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Study of UV Cu + Ne – CuBr laser lifetime by statistical methods

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Abstract. **On the basis of a large amount of experimental data, statistical investigation of the average lifetime of a UV Cu + Ne – CuBr laser depending on ten input physical laser parameters is carried out. It is found that only three of the parameters have a substantial influence on the laser lifetime. Physical analysis and interpretation of the results are provided.**

Keywords: *ultraviolet copper bromide vapour laser, laser lifetime, hierarchical cluster analysis, factor analysis, principal component regression.*

1. Introduction

The output power of laser radiation is one of the most important characteristics of different lasers including metal and metal compound vapour lasers. Using the known data one can determine the degree of influence of independent physical parameters, such as the input electric power, geometric dimensions, type of the construction materials used, kind and pressure of gases, discharge temperature, energy of electrons, etc., on the output power. The other often studied output characteristic of laser operation is the efficiency.

However, such a laser characteristic as the lifetime is not thoroughly studied, although it is exclusively important in laser exploitation because it determines the laser reliability.

The aim of the present paper is to reveal the key physical parameters that affect the average lifetime of the laser by classifying them by means of the classical statistical methods, such as the cluster analysis, the factor analysis and the principal component regression analysis.

For the concrete solution of the problem, the copper halide vapour laser emitting in the UV spectral region was chosen [1 – 3]. The experimental data on the lifetime for the lasers of this type, presented in papers [1 – 3], were used in the statistical analysis performed.

The studies were carried out using the SPSS statistical software package [4].

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2. General description of UV Cu + Ne – CuBr laser

The laser in question has been developed since 1999 at the Laboratory of Metal Vapour Lasers of the Georgi Nadjakov Institute of Solid State Physics, Bulgarian Academy of Sciences [5]. The laser produces radiation in the UV spectral region at the wavelengths of 248.6, 252.9, 259.7, 260.0 and 270.3 nm. The output power of 1.3 W was obtained experimentally at all five wavelengths and the power of 0.85 W was obtained at $\lambda = 248.6$ nm. It was established that the addition of small amounts of hydrogen (0.02 – 0.04 Torr) increases the laser generation by almost two times [3].

The scheme of the laser tube is shown in Fig. 1. As a source of steady-state UV radiation, the UV Cu + Ne – CuBr laser finds practical use in information recording, high-quality cutting and drilling of various materials with minimal mechanical and thermal damage; it is also widely applied in chemistry, medicine, spectroscopy, and other fields of science and engineering [6].

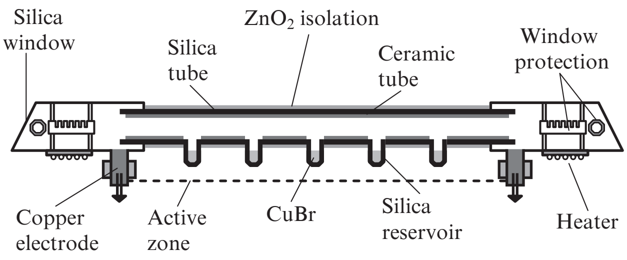

Figure 1. Scheme of the UV Cu + Ne – CuBr laser tube.

3. Initial data

We consider the following ten independent input laser parameters (variables): the inner diameter of the laser tube D (mm), the inner diameter of apertures D_r (mm), the active zone length (separation between the electrodes) L (cm), the input electric power P_{in} (W), the electric power per unit length P_L (kW cm^{-1}) with the losses taken into account, the repetition frequency P_{rf} (kHz) of the electric pulses, the pressure p_{Ne} (Torr) of the buffer neon gas, the pressure p_{H_2} (Torr) of the additional hydrogen gas, the equivalent capacity C (nF) of the capacitor battery, and the temperature T_r (°C) of the reservoir containing CuBr.

The laser lifetime L_{time} is considered to be a dependent variable.

In the study we used the values of the above variables from $n = 238$ experiments. The data are published in Refs [1–3].

4. Classification of parameters by cluster analysis

The method of cluster analysis consists of a variety of computational procedures according to which a certain sample of data or objects is divided into non-intersecting subsets referred to as clusters. The data may be classified with respect to variables or observations. We will use the first version. The grouping of similar objects into clusters is carried out according to the chosen criterion of 'similarity' ('likeness'). The aim of the classification of variables is to obtain an optimal grouping, in which the objects in each cluster are similar, while the clusters differ strongly from each other.

To quantify the concept of similarity the notion of metric (distance) in a vector space is used. The presence of similarity or difference between two points in the space is determined by the distance between them. The larger the distance, the greater the difference, and vice versa. Usually the metric is defined as the squared Euclidean distance between two vectors [4].

The methods of cluster formation in the so called hierarchical approach include agglomerative (joining) and divisive (separating) methods. We shall use the hierarchical agglomerative method, which is recommended for small samples ($n \leqslant 500$). At each stage the clusters separated by the minimal distance are joined into one new cluster. The joining of clusters is implemented using the chosen method of clustering, i.e., by calculating a certain type of distance between the clusters, depending on the given metric. The mostly used clustering methods are the method of average linkage clustering and the method of the closest neighbour (the single-linkage method). A more detailed description of the stages of the cluster analysis and its implementation using the SPSS package can be found, e.g., in Ref. [4].

In order to determine the optimal number of clusters, it is convenient to use the graphical representation of the solutions in the form of a dendrogram [4]. For the variables studied here the dendrogram is presented in Fig. 2. The optimal solution is obtained with the number of clusters $k = 3$, when the greatest separation of clusters is observed (more than five scale marks, the total standard scale length being 25 units). Thus, using the method of the cluster analysis we arrive at the optimal grouping of all variables in the following three clusters:

$$\text{cluster 1: } \{D, D_r\},$$

$$\text{cluster 2: } \{P_{in}, L, P_L, P_{rf}, p_{Ne}, T_r\} \quad (1)$$

$$\text{cluster 3: } \{p_{H_2}, C, L_{time}\}.$$

From Fig. 2 and the optimal solution (1) one can conclude that the dependent variable L_{time} shows the strongest similarity with the variables p_{H_2} and C. It may be expected that these variables affect the lifetime of the UV Cu + Ne–CuBr laser stronger than the other ones.

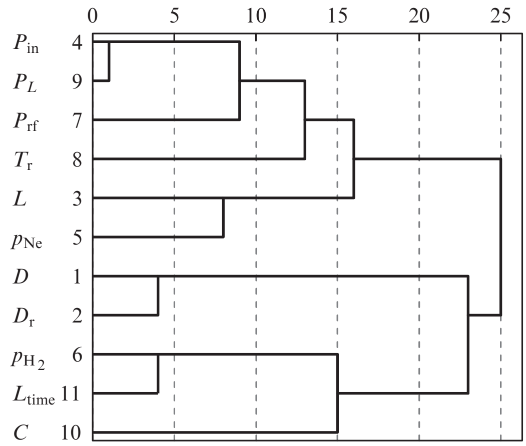

Figure 2. Cluster dendrogram of ten independent variables and the laser lifetime L_{time} obtained by the average linkage (between groups) method with the metric defined as the squared Euclidean distance. The horizontal axis shows the distance between clusters in relative scale from 0 to 25; 0 corresponds to the minimal distance at the first stage, and 25 corresponds to the maximal distance at the last stage.

5. Classification of parameters by means of factor analysis

Factor analysis is a powerful tool of studying the structure of relations between variables and reducing the dimensionality of the given multidimensional data under the condition of minimal loss of the initial information. The method is used in the case of collinear or almost collinear variables that are joined into groups (factors) depending on the degree of correlation between them. The factors themselves are orthogonal to each other or correlate very weakly. Such an approach allows the reduction of the number of initial variables and the use of the obtained factors as artificial variables (instead of the initial ones) in further analysis.

Similar to the cluster analysis, the factor analysis is implemented in several stages, described below together with the obtained results.

5.1. Calculation of the correlation matrix

The first stage of the factor analysis includes the standardisation of the given values of the variables by calculating their z-values (making them dimensionless) [4] and the calculation of the correlation matrix. Also the conditions of the multicollinearity presence are determined and the testing of the method adequacy is performed. The correlation matrix for all ten variables and the dependent variable L_{time} is presented in Table 1. When the absolute value of a certain coefficient of the matrix is greater than 0.5, it is assumed that the corresponding two variables are correlated, the closer the coefficient to 1, the more collinear these two variables. From the results of Table 1 the presence of correlation coefficients with large moduli is seen. Since the determinant of the matrix (excluding L_{time}) is very small (6.27×10^{-7}), a strong multicorrelation between the variables is present. Moreover, in the considered case the formal statistical Kaiser–Meyer–Olkin criterion for the sample adequacy is $0.552 > 0.5$, and the Bartlett sphericity coefficient is equal to zero. This confirms the adequacy of factor analysis [4].

From the correlation matrix (Table 1) it is seen that the variable L_{time} for the lifetime of the laser strongly correlates

Table 1. Correlation matrix containing pair correlation coefficients of all variables of the UV Cu + Ne–CuBr laser.

	D/mm	D_r/mm	L/cm	P_{in}/kW	p_{Ne}/Torr	p_{H_2}/Torr	P_{rf}/kHz	T_r/°C	P_L/kW cm^{-1}	C/nF	L_{time}
D	1.000	0.789	−0.654	−0.465	−0.842	−0.164	−0.035	0.023	−0.312	−0.095	−0.166
D_r	0.789	1.000	−0.772	−0.363	−0.870	−0.171	−0.030	0.134	−0.135	−0.100	−0.174
L	−0.654	−0.772	1.000	0.485	0.593	0.040	0.055	−0.013	0.192	0.023	0.050
P_{in}	−0.465	−0.363	0.485	1.000	0.364	−0.169	0.533	0.257	0.951	−0.079	−0.181
p_{Ne}	−0.842	−0.870	0.593	0.364	1.000	0.054	0.122	0.042	0.209	0.031	0.057
p_{H_2}	−0.164	−0.171	0.040	−0.169	0.054	1.000	−0.395	−0.835	−0.202	0.452	0.783
P_{rf}	−0.035	−0.030	0.055	0.533	0.122	−0.395	1.000	0.440	0.576	−0.242	−0.389
T_r	0.023	0.134	−0.013	0.257	0.042	−0.835	0.440	1.000	0.298	−0.483	−0.848
P_L	−0.312	−0.135	0.192	0.951	0.209	−0.202	0.576	0.298	1.000	−0.095	−0.220
C	−0.095	−0.100	0.023	−0.079	0.031	0.452	−0.242	−0.483	−0.095	1.000	0.067
L_{time}	−0.166	−0.174	0.050	−0.181	0.057	0.783	−0.389	−0.848	−0.220	0.067	1.000

only with p_{H_2} (0.783) and T_r (−0.848), the second dependence being inversely proportional. It is possible to conclude that the increase in the hydrogen pressure p_{H_2} leads to the increase in L_{time}, while the growth of the copper bromide reservoir temperature T_r reduces L_{time} and vice versa.

5.2. Extraction of factors and the choice of their number

At the second stage of factor analysis the factors are extracted. The sense of the procedure is the transformation of the initial correlated variables into a set of non-correlated variables. Usually for this aim the method of principal components is used [4]. In our case, using this method ten directions (components) that contain nonoverlapping fractions of the total variance are calculated from the data matrix for ten independent variables.

The specific feature of the factor analysis is the determination of the number of factors from these ten components. The factors themselves are calculated by transforming the matrix of all data into a product of two matrices, one of which has the smaller second dimension, equal to the chosen number of factors, neglecting a certain error matrix. From all components three factors were selected. They determine 80.27% of the total variance of the tested data. Among them the first factor determines 38.34%, the second 28.67%, and the third 13.26% of the total variance.

5.3. Rotation of factors and calculation of factor scores

At the next stage the rotation of the factors is implemented with the aim of creating a simplified structure and separating the factors. The grouping of variables should be performed such that each variable could dominate in a single factor only.

Usually for an orthogonal rotation the Varimax method is used, while for an oblique one the direct oblimin method [4] is applied.

With the aim of classification we first make use of the Varimax method for all variables, i.e., ten initial parameters and the dependent variable L_{time}. The procedure of rotation with three factors yields the matrix of rotated components, presented in Table 2, where the coefficients (factor loadings) smaller than 0.5 are omitted.

The factor loadings in Table 2 should be considered as coefficients of correlation between the variables and the factors. Thus, the variables L_{time}, T_r, and p_{H_2} demonstrate the strongest correlation with the second factor, the correlation value for L_{time} amounting to 0.837, T_r correlates with the negative sign (−0.942), for p_{H_2} the correlation is 0.927, and for C it is 0.535. As mentioned above, each variable can dominate only in one factor, and in other factors it should be smaller than a certain threshold value (usually 0.5 [4]). If it is not so, then it is necessary to choose a different number of factors or a different method of rotation. In our case the three-factor rotated solution from Table 2 is quite satisfactory. As a result we obtain the following factor grouping:

Table 2. The matrix of rotated components, including L_{time}.

Variables	Factors		
	F'_1	F'_2	F'_3
D_r	−0.946		
p_{Ne}	0.915		
D	−0.888		
L	0.823		
T_r		−0.942	
p_{H_2}		0.927	
L_{time}		0.837	
C		0.535	
P_L			0.959
P_{in}			0.899
P_{rf}			0.699

Note. Two methods were used: the extraction method (the method of principal components) and the rotation method (Varimax with Kaiser normalisation).

$$F'_1 = \{D_r, P_{Ne}, D, L\},\quad F'_2 = \{T_r, p_{H_2}, L_{time}, C\},\quad F'_3 = \{P_L, P_{in}, P_{rf}\}. \tag{2}$$

Three factors (2) reflect 78.63% of the total sample variability. The influence of individual factors after the rotation is the following: the first factor determines 35%, the second 31.12%, and the third 12.51% of the total variance.

As mentioned above, the variable L_{time} together with the variables T_r, p_{H_2} and C is grouped in the second factor. Therefore, there is a high degree of correlation between these variables. For T_r and p_{H_2} this is confirmed by the corresponding component of the correlation matrix (see Table 1). At first sight the presence of the quantity C in the second factor is unexpected. Its coefficient with L_{time} in the correlation matrix is equal to 0.067, and C does not affect L_{time} directly. Apparently, the influence of C has a more complex nonlinear nature. The quantity C can essentially affect L_{time} via its interaction with T_r and p_{H_2}. This is possible, e.g., due to the influence of new second-order nonlinear quantities like T_r, C and p_{H_2}, C. At the present stage it is not possible to answer this question directly, since the statistical analysis proposed here is linear. We also do not consider the possible more complex second-order nonlinear influence, which can be studied if each of the independent variables can participate in the interaction with other variables not only linearly, but being raised to the second and higher powers.

Since we need to find the dependence of $L_{\rm time}$ on ten initial laser variables, let us repeat the described procedure of the factor analysis, omitting $L_{\rm time}$. Using the Varimax method, we arrive at the factor solution

$$F_1 = \{D_{\rm r}, p_{\rm Ne}, D, L\},\quad F_2 = \{P_L, P_{\rm in}, P_{\rm rf}\},\quad F_3 = \{T_{\rm r}, p_{\rm H_2}, C\}. \quad (3)$$

The relative influence of the factors (3) changes after the rotation, namely, the first factor determines 33.64%, the second 23.88% and the third 22.75% of the total variance. The total influence of the three factors equals 80.27%.

At the final stage of the factor analysis the factor scores are calculated and three new (factor) variables F_1, F_2 and F_3 are obtained, each having the dimension n, which may be used in further statistical analyses.

6. Classification of laser parameters by the method of multiple regression

Multiple regression analysis is used to construct the models for quantitative description of relations between several independent variables (regressors) $x_1, x_2, \ldots, x_p$ and one or more variables y depending on them, where all variables are vectors with the dimension n. Under the condition of normally distributed y, the linear regression equation with respect to the regression coefficients has the form

$$\hat{y}_i = b_0 + b_1 x_{i1} + b_2 x_{i2} + \ldots + b_p x_{ip}, \quad i = 1, 2, \ldots, n, \quad (4)$$

where $y_i = \hat{y}_i + \varepsilon_i$, $i = 1, 2, \ldots, n$, $b_0, b_1, \ldots, b_p$ are the regression coefficients; and $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$ is the error vector.

To reveal the role of every quantity the transformation with the use of z-values is applied. In this case the standardised form of the regression equation is obtained:

$$\hat{y} = \beta_1 z_1 + \beta_2 z_2 + \ldots + \beta_p z_p. \quad (5)$$

Here $\beta_1, \beta_2, \ldots, \beta_p$ are the standardised coefficients, showing the relative influence of each independent variable on the dependent variable. The determination of regression coefficients is carried out using the well-known least squares method, after which their statistical significance is studied [4].

As it was already mentioned, the main goal of the present paper is to determine the degree of influence of each of the ten independent quantities on the dependent quantity $L_{\rm time}$. For this aim we should find the standardised equation of type (5) using the regression analysis.

6.1. Application of the method of principal component regression

In order to get the coefficients in regression equations (4) and (5) it is desirable that the independent variables in the right-hand sides of these equations were not correlated with each other. In particular, they may be mutually orthogonal. The latter is true for the factor variables F_1, F_2, and F_3 from Eqn (3), obtained by using the Varimax rotation method, which yields the standardised equation in the form

$$\hat{L}_{\rm time} = \beta_1 F_1 + \beta_2 F_2 + \beta_3 F_3. \quad (6)$$

After completing the regression analysis with the factors F_1, F_2, and F_3, we obtain the estimates summarised in Table 3. Using the standardised coefficients $\beta_{1,2,3}$, we write the desired equation in the form

$$\hat{L}_{\rm time} = 0.131 F_1 - 0.245 F_2 + 0.651 F_3. \quad (7)$$

Table 3. Regression coefficients with the factors $F_{1,2,3}$ in the linear regression model ($L_{\rm time}$ is a dependent variable).

Nonstandardised coefficients		Standardised coefficients	Significance
b_i	Standard error	β_i	
118.655	10.538	–	0.000
30.056	10.560	0.131	0.005
–56.014	10.560	–0.245	0.000
148.789	10.560	0.651	0.000

It is seen that the maximal coefficient ($\beta_3 = 0.651$) belongs to F_3, joining the variables $T_{\rm r}$, $p_{\rm H_2}$ and C. Therefore, these variables exert the maximal influence on the quantity $L_{\rm time}$. The results obtained from the regression equation (7) confirm the solution of the factor analysis (see Table 2).

7. Results of statistical analyses and their physical interpretation

To determine the degree of influence of ten independent quantities on the lifetime of the UV Cu + Ne–CuBr laser in the present paper we used three statistical methods, namely, the cluster, factor and regression analysis. Each of the methods yields a specific answer to the posed question. The aim of the present section is to generalise the obtained results and to interpret their physical meaning.

The basis of classification of the input parameters with respect to the degree of their influence on the laser lifetime is their grouping by means of the factor analysis using the correlation criterion. As a result, it was established that the lifetime is mainly determined by the following three quantities: $T_{\rm r}$, $p_{\rm H_2}$ and C. This conclusion is also confirmed by the results of regression analysis. The cluster analysis also partially confirms the obtained result, since the quantities $p_{\rm H_2}$ and C enter the third cluster of Eqn (1) together with $L_{\rm time}$.

The influence of the quantity $T_{\rm r}$ on the laser lifetime is well known, and this influence is negative. The increase in the copper bromide reservoir temperature leads to the growth of the concentration of the copper halide molecules and the output power, which, in turn, increases the loss of the active substance. The latter can occur via the sputtering of the substance and filling the cold parts of the laser tube with it, as well as the accumulation of the substance in the vicinity of output mirrors, where the temperature is minimal. The increase in the temperature always leads to thermochemical degradation of the substance, which becomes incapable of the further participation in laser oscillation. Since the laser tube is sealed off and the loss of the substance cannot be compensated, the output laser power degrades with time.

The influence of the equivalent capacity C of the capacitor battery is more complex and ambiguous. From the obtained results it is seen that the laser lifetime L_{time} is directly proportional to this quantity. The obtained result is open to discussion. With increasing capacity C, the energy stored in the capacitor battery also increases, as well as the discharge time constant. The release of the accumulated energy during a very short interval of time subjects the tube to serious physical deformations, which causes its damage. The electrodes are sputtered, thus littering the active laser medium. The results of the analysis show that the increase in the discharge time of the capacitor battery may have a positive effect on the tube lifetime. We consider these results as preliminary ones. A more precise estimate of the influence of the equivalent capacity C of the capacitor battery can be achieved after a nonlinear analysis of the interaction of this quantity with the other ones, e.g., T_r and p_{H_2}, with which it correlates (see Table 1).

One should mention the positive influence of increasing the pressure p_{H_2} of the additional hydrogen gas. The growth of p_{H_2} is known to increase the laser oscillation output power. Moreover, it appears that the increase in p_{H_2} also prolongs the laser lifetime. This fact is somewhat unexpected. As well known, the hydrogen, being used as an electronegative gas, increases the loss of electrons in the discharge, which leads to the change in the discharge voltage–current characteristic. At the same applied electrical power the voltage between the electrodes grows and the current through the tube decreases. The decrease in the discharge current is always favourable for increasing the lifetime of the laser tube. The sputtering of the electrodes becomes slower, the probability of the development of thermal and electric ionisation instability decreases. The processes of the laser medium degradation are slowed down. The combination of these conditions provides a certain increase in the average lifetime of the laser tube. Thus, it can be recommended to add hydrogen in order to provide longer lifetime of the laser tube.

8. Conclusions

We have proposed for the first time a new parametric method of estimating the influence of ten independent quantities on the lifetime of the UV Cu + Ne–CuBr laser. Using several statistical methods (cluster, factor and regression analysis) it is found that only three of all these quantities exert essential influence on the laser lifetime. The physical analysis of the obtained results is carried out and the physical interpretation is given.

The proposed approach can be extended over the study of other types of lasers.

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