

CONTROL OF LASER RADIATION PARAMETERS

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Theoretical analysis of phase locking in an array of globally coupled lasers

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Abstract. **A model of an array of globally coupled fibre lasers, with the same fraction of the total output beam power injected into each laser, is considered. Phase self-locking of the laser array makes it possible to increase the brightness of the total output beam without any devices for controlling the phases of output beams, which significantly complicate the laser system. The spread of the laser optical lengths is several hundreds of wavelengths (or even more); within the theory of hollow cavities, this spread should lead to a fast decrease in the total power with an increase in the number of lasers. The presence of the active medium may reduce this drop to a great extent due to the self-tuning of the laser array radiation wavelength to a value providing a maximum gain for the array lasing mode. The optical length of each element is assumed to be random. The increase in the phase-locking efficiency due to the gain saturation is explained based on the probabilistic approach. An iterative procedure is developed to find the array output power in the presence of steady-state phase locking. Calculations for different values of small-signal gain and the output-power fraction spent on global coupling are performed. It is shown that, when this fraction amounts to ~20%–30%, phase locking of up to 20 fibre lasers can be implemented with an efficiency as high as 70%.**

Keywords: *laser array, fibre amplifier, phase locking, global coupling.*

1. Introduction

Currently the output power of fibre lasers with a beam quality close to the diffraction one reaches 10 kW due to the use of multistage amplifiers and a tandem pump circuit [1, 2]. The necessity of maintaining single-mode lasing limits the lasing-mode area. Such nonlinear processes as stimulated Brillouin scattering, stimulated Raman scattering, and self-focusing [3] restrict the maximum intensity. A further increase in the total power of an output beam with diffraction divergence can be implemented by designing arrays of amplifiers or lasers with subsequent beam addition on a common output aperture.

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To date, a number of architectures for adding beams emitted by array elements have been investigated. We should note as an individual class systems with spectral beam addition [4], in which each element generates at its intrinsic frequency and the beams from individual elements are added on a common diffraction grating to form a single output beam.

Phase locking of laser sources at the single frequency of the array collective mode makes it possible to obtain lasing at a single wavelength; it is believed to fit better for phase locking of two-dimensional arrays. There are two main approaches to beam addition in an array: (i) active control of the parameters of each laser beam and (ii) phase self-locking of the entire array. The phase-locking methods based on active control are compatible with output-beam control systems and are convenient in use. However, the additional optical equipment and electronic control blocks make the system more complicated and expensive [5]. Currently, passive phase locking, which is based on the internal properties of system, is being actively investigated as one of alternative ways for developing high-power laser systems.

Passive phase locking is implemented either due to the diffraction coupling in multicore fibres or by coupling fields in an external spatial device (see reviews [6, 7]). In multicore lasers [8, 9], an increase in the multicore diameter gives rise to a spectrum of competing supermodes that are retained in the multimode fibre. As a result, the optical quality of the total beam deteriorates. The question about the main factors limiting the size of a laser array with a supermode selected by an external filter remains open to a certain extent.

To implement stable phase locking, one must, on the one hand, exclude independent lasing of individual lasers and, on the other hand, suppress the competition between the array supermodes. It was shown in [6] for a small spread of optical lengths of individual cavities that, when lasing occurs at the fundamental in-phase supermode, global coupling [10] suppresses other supermodes to a great extent. Global coupling can be implemented in different ways. One of them is coupling through fibre X couplers (2×2 coupler) [11–13], in which one of the outputs is used to organise feedback, while the other provides out-of-phase field loss. The drawbacks of this architecture are the extraction of radiation from the entire array into a single-mode fibre and instability of lasing in time [14]. Fourier coupling [15], which is based on the diffraction of output laser beams in a semiconfocal cavity composed of a reflecting mirror and end faces of emitters forming a periodic grating, is characterised by a degree of supermode selection close to that provide by global coupling. It appears realistic to use adders and couplers for a fibre laser array to add N beams in the coupling system with their subsequent distribution over the laser array [16].

The main factor impeding passive phase locking of active elements is the spread of the optical lengths of elements (OLEs), which leads to a difference in the phase shifts of the propagating radiation and to destructive interference of fields (Vernier effect [13]). Nevertheless, a high degree of phase locking was obtained at a random OLE spread of few centimetres in a number of experiments [12, 13, 17]. The efficiency of field locking in an array composed of nonidentical elements is provided by several factors, the main of which is the self-tuning of the supermode wavelength within the gain band to the value corresponding to the maximum difference between the gain and loss [18–20]. The selection of lasing modes from the spectrum of amplified spontaneous radiation was numerically investigated for arrays composed of two and four fibre amplifiers, coupled through X couplers [21]. Kouznetsov et al. [22] predicted that stable phase locking can be performed for no more than eight elements. The average locking efficiency reduced to ∼50% in an experiment with a 16-channel fibre array [23]. Numerical calculation of the beam-addition efficiency with application of a multipath interferometer [24] showed that the average addition efficiency for eight channels is 75% at a spread of 25%–30%; this result is in agreement with experiment [14].

In contrast to the studies on phase locking of gas lasers, each having an intrinsic cavity [6, 7], the analysis of a fibre laser array was performed with consideration of a system of single- or double-pass amplifiers in a common cavity. It was shown in [25, 26] that, in the case of an injection-locked laser array, the nonlinear dependences of the output radiation power and phase on the external signal frequency allow one to improve the locking efficiency and increase the number of locked elements.

The dependences of the gain and refractive index of laser media on the field intensity give rise to correlations between the phases and power distribution over the array elements. It was shown for the first time in [27] (within a simplified model of a laser array with nearest neighbours coupled) that the Kerr nonlinearity of the refractive index may provide stable phase locking. Cheo et al. [9] reported spontaneous selection of an in-phase supermode in a fibre laser with seven active cores. In [28] this effect was ascribed to the influence of the resonant part of the nonlinear refractive index, related to the polarisability of excited ions [29]. Later on, detailed numerical calculations [30, 31] showed that the refractive-index nonlinearity plays a less important role in the selection of the in-phase supermode than the gain saturation. Previously, our theoretical analysis [25] revealed that, even in the absence of refractive-index nonlinearity, the gain saturation significantly increases the efficiency of coherent beam addition in a fibre laser array with a random OLE spread, with the same signal injected into each laser.

In this paper, we report the results of studying the joint influence of gain saturation and resonant nonlinearity of refractive index on the efficiency of laser beam addition. A calculation based on a closed model through numerical iterations for an array composed of 20 fibre lasers showed that, when the laser-medium nonlinearity is completely taken into account, the phase-locking efficiency may reach 70%. Section 2 contains the main equations describing a fibre laser array with global external coupling. In Section 3 we report the results of numerical simulation of a laser array with the same signal injected into each laser. The role played by the gain saturation and resonant refractive-index nonlinearity in the phase locking is discussed. The results of calculating the laser array power within the closed model are presented in Section 4.

2. Numerical model of an array of globally coupled fibre lasers

The model design under consideration is an array of single-mode fibre laser elements (Fig. 1). Each element is an active fibre, the length of which is a random value. It is assumed that one of the fibre end faces is an ideal reflector. The second end face plays the role of a semitransparent mirror. Most of the radiation emitted within each element is extracted outside, while the remaining part is directed to the device implementing global coupling between the elements. It is assumed that this device performs coherent addition of all beams, after which the total beam is split into N identical beams to be injected into the array elements through the fibre end faces. There is a certain relationship between the injected signal power and the frequency detuning from the resonance, which ensures stable controlled lasing [32]. If there is a radiation wavelength at which the ranges of injection locking for all lasers are overlapped, one would expect stable phase-locked lasing at this wavelength for all array elements.

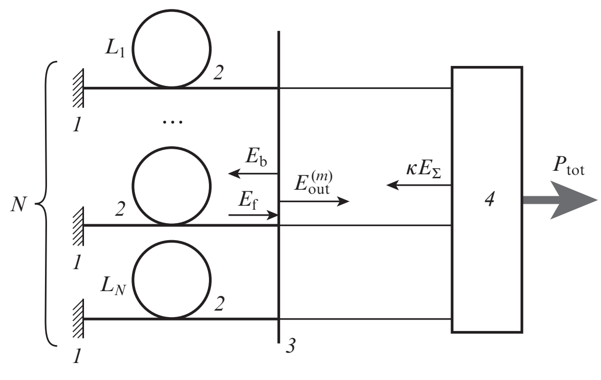

Figure 1. Schematic of an array of globally coupled fibre lasers: (*1*) highly reflecting mirror, (*2*) active fibres, (*3*) output mirror with reflectance r, and (*4*) system for feedback and radiation extraction.

Let us consider the gain saturation in the medium within a very simple model. The dependence of the local gain on intensity is given by the expression

$$g(x, y, z) = g_0/[1 + I(x, y, z)],$$

where I is the field intensity normalised to the saturation intensity and g_0 is the local small-signal gain, which (for simplicity) is assumed to be constant within the active core. Separate calculations showed that the saturation of the mode gain (i.e., the gain averaged over the transverse profile of mode intensity) for a single-mode fibre is described with a high accuracy by the formula $g(z) = g_0/(1 + P)$, where $P = P_f + P_b$ is the power of single-mode radiation normalised to the saturation power obtained in separate calculations. The total beam power in a fibre, with interference of counterpropagating waves neglected, is the sum of their powers $P_{f(b)} = |E_{f(b)}|^2$, where $E_{f(b)}$ are the amplitudes of the waves propagating in the forward (f) or backward (b) directions along the fibre. The effects related to the change in the field polarisation are neglected within this model.

Generally, the fibre refractive index depends on the radiation intensity. In the majority of fibre lasers the phase shift caused by the Kerr effect is small ($n_2 \approx 3\times10^{-20}$ m^2 W^{-1}). The resonant nonlinearity of the refractive index may play a more important role. This component, expressed in terms of the gain through the Kramers–Kronig relation, is proportional to the population inversion [28, 29]. The corresponding phase shift can approximately be written as

$$\alpha\int_0^L g\mathrm{d}z,$$

where L is the fibre length; the optical nonlinearity coefficient α depends weakly on the wavelength within the spectral gain band.

Within the above assumptions the amplitudes of the waves propagating along a single-mode fibre are determined by the equations

$$\frac{\mathrm{d}E_{\mathrm{f(b)}}}{\mathrm{d}z} = \pm\beta E_{\mathrm{f(b)}} \pm \frac{1}{2}g(z)(1+\mathrm{i}\alpha)E_{\mathrm{f(b)}} \pm \mathrm{i}kn_2PE_{\mathrm{f(b)}}, \quad (1)$$

where β is the mode propagation constant for radiation with a wave vector modulus k; the plus and minus signs refer, respectively, to the forward and backward directions. Having separated the real and imaginary parts, one can obtain the equations for the powers of counterpropagating waves,

$$\frac{\mathrm{d}P_{\mathrm{f(b)}}}{\mathrm{d}z} = \pm g(z)P_{\mathrm{f(b)}} \quad (2)$$

and the expression for the phase delay per run due to the nonlinear part of the refractive index [33],

$$\varphi_{\mathrm{nl}} = \alpha G + \frac{n_2kL}{G_0}P_{\mathrm{b}}(L)$$

$$\times\{\exp(2G) - 1 + P_{\mathrm{b}}(L)\exp(-2G)[2G + \sinh(2G)]\}, \quad (3)$$

where

$$G = \int_0^L g(z)\mathrm{d}z; \;\; G_0 = g_0L; \;\; P_{\mathrm{b}}(L) = \frac{\exp(2G)-1}{G_0 - G}.$$

In the absence of loss in the fibre, the powers of the injected (P_{inj}) and output (P_{out}) beams are related by the expression

$$P_{\mathrm{out}} = |E_{\mathrm{out}}|^2 = P_{\mathrm{inj}} + G_0 - G. \quad (4)$$

The total gain G in a laser controlled by an external signal should be below the threshold, i.e., $r\exp G < 1$, where r is the amplitude reflectance from the output fibre end face. In the opposite case the laser can generate not only at the external-signal frequency but also at the cavity eigenfrequencies, which will lead to a complex lasing dynamics of the entire array. We will restrict ourselves to establishment of the conditions under which steady-state generation of a phase-matched field can be implemented with some probability. The critical power of the external signal, with excess of which the lasing of a controlled laser is stably injection-locked, is determined by the expression [32]

$$P_{\mathrm{cr}} = \frac{4r^2}{t^4}(G_0 - G_{\mathrm{th}}),$$

where $t = \sqrt{1-r^2}$, and $G_{\mathrm{th}} = \ln(1/r)$ is the total threshold gain. For a typical value of reflectance from the fibre end face, $r = 0.2$, $G_{\mathrm{th}} \sim 1.61$. In the mode of stable injection locking of a laser by external signal, the output field amplitude can be written as

$$E_{\mathrm{out}} = \frac{\exp G - r\exp(-2\mathrm{i}\varphi)}{\exp(-2\mathrm{i}\varphi) - r\exp G}\sqrt{P_{\mathrm{inj}}}, \quad (5)$$

where $\varphi = \beta L + \varphi_{\mathrm{nl}}$. We should note that the spectrum of output radiation contains resonant peaks at the cavity eigenfrequencies ($\varphi = l\pi$, where l is an integer). Note also that the refractive-index nonlinearity may lead to ambiguity of the solutions to Eqn (5) (see, for example, [34]).

Expressions (3)–(5) allow one to calculate the output field amplitude for each element with the following specified parameters: radiation wavelength, OLE, injected-signal power, small-signal gain, and coefficients of the refractive-index nonlinearity. For a laser array with a fixed random sample of OLE values, the fields of the output beams are determined from formulas (3)–(5). The total field at the coupling-device output is found from the formula

$$E_{\Sigma} = \frac{1}{\sqrt{N}}\sum_{m=1}^{N}E_{\mathrm{out}}^{(m)}, \quad (6)$$

where m is the laser number.

The power of the signal formed in the global-coupling system and injected into each element is given by the expression $P_{\mathrm{inj}} = |\kappa E_{\Sigma}|^2/N$, where κ is the feedback coefficient. Thus, the coupling between the injected-field amplitudes after the nth circular roundabout of the entire system can be described by the expression

$$E_{\mathrm{inj}}^{(n+1)} = \frac{\kappa}{\sqrt{N}}\sum_{m=1}^{N}\frac{\exp G_m - r\exp(-2\mathrm{i}\varphi_m)}{\exp(-2\mathrm{i}\varphi_m) - r\exp G_m}E_{\mathrm{inj}}^{(n)}. \quad (7)$$

If Eqn (7) has a solution, the stable radiation power can be found using iterations. For a random OLE sample, the parameters of array elements are fixed. The additional common parameters are the small-signal gain G_0 and the fraction κ^2 of the total output power spent to implement feedback. Before finding a self-consistent solution to the system of equations (3)–(7), it is reasonable to study the properties of the sum in the right-hand side of Eqn (7). The radiation wavelength, which enters the propagation constant, significantly affects the result of addition of the output beams of the elements. As was shown in a number of studies [13, 17, 18, 21, 22], devoted to the properties of the total field of coherent beams transmitted through media with different optical lengths, the total field of the beams summed on the common aperture depends strongly on the wavelength.

3. Influence of the refractive-index nonlinearity on the field-addition efficiency

The possibility of phase locking of an array of globally coupled lasers is primarily determined by the efficiency of field addition in the feedback circuit. It was suggested by some researchers [12, 17, 35] that the resonant nonlinearity of refractive index may be the main factor positively affecting the phase locking of array lasers. We showed in [25] that the gain saturation, with refractive-index nonlinearity neglected, improves significantly the phase-locking efficiency for a globally coupled array.

To reveal the peculiar role of the refractive-index nonlinearity, we will consider the same system of single-mode fibre lasers as that analysed in [25]. The lengths of amplifying fibres are set by the expression $L_m = 10 + 0.1m + \delta l_m$ (in m), where δl_m is a random detuning of the fibre length for an mth laser. The refractive index of the medium is 1.5.

We took the value $P_{\rm cr}/2$ to be the characteristic power of the injected signal. At this power lasers are injection-locked by an external signal at a phase-delay detuning from the resonance by $|\varphi_m| < \pi/4$. If phase detunings do not fall in this range, the gain remains higher than the threshold value, and a laser can generate independently.

Numerical calculations were performed in the following order. First, the amplitude and phase of the output field were determined numerically for each element from the system of transcendental equations (3)–(5) at a specified injected-radiation wavelength from the range $[\lambda_0 - \delta\lambda_{\rm max}, \lambda_0 + \delta\lambda_{\rm max}]$ (λ_0 = 1.05 μm and $\delta\lambda_{\rm max}$ = 2 nm). The output field E_Σ of the entire array was found from formula (6). The spectrum of this field has a complicated structure [25] with three different scales, related to the following factors: (i) random spread of fibre lengths on the order of few millimetres; (ii) a regular increment in the fibre length, multiple of 10 cm; and (iii) beatings of modes with different longitudinal numbers. The relative difference in the longitudinal-mode frequencies changes by few tenths of percent from laser to laser.

We found the wavelength within the gain band (4 nm) at which the efficiency of addition of output laser field, $\eta = |E_\Sigma(\delta\lambda)|^2/P_0$ (P_0 is the total power in the absence of random spread of laser lengths), was maximum for each random sample of fibre lengths $\{L_m\}$. The number of injection-locked lasers, i.e., the lasers satisfying the condition $G < G_{\rm th}$, was determined for this wavelength. The calculation results for 200 samples of laser lengths at a fixed rms spread of fibre lengths $\langle\delta\lambda_m\rangle$ were averaged.

The results of sample averaging are presented in Fig. 2 for a small-signal gain G_0 = 3.7. In the absence of refractive-index nonlinearity, the mean efficiency monotonically decreases with an increase in the array size and the number of injection-locked lasers first reaches a maximum (~6) and then decreases to 2–4. Thus, the maximum size of array in which phase locking can be implemented is on the order of ~7–8 lasers in this case. The resonant refractive-index nonlinearity at any sign of coefficient α leads to some increase in the phase-locking efficiency of fields and radically changes the behaviour of the number of injection-locked lasers ($N_{\rm L}$). This number monotonically increases with an increase in the array size up to N = 20.

To reveal the role of refractive-index nonlinearity, we will first analyse the effect of the decrease in the number of injection-locked lasers with an increase in the array size in the model disregarding the refractive-index nonlinearity. Since this effect arises at averaging over random samples, it is natural to apply the probability theory. In the asymptotic limit of a large array, the single-pass phase shift in a separate laser can be equated to $l\pi$, with a random detuning uniformly distributed over the interval $(0, \pi)$. A variation in wavelength in the spectral gain band leads to a change in random combinations φ_m (note that the integer l does not affect the total field). The field-addition efficiency is high only when the phase detunings in most of lasers have close values. Generally, the central phase detuning is nonzero. Thus, we are interested in the probability of the situation where the phase detunings in all lasers fall in the interval $(\varphi_{\rm c} - \delta\varphi, \varphi_{\rm c} + \delta\varphi)$ ($\varphi_{\rm c}$ is the midpoint of the interval and $2\delta\varphi$ is its width). In the limit of a large-size array [33], the summation of laser fields can be replaced by integration and the addition efficiency can be defined as $\eta = |E_{\rm av}(\varphi_{\rm c}, \delta\varphi)|^2/P_0$, where

$$E_{\rm av}(\varphi_{\rm c}, \delta\varphi) = 2\delta\varphi^{-1}\int_{\varphi_{\rm c}-\delta\varphi}^{\varphi_{\rm c}+\delta\varphi} E_{\rm out}(G_0, \varphi)\,{\rm d}\varphi.$$

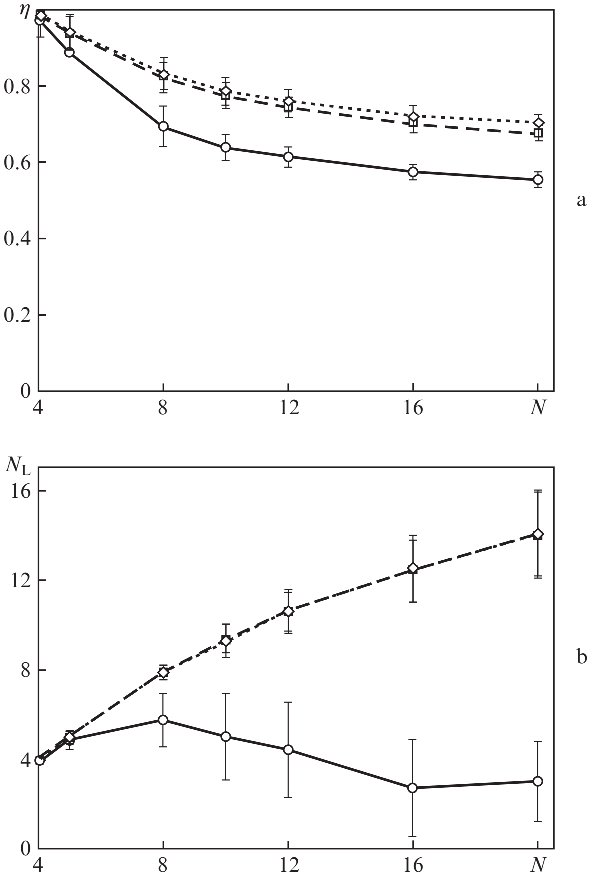

Figure 2. Dependences of (a) the phase-locking efficiency η and (b) the number of injection-locked lasers $N_{\rm L}$, averaged over 200 random realisations, on the number of lasers in the array, N. The rms spread of fibre lengths $\langle\delta l_m\rangle$ = 1 mm; G_0 = 3.7; $P_{\rm inj} = P_{\rm cr}/2$; the spectral gain bandwidth is 4 nm; α = (solid lines) 0, (dashed lines) 1, and (dotted lines) –1.

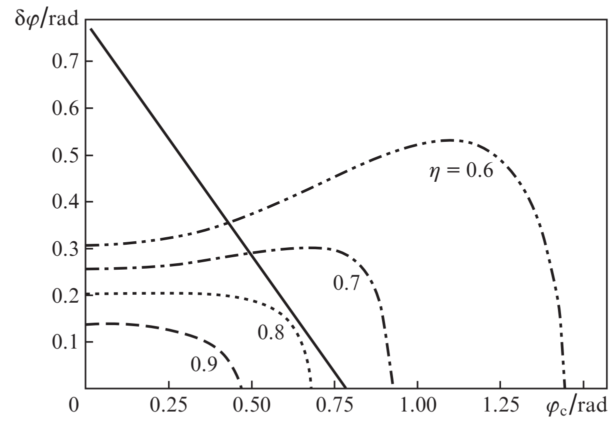

Figure 3. Isolines of phase-locking efficiency for an array of globally coupled lasers at G_0 = 3.7 and $P_{\rm inj} = P_{\rm cr}/2$. The condition for frequency locking of lasing for each laser is fulfilled under the straight line.

Here, $E_{\text{out}}(G_0,\varphi)$ is found by solving Eqns (3)–(5). At an injection power below critical, it is also necessary to take into account the limitation on the phase detunings φ_m, which is related to the stability of lasing controlled by external signal [36].

Figure 3 shows isolines of the field-addition efficiency η in the $\varphi_c, \delta\varphi$ plane at an injected-signal power equal to half of critical power in the absence of refractive-index nonlinearity ($\alpha = 0$). A segment of a straight line described by the equation $\delta\varphi = \pi/4 - \varphi_c$ limits from above the region of states in the $\varphi_c, \delta\varphi$ plane that corresponds to the frequency locking by external signal. The ratio of the area between the two η isolines to the total area in Fig. 3 (equal to $\pi^2/4$) determines the probability for the field-addition efficiency to lie in the range of its values for the two corresponding isolines.

The plots in Fig. 3 indicate that, at a field-addition efficiency $\eta \geqslant 0.8$, the lasing in all lasers is controlled by external signal. When η decreases to values smaller than 0.8, the area of the region of possible states of phases φ_m in which lasing stops being injection-locked begins to rapidly increase, while the midpoint of the averaging interval in which the locking width $\delta\varphi$ is maximum significantly deviates from zero. The reason is that the decrease in the field-addition efficiency is caused by the increase in the phase detunings of the output laser fields. At a low addition efficiency the probability for the lasers only slightly detuned from resonance to make the main contribution to the total field decreases, while the probability of forming a group of lasers with similar frequencies at detunings φ from exact resonance increases.

A part of the region in Fig. 3 under the isoline of specified efficiency is below the straight line $\delta\varphi = \pi/4 - \varphi_c$. The area of this part determines the fraction of array lasers that are in the mode of stable injection-locked lasing, N_L/N, provided that the field-addition efficiency exceeds the specified value. The thus found fraction of injection-locked lasers as a function of efficiency is shown by squares in Fig. 4, which also presents the results of direct statistical calculation for arrays with the same parameters as in Fig. 2, with averaging over 200 random realisations. It can be seen that the predictions of the asymptotic theory are in good agreement with the results of direct calculation.

The consideration of the resonant nonlinear phase shift $\Delta\varphi_m = \alpha G_m$ in the model leads to a small increase (by less than 15%) in the phase-locking efficiency at any sign of nonlinearity and to a significant increase in the fraction of injection-locked lasers. The number of such lasers continues to grow even in a large array (Fig. 2b). It can be seen in Fig. 4 (dashed line) that, at a fixed efficiency, an increase in the fraction of injection-locked lasers due to the resonant nonlinearity of the refractive index is within the statistical error. Thus, specifically the increase in the field-addition efficiency at the output of the laser array, with allowance for the resonant refractive-index nonlinearity in the model, plays a key role in the enlargement of phase-locked array.

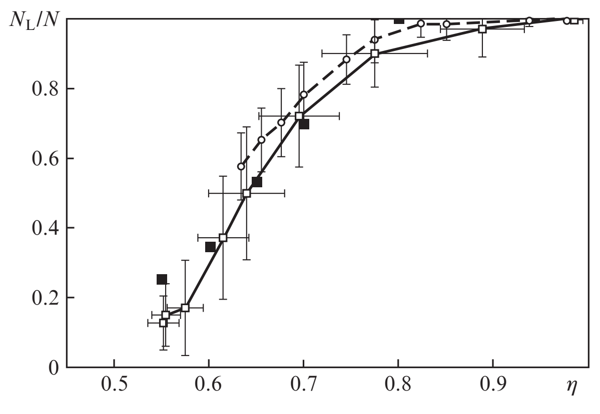

Figure 4. Dependence of the fraction of injection-locked lasers on the phase-locking efficiency for an array with the same parameters as in Fig. 2. The solid and dashed lines correspond to $\alpha = 0$ and 1, respectively; black squares show the result of calculation within the semi-analytical model.

Until now, we studied the addition efficiency for the fields emitted by an array of fibre lasers, with an external signal of specified power at a specified wavelength, fed into each laser. In a globally coupled array, this signal is formed by splitting off a fraction κE_Σ of the total output field. In the next section, we will analyse a closed model for a system with a specified feedback coefficient κ.

4. Model of phase locking of an array with a specified feedback

Let us now consider a globally coupled system, in which the power injected into each laser is determined from the formula $P_{\text{inj}} = |\kappa E_\Sigma|^2/N$. It is natural to begin with the array radiation wavelength equal to the wavelength at which the field-addition efficiency at specified parameters of array elements is maximum within the gain band. The next step is the iterative calculation of the output power of a phase-locked laser array. As an example (which is of independent interest), we took an array of 20 globally coupled fibre lasers. The spectral response and the total power of an array with a fixed random OLE sample in the vicinity of the maximum spectral response are compared in Fig. 5. Both values are normalised to the maximum in the resonance. The asterisks denote the boundaries of the locking spectral range (i.e., the range beyond which some part of array lasers generate independently). It can be seen that the positions of the maxima of both values coincide. The resonant curve for the power is narrower than for the spectral response. The consideration of the refractive-index nonlinearity at a small excess of the lasing threshold ($G_0 = 2$) makes the spectral response slightly asymmetric and significantly distorts the spectral dependence of the output power (Fig. 5b). At large values of the small-signal gain, iterations cease to converge, because the dependence of the eigenfrequencies of cavities on the power circulating in them becomes multivalued.

Note that the fraction of the output power spent on coupling between elements is an important parameter. An increase in this fraction makes coupling stronger and thus increases the probability of implementing phase locking for all lasers. The increase in this parameter is limited by two factors: nonlinear effects in adder and the decrease in the output power. The nonlinear effects in the adder are not analysed here, because its design must be specified to this end.

Figure 6 shows the power of a 20-laser array before the global-coupling system ($P_\Sigma = |E_\Sigma|^2$) and the output power ($P_{\text{tot}} = 1 - \kappa^2 P_\Sigma$) for different values of the parameter κ^2, at $\alpha = 0$, $G_0 = 3.7$, and a random sample of laser lengths, as for Fig. 5. The histogram shows the fraction of injection-locked lasers. The output power at specified array parameters and a fixed random sample is maximum at the fraction of power fed to the global-coupling device $\kappa^2 \sim 0.2$–0.3. At smaller κ values some lasers begin to generate independently, while at larger κ the output power decreases. Note that the value

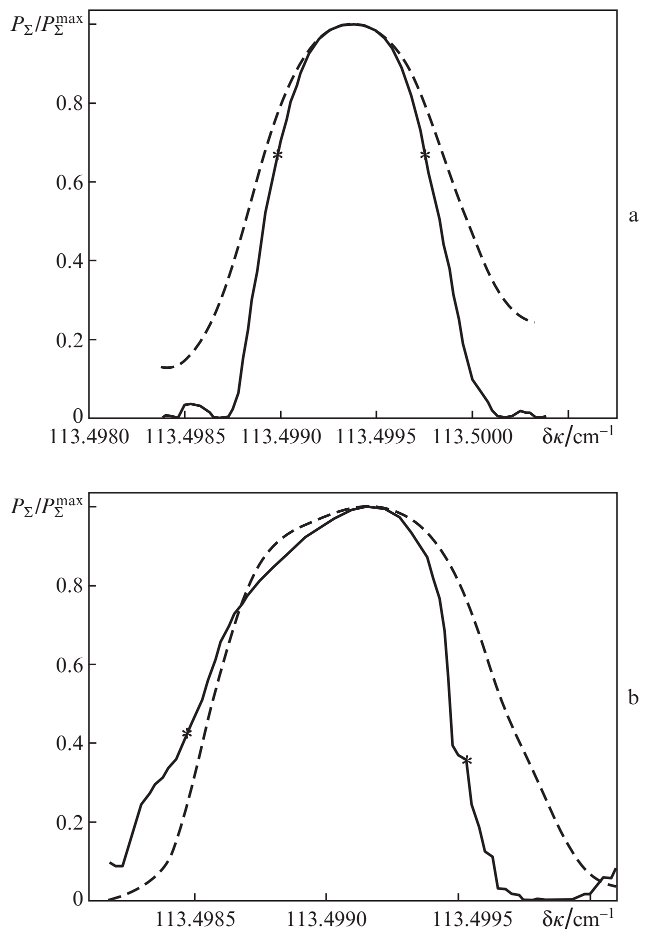

Figure 5. Dependences of the total power (solid line, $\kappa = 0.45$) and the spectral response at a fixed injection power (dashed line, $P_{inj} = P_{cr}/2$) for a 20-laser array on the frequency detuning at $G_0 = 2$, $\langle \delta l_m \rangle = 1$ mm, $2\delta\lambda_{max} = 4$ nm, and α = (a) 0 and (b) 1.

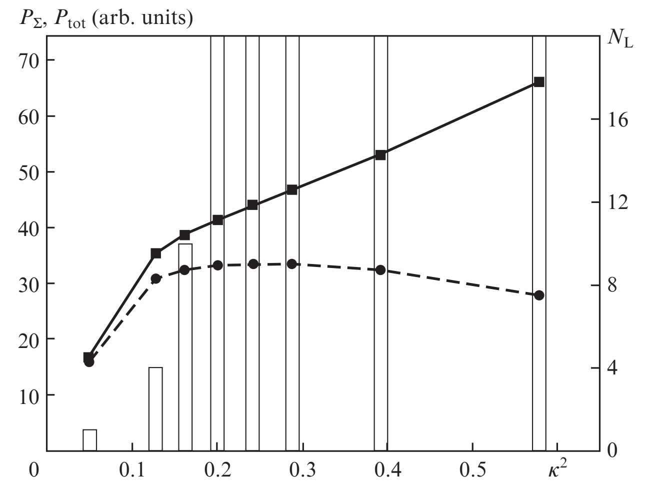

Figure 6. Dependences of (solid line) the power P_Σ of a 20-laser array before the feedback system, (dashed line) the total output power P_{tot}, and (histogram) the number of injection-locked lasers N_L on the fraction of the power spent on feedback at $G_0 = 3.7$ and $\alpha = 0$.

$\kappa^2 = 0.2$ corresponds to the power $P_{inj} = 0.42$ injected into each element, which exceeds the critical power for the specified parameters ($P_{cr} = 0.362$). The consideration of the refractive-index nonlinearity with $\alpha = 1$ at $G_0 = 3.7$ and $\kappa^2 = 0.2$ in the model leads to an increase in the beam-addition efficiency from 0.53 at $\alpha = 0$ to 0.64 at $\alpha = 1$. An increase in the efficiency leads also to a growth of the power injected into each laser; therefore, the output power increases by approximately 20% due to the refractive-index nonlinearity.

Figure 7 shows the dependence of the output power on the integral small-signal gain G_0 for the same array realisation as in Figs 5 and 6 (20 lasers, $\alpha = 0$), at two values of the feedback coefficient. The cost of phase locking can be understood by comparing the output powers of systems with specified and zero OLE spreads. The efficiency of a system, i.e., the ratio of the output power of a laser array to the power of the same array but in the absence of the OLE spread, increases with an increase in the small-signal gain and reaches 59% at $G_0 = 5.2$.

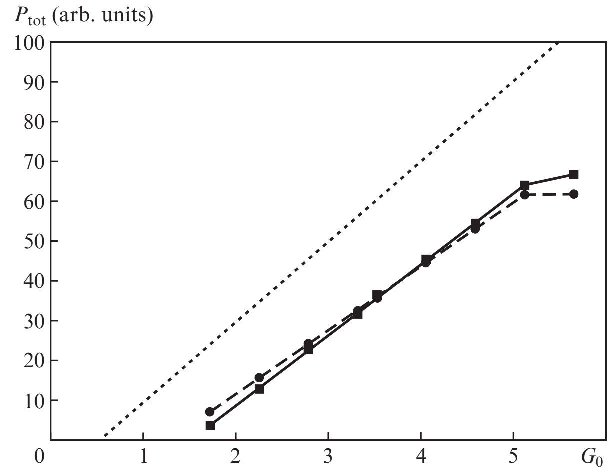

Figure 7. Dependence of the output power of a 20-laser array on the total small-signal gain at κ^2 = (solid line) 0.2 and (dashed line) 0.3. The dotted line shows the output power in the absence of OLE spread at $\kappa^2 = 0.2$.

The consideration of the refractive-index nonlinearity with $\alpha = 1$ in the model discussed here increases the efficiency to 70%, a value much higher than the predicted and observed phase-locking efficiency (54%) for a system composed of fibre amplifiers with X couplers [23]. The result of a further increase in the small-signal gain is that the consideration of optical nonlinearity leads to the absence of convergence of iterations in solution of Eqn (6), which may indicate unstable steady-state array lasing. It should be emphasised that the calculation results in Figs 5–7 correspond to some fixed OLE sample with an rms spread of 1 mm. Figures 2 and 4 give some idea of variations in the output characteristics due to a change in the sample of random OLE values.

5. Conclusions

The development of a self-consistent model of a fibre laser array with an external global-coupling system made it possible to calculate the attainable output power of the array in the regime of phase self-locking. This model takes into account the presence of many longitudinal modes in the cavity of an individual laser, refractive-index nonlinearity, and nonuniform field distribution over the cavity length. Each array element is considered as a laser controlled by injection of a signal from the global-coupling system. The range of values of the squared feedback coefficient κ^2 in which stable phase locking of an array of globally coupled fibre lasers can be implemented was found to be $\sim$0.2–0.3. The possibility of attaining a phase-locking efficiency of 70% for a 20-laser array was predicted; this efficiency significantly exceeds the previously reported values. We explain this difference by the fact that our model considers an array of lasers, each of which has an intrinsic cavity, rather than a system of amplifiers in a common cavity. The approach implemented here can be used (with insignificant modifications) to describe laser arrays with coupling via X couplers [12, 13] or with Fourier coupling [15]. The subject of further study is the analysis of the dynamic stability of phase locking, especially in the presence of refractive-index nonlinearity, as well as the analysis of regimes with partial phase locking of a laser array.

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