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Lasing thresholds of helical photonic structures with different positions of a single light-amplifying helix turn

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Abstract. **Numerical simulation is used to assess the lasing threshold of helical structures of cholesteric liquid crystals (CLCs) in which only one turn amplifies light. This turn is located either in the centre of symmetric structures of various sizes or in an arbitrary place in asymmetric structures of preset size. In all cases, we find singularities in light amplification by a one-dimensional CLC structure for the most important band-edge modes (m1, m2 and m3) and plot the threshold gain coefficient k_{th} against the position of the amplifying turn. For the symmetric structures, the lasing threshold of the m1 mode is shown to vary linearly with the inverse of the square of the cavity length. Moreover, modes with a lower density of photonic states (DOS) in the cavity may have a lower lasing threshold. This can be accounted for by the dependence of the density of photonic states on the position of the amplifying turn and, accordingly, by the nonuniform electromagnetic field intensity distribution along the cavity for different modes. In the asymmetric structures, the same field energy distribution is responsible for a correlation between k_{th} and DOS curves.**

Keywords: *lasing, light amplification, cholesteric liquid crystals, simulation.*

1. Introduction

Cholesteric liquid crystal (CLC) lasers have been the subject of intense studies [1] because the helical periodic structures of CLCs are very similar in essence to classic photonic crystals, including three-dimensional (3D) ones. In particular, the cholesteric blue phase is 3D. It is because of the chiral symmetry of the cholesterics that new photonic effects would be expected in them. In most studies concerned with liquid crystal lasers, use is made of a uniform dye distribution over the entire cholesteric structure. Under pumping, the dye ensures spontaneous emission amplification and the CLC acts as a spatially distributed feedback cavity. As shown earlier for lasers with such feedback [2], the gain coefficient corresponding to the lasing threshold, k_{th}, decreases with increasing cavity length, d, according to the relation $k_{th} \propto d^{-3}$. As a rule, lasing occurs at the frequency of one of the electromagnetic modes at the edges of the band gap (stop band). Other studies addressed the influence of defects in the helical structure of CLCs on their lasing. Possible defects are a phase jump in the helix [3, 4] and the insertion of a thin isotropic [5, 6] or nematic [7] layer, including one with a controlled phase delay [8], into the CLC structure. Such defect layers typically produce one or a few very narrow spectral transmission bands in the band gap.

In this work, we use numerical simulation to examine the effect of the position of a single light-amplifying turn in a cholesteric helix on optical amplification and lasing conditions in CLC structures. Such a 'hot' turn does not disturb the phase of the helix and, according to simulation results, nor does it produce extra spectral features in the band gap. For this reason, as shown below a 'hot' turn may act as a probe of the distribution of the electromagnetic field of eigenmodes or the density of photonic states when it is displaced along a 'cold' (without amplification) CLC structure. From the practical viewpoint, such studies can be useful in optimising the arrangement of dye layers in order to achieve the lowest possible single-mode lasing threshold in liquid crystal lasers.

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2. Simulation

In simulation, we used an algorithm developed earlier by exactly solving Maxwell's equations for layered media using 4×4 Berreman matrices [9]. We consider test light beam amplification in two types of structures (Fig. 1), whose cross-sectional dimensions are taken to be infinite. For definiteness, let a test beam of white light with a normalised intensity I_0 = 1 pass from right to left exactly along the z axis of a cholesteric helix with a pitch P_0 = 0.4 μm and principal refractive indices $n_{\parallel}$ = 1.6 and $n_{\perp}$ = 1.5 (the symbols ∥ and ⊥ refer to the

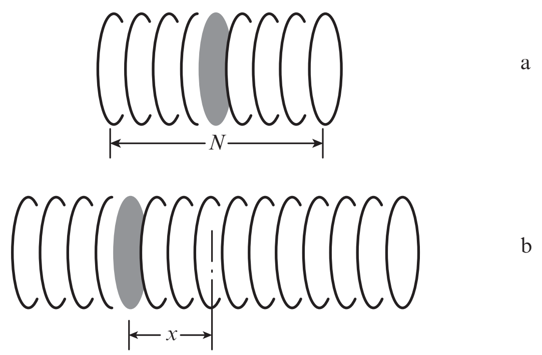

Figure 1. (a) Symmetric cholesteric structure with a number of helix turns, N, from 5 to 33 and a single light-amplifying ('hot') turn in the centre; (b) asymmetric structure with a fixed number of turns N = 30 (an arbitrary number of turns is shown) and a single 'hot' turn whose position x in the 'cold' CLC structure is varied without disturbing the phase of the helix. Distance x is measured from the centre of the structure.

local director orientation in the helical structure). Only a 'hot' turn, inserted into the cholesteric with no phase discontinuities, has wavelength-independent principal gain coefficients $k_{\parallel}$ and $k_{\perp}$ ($k_{\parallel} = 4k_{\perp}$). At the CLC output, the amplified light intensity is I. The gain of the entire structure is $K(k_{\parallel}) = I/I_0$. To minimise boundary effects, the structures under investigation are immersed in an isotropic medium with a refractive index n = 1.55. One type of structure (Fig. 1a) comprises five symmetric CLC structures of different lengths (number of turns N = 5, 9, 17, 25 and 33), each having one 'hot' turn in its centre. The other, asymmetric type of structure (Fig. 1b) has a constant number of turns, $N = 30$, with a variable position x of a single 'hot' turn to the left or right of the centre at $x = 0$.

3. Symmetric structures with a 'hot' turn in their centre

Consider first the former, symmetric type of structure. For clarity, the inset in Fig. 2 shows a characteristic transmission spectrum of a CLC consisting of 25 turns with the parameters indicated above. The gain coefficients $k_{\parallel}$ and $k_{\perp}$ are zero. The three band-edge modes (m1, m2 and m3) of interest here are situated at the long-wavelength edge of the band gap and, when there is no amplification, their wavelengths are 643.8, 652.8 and 664.6 nm, respectively. Let us consider now one turn in the centre and gradually increase $k_{\parallel}$ and $k_{\perp}$ of this turn, whereas the 12 turns to the right and 12 turns to the left of it remain 'cold'. As a result, we observe a sharp increase in the test beam gain $K(k_{\parallel})$ for the three band-edge modes, and the peaks in the curves correspond to the onset of lasing at $k_{\parallel} = k_{\parallel\mathrm{th}}$ (with decreasing sampling increment, we observe test beam gain singularities at these points). The corresponding data are presented in Fig. 2 as semilog plots. It is known that, if the entire layer of a CLC can amplify, the first band-edge mode m1 has the lowest lasing threshold. This is so in our case as well (Fig. 2). Note, however, that the m3 mode has a considerably lower threshold than does the m2 mode. This result is paradoxical because the density of states and, accordingly, the cavity finesse for the m2 mode exceed those for the m3 mode. An explanation for this is presented in Fig. 3, which qualitatively illustrates the electromagnetic field intensity (E^2) distribution along the length of the structure for the three band-edge modes in classic photonic crystals [3] and CLCs [10]. Indeed, in the central part of the structure, where the 'hot' turn is located, the E^2 of the m3 mode exceeds that of the m2 mode, which is responsible for the lower lasing threshold of the third mode. It is of interest to note that, in shorter symmetric structures, lasing on the m3 mode does not develop at $k_{\parallel}$ values of up to ~12 μm^{-1}.

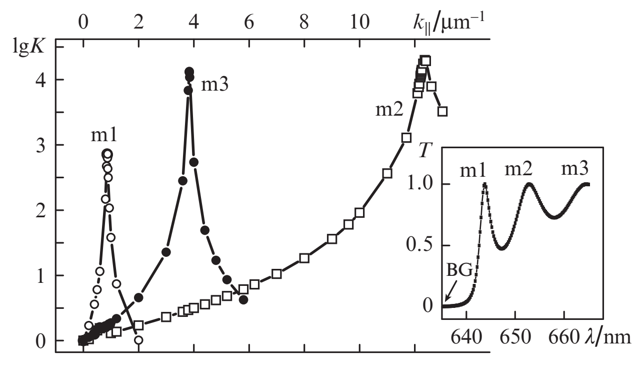

Figure 2. Semilog plot of the test beam gain $K = I/I_0$ against the gain coefficient $k_{\parallel}$ of the central, 'hot' turn (**1**) for a 12-**1**-12 symmetric 25-turn CLC structure. The maxima in lg$K$ lie at $k_{\parallel} = k_{\parallel\mathrm{th}}$ singularities corresponding to the onset of lasing on the modes chosen. Inset: transmission spectrum of the CLC structure with no amplification. Only part of the band gap (BG) and the edge modes (m1, m2 and m3) at its long-wavelength edge are shown.

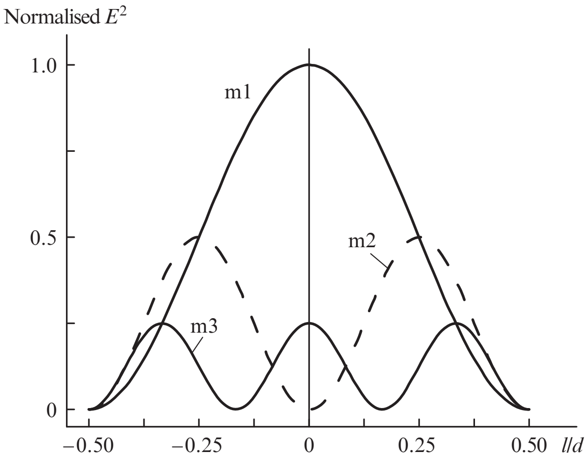

Figure 3. Electromagnetic field intensity (E^2) distributions along the length of a CLC structure for the m1, m2 and m3 band-edge modes (d is the length of an arbitrary structure).

Reducing the total length of the symmetric structures from d = 13.2 μm (33 turns) to d = 2 μm (5 turns), we obtain a set of five $K(k_{\parallel})$ curves for either mode. In the case of the m2 mode, all the singularities corresponding to different CLC cavity lengths are concentrated in a relatively narrow range of gain coefficients: $k_{\parallel}$ = 10–13 μm^{-1}. For this reason, Fig. 4 presents lg$K(k_{\parallel})$ data only for the m1 mode, and the threshold gain coefficient $k_{\parallel\mathrm{th}}$ increases with decreasing cavity length. We observe a well-defined linear relation between $k_{\parallel th}^{-1}$ and the square of the cavity length (d^2): $k_{\parallel th}^{-1}/d^2 = 0.16 + 0.01d^2$ (in

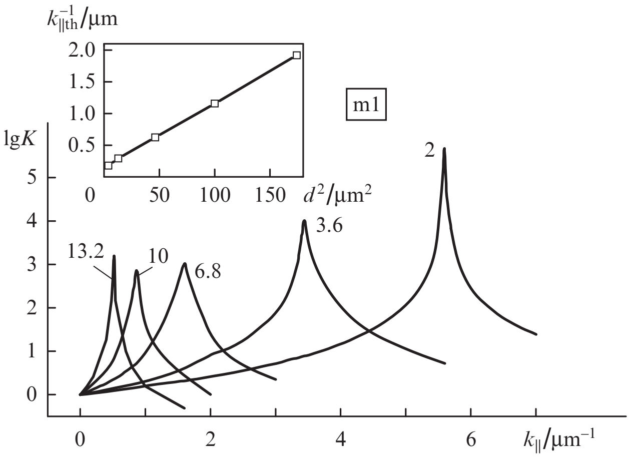

Figure 4. Singular dependences of the test beam intensity gain on the gain coefficient $k_{\parallel}$ of the 'hot' turn in the centre of the CLC layer for the m1 band-edge mode. The numbers at the curves specify the length of the structures. Inset: inverse threshold gain coefficient $k_{\parallel\mathrm{th}}^{-1}$ against the square of the length of the symmetric CLC structures.

inverse microns) (Fig. 4, inset). Compare now the $k_{\|\mathrm{th}}(d) \propto d^{-2}$ data obtained for these structures (cavities with an amplifying central turn) to the well-known relation $k_{\mathrm{th}}(d) \propto d^{-3}$ obtained previously for one-dimensional photonic structures with a uniform gain throughout the cavity [2]. Note that the exponents differ exactly by unity (−2 and −3, respectively).

4. Asymmetric structures with a relocatable 'hot' turn

Consider now asymmetric structures of constant length d = 12 μm (30 turns). The light-amplifying turn is situated at different distances x from the centre of the structure. There are eight such structures, with the 'hot' turn located to the left of their centre ($x < 0$) at intervals through each 'cold' turn from $x = 0$ to the left edge of the structure. Simulation showed that the optical properties of structures having a 'hot' turn to the right of their centre ($x > 0$) were completely identical to those of the eight 'left-handed' structures. In view of this, only the properties of the nine structures with $x \leqslant 0$ were examined in detail.

Figure 5 shows the gain as a function of the gain coefficient $k_{\|}$ of the 'hot' layer in a CLC for the m1 and m2 band-edge modes. All nine have singularities. Only seven curves are left in each panel in order not to clutter Fig. 5. Note that, in the case of the m1 mode (Fig. 5a), the $k_{\|\mathrm{th}}(x)$ threshold, corresponding to a singularity, steadily increases with increasing distance from the 'hot' turn to the centre of the structure (from $|x| = 0$ to $|x| = 6$ μm), whereas for the m2 mode (Fig. 5b) there is a minimum ($k_{\|\mathrm{th}}$ first decreases as $|x|$ increases from 0 to 3.2 μm, and then, by contrast, increases with an increase in $|x|$ from 3.2 to 6 μm). This is more clearly shown in Fig. 6a for the 17 structures with $x < 0$, $x = 0$ and $x > 0$ (accordingly, each curve has 17 data points). Note that, in some of the asymmetric structures, an increase in $k_{\|}$ was observed to result in repeat singularities such as beating at a frequency intermediate between the band-edge mode frequencies, whose origin is not yet clear. However, we detected no additional transmission bands (defect modes) in the band gap of the symmetric or asymmetric structures containing a light-amplifying turn.

As seen in Fig. 6a, the $k_{\|\mathrm{th}}(x)$ data can be correlated with the density of photonic states (DOS) for the two band-edge modes. To this end, we should return to the many amplified

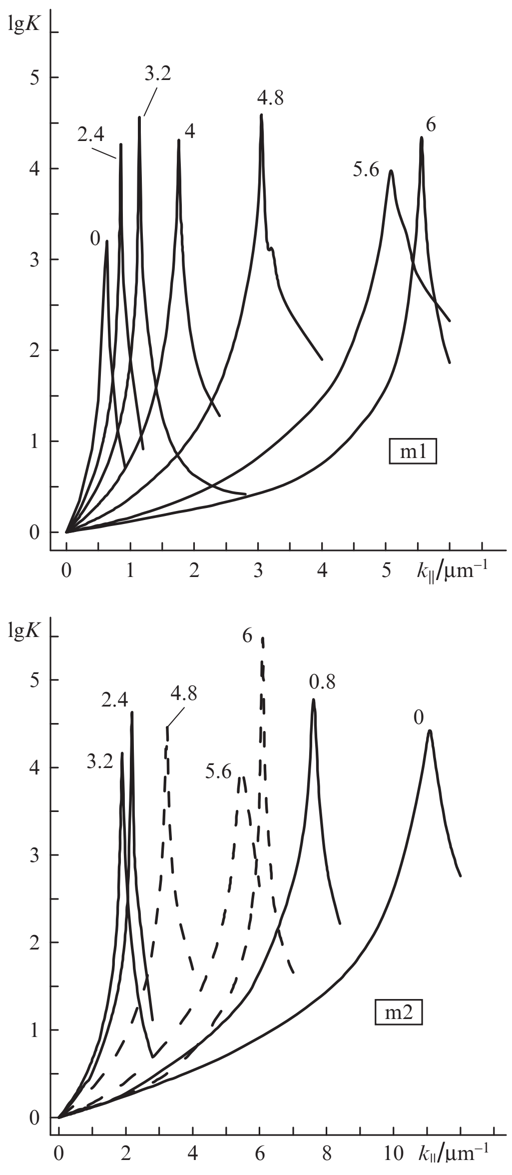

Figure 5. Singular dependences of the optical gain on the gain coefficient $k_{\|}$ of a 'hot' layer that is displaced along a 'cold' CLC structure for the (a) m1 and (b) m2 band-edge modes of some asymmetric structures. The numbers at the curves specify the distance $|x|$ from the centre of the structure to the position of the 'hot' turn (in microns). Note the increase in $k_{\|\mathrm{th}}$ for $|x| > 3.2$ μm in Fig. 5b (the corresponding data are represented by dashed lines).

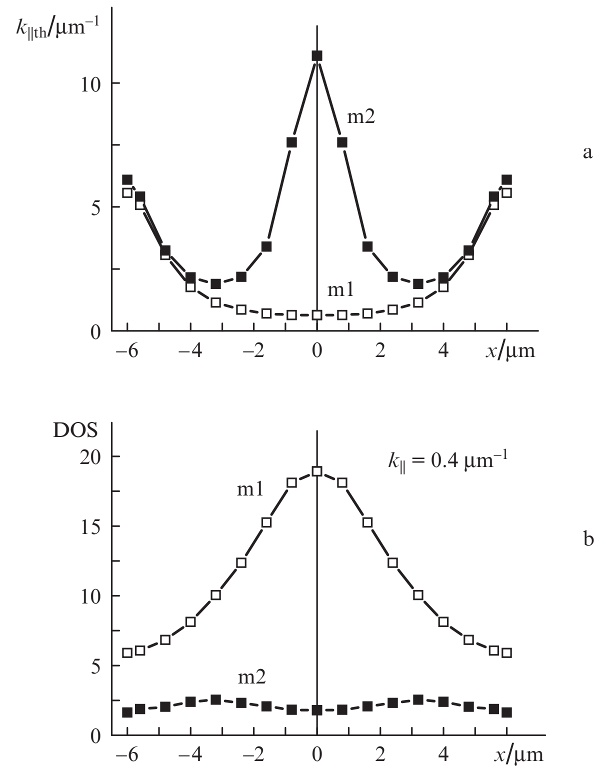

Figure 6. (a) Threshold gain coefficient $k_{\|\mathrm{th}}$ as a function of distance x from the 'hot' turn to the centre of the CLC structure (see Fig. 1b) for the m1 and m2 modes; (b) density of photonic states as a function of x for the m1 and m2 modes at a gain coefficient $k_{\|} = 0.4$ μm^{-1}, which is smaller than the smallest coefficient $k_{\|\mathrm{th}}$ for the m1 and m2 modes. Each data point corresponds to one position of the 'hot' turn in the CLC layer.

light intensity spectra for all the asymmetric structures (these spectra are not presented because of space limitations). Let us consider a group of spectra with an arbitrary fixed $k_{\|}$ coefficient smaller than any threshold value (e.g. $k_{\|} = 0.4\ \mu m^{-1}$) and find $\Delta\lambda$ (full width at half maximum) of the m1 and m2 spectral modes from these spectra. Next, we convert to frequency increments,

$$\Delta\omega = -\frac{2\pi c_0 \Delta\lambda}{\lambda^2}$$

(where c_0 is the speed of light in vacuum), and write down the DOS in the form [11]

$$\mathrm{DOS} = \frac{\partial k(\omega)}{\partial\omega} = \frac{1}{v_g} = \frac{\Delta\tau}{d} \approx \frac{2}{\Delta\omega d}.$$

Here, k is the wave vector of the light in the CLC; v_g is the group velocity of the light; and $\Delta\tau$ is the group propagation time (dwell time) of the light through the structure under investigation (of length d). On the right-hand side of this relation, we use the uncertainty principle for a Lorentzian spectral line: $\Delta\tau\Delta\omega \approx 2$ [1]. As a result, we obtain DOS vs. x curves for the m1 and m2 band-edge modes at $k_{\|} = 0.4\ \mu m^{-1}$ (Fig. 6b).

5. Conclusions

It is worth noting that the dependence of the DOS on the position of the light-amplifying turn correlates well with the field intensity distributions in Fig. 3 (see also Ref. [1]). Moreover, these distributions are strikingly similar to the theoretical picture of the variation in the intensity of light propagating from a point source moving in a one-dimensional photonic crystal [12]. We think that it is the increased density of states, the inverse of which determines the group velocity of light, which is responsible for the increased local electromagnetic field intensity.

In conclusion, note that the main result of this study is the finding that the density of states and, accordingly, the group velocity of light depend on the x-coordinate in a limited CLC layer. The spatial group velocity distribution, which determines the field intensity distribution, varies from mode to mode. In particular, the m1 mode has the maximum density of states and minimum group velocity in the centre of the CLC layer, whereas the group velocity of the m2 mode has a maximum in the centre of the layer and its minimum values correspond to a distance of a quarter of the thickness of the layer from its centre. The same DOS (and field intensity) distribution is responsible for the variation of the gain coefficient $k_{\|th}$ with the distance x from the position of the 'hot' turn to the centre of the CLC structure for the m1 and m2 modes. Thus, a 'hot' turn that can be displaced along a CLC structure does probe its properties. Another important result of this study is that we have found the relation $k_{\|th}(d) \propto d^{-2}$, which describes the threshold for lasing on the m1 band-edge mode in a wide range of thicknesses of symmetric CLC structures having an amplifying layer (helix turn) in their centre. We believe that the present results can be useful in further studies and particularly in designing liquid crystal lasers.

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