $f_p/2$  is absent. With increasing noise intensity  $D$ , the laser response to noise considerably increases in the frequency region close to the relaxation frequency  $f_r$  and to the doubled relaxation frequency (Fig. 3b). This increase in the noise response is explained by prebifurcational noise rise [12]. Investigations performed in this work showed that an increase in the noise intensity (in the region of  $D > 1.5$ ) causes, along with the parametric amplification of the noise response, the stochastic excitation of the  $f_p/2$  subharmonic of the modulation signal. In this region, the amplitude  $S(f_p/2)$  of the spectral component at the subharmonic frequency differs from zero and dominates in the radiation intensity spectrum. As far as we know, the effect of the stochastic excitation of the modulation signal subharmonic has not been studied previously.



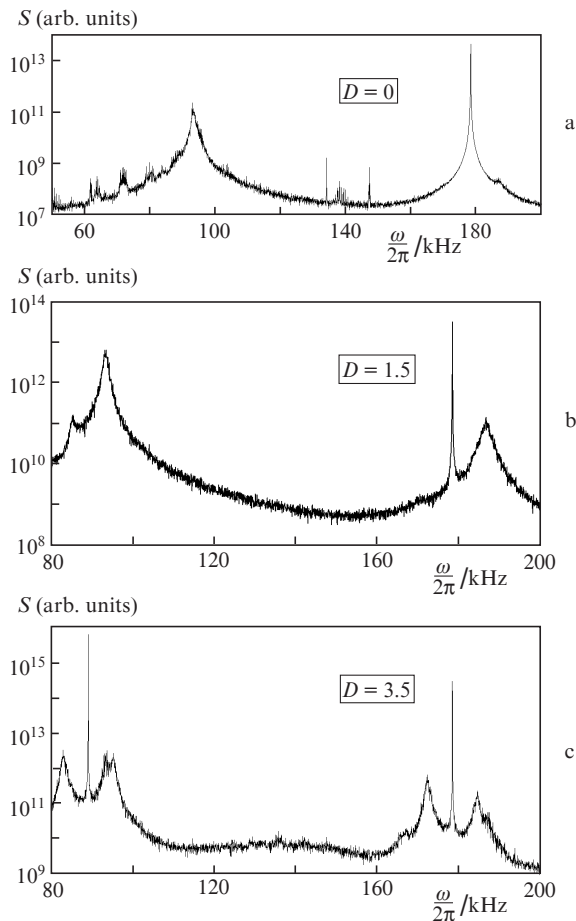
**Figure 2.** Oscillograms of the laser radiation intensity at the external noise intensities  $D =$  (a) 1, (b) 4 and (c) 5.

### 3.2. Stochastic resonance

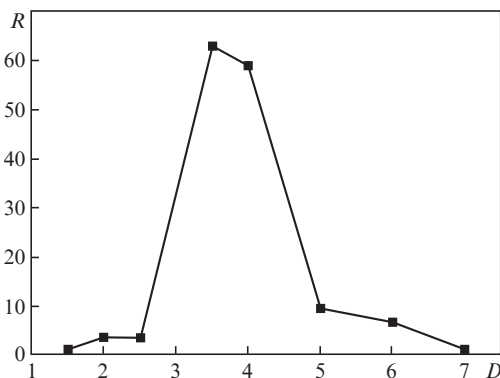
We experimentally found that the laser response at the modulating signal subharmonic  $f_p/2$  due to stochastic excitation increases with increasing noise intensity. From the  $S(\omega)$  spectra measured at different noise intensities  $D$ , we determined the dependence of the signal-to-noise ratio  $R$  at the  $f_p/2$  frequency on  $D$  (Fig. 4). The shape of this dependence is typical for SR, in particular, the signal-to-noise ratio at the half frequency reaches a maximum at the optimal power of the noise pump component and tends to zero at low and high powers.

In previous studies, SR was observed at the fundamental and higher harmonics of the modulation signal. In contrast, SR in the laser studied in this work was observed at a subharmonic of the modulation signal rather than at its higher harmonics. In [4], SR in a solid-state laser was observed at the fundamental harmonic of the modulation signal in the absence of bistability. The SR at the modulation signal subharmonic studied in this work appears only in the case of a bistable response of the laser to the periodic pump modulation signal. Note that the optimal noise intensities in the absence [4] and presence (this work) of bistability are close to each other (the difference is about 50%). Similar to [4], the laser response to the pump modulation by a periodic signal and noise is analogous to the response of a nonlinear Toda oscillator. Based on the investigations performed in this work, we can assert that the effects observed by us (stochastic excitation of the modulation signal subharmonic and SR at the subharmonic) may also occur in other nonlinear oscillators.

The possibility of SR at a subharmonic strongly depends on the periodic signal amplitude. The dependence  $R(D)$  given



**Figure 3.** Radiation intensity spectra  $S(\omega)$  at different external noise intensities.



**Figure 4.** Dependence of the signal-to-noise ratio  $R$  at the  $f_p/2$  subharmonic frequency on the noise intensity.

in Fig. 4 is obtained at the pump modulation depth  $h = 0.05$ . SR was also observed at a twofold lower modulation depth, but the maximum signal-to-noise ratio was considerably smaller ( $R_{\max} = 3$ ).

Thus, we experimentally observed stochastic excitation of a subharmonic of the periodic signal modulating the pump power in a solid-state laser. In the studied range of laser parameters, there exist a bistability of the laser response to the periodic modulation signal, i.e., two states (branches I and II) are observed. Our investigations showed that the sto-

chastic resonance in the laser occurs at the  $f_p/2$  subharmonic of the periodic modulation signal.

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