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Influence of the axicon characteristics and beam propagation parameter M^2 on the formation of Bessel beams from semiconductor lasers

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Abstract. **We study the peculiarities of the formation of Bessel beams in semiconductor lasers with a high propagation parameter M^2. It is shown that the propagation distance of the Bessel beam is determined by the divergence of the quasi-Gaussian beam with high M^2 rather than the geometric parameters of the optical scheme. It is demonstrated that technologically inevitable rounding of the axicon tip leads to a significant increase in the transverse dimension of the central part of the Bessel beam near the axicon.**

Keywords: *Bessel beams, quasi-Gaussian beams, axicon, beam propagation parameter, semiconductor laser.*

1. Introduction

Bessel beams have long attracted the attention of researchers because of their spatial invariance, i.e., the possibility of long-distance propagation without divergence [1–3], which holds great promise for use in devices for the manipulation of micro-and nano-objects (in the so-called optical tweezers) and for the control of micromachines, and other applications [4]. In the projection on a plane, which is perpendicular to the propagation axis, the Bessel beams appear as a bright spot surrounded by a system of concentric rings and their profile is described by the zero-order Bessel function of the first kind, which gave its name to such beams.

In practice, the Bessel beams are produced by the interference of converging rays when a collimated Gaussian beam passes through a conical lens (axicon). The diameter of the central spot (the central lobe of the Bessel beam) is determined by the angle of the axicon, and may be of the order of the radiation wavelength. Bessel beams have a finite distance of propagation, depending on the diameter of the cross section (aperture) of the initial beam (Fig. 1). In this case, the distance of the Bessel beam propagation can reach several meters. Another important feature of Bessel beams is ability of the central lobe self-healing after meeting with an obstacle, which (when used in optical tweezers) provides an opportunity to manipulate not one, but several microscopic objects simultaneously [5]. Application of Bessel beams greatly increases the working distance between the object being manipulated and the focusing optics; in this case, fine adjustment is not required, which makes such systems more flexible and more attractive from the practical standpoint.

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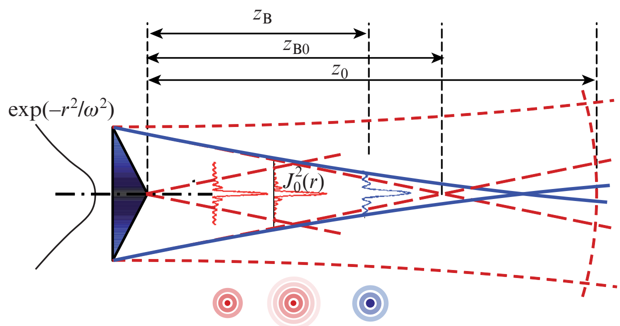

Figure 1. Propagation of the Bessel beam formed from a quasi-Gaussian beam with a high parameter M^2: z_B is the Bessel beam propagation distance due to the beam divergence; z_{B0} is the geometric propagation distance; z_0 is the Rayleigh length.

According to a popular belief, for the Bessel beams to be formed, one needs light sources with a high spatial coherence, such as gas and solid-state lasers, which makes optical tweezers a very complex and expensive tool. However, as was shown recently, the temporal coherence of a light source has a much smaller effect on the formation of Bessel beams than its spatial coherence [6]. This allowed the generation of Bessel beams using surface- and edge-emitting semiconductor lasers [7, 8], including a curved-grating distributed-Bragg reflector laser [9]. Furthermore, we demonstrated superfocusing of multimode radiation from semiconductor lasers and light-emitting diodes [10], and the possible application of the Bessel beams produced by semiconductor lasers for optical manipulation of microscopic (including biological) objects [11].

The present work is devoted to a detailed study of the formation of Bessel beams by semiconductor lasers with a high propagation parameter M^2, as well as to investigation of the effect of technologically inevitable rounding of the axicon tip on the transverse dimension of the central lobe of the Bessel beam.

2. Influence of the propagation parameter M^2 on the formation of Bessel beams

In general, the quality of the laser beam is usually described by the propagation parameter M^2 [12, 13], which is defined as the ratio of the beam divergence to the divergence of an 'ideal' Gaussian beam (i.e., a beam with $M^2 = 1$), corresponding to the diffraction limit. Similarly, the parameter M^2 determines how many times the focal spot of the beam is greater than the spot produced by focusing the ideal Gaussian beam with the same optical system. The parameter M^2 is useful because it allows one to describe quasi-Gaussian beams using the mathematical apparatus developed for Gaussian beams; in this case, use is made of a simple replacement $\lambda \to M^2\lambda$, i.e., the wavelength is increased numerically by M^2 times.

When a Bessel beam formed from a collimated multimode quasi-Gaussian beam propagates, the diameter of the beam central lobe gradually increases due to large divergence of the forming quasi-Gaussian beam, which leads to an increase in the transverse dimension of the Bessel beam kernel and can limit the propagation distance z_B of the beam formed (Fig. 1). In order to evaluate the effect of the beam divergence, it is necessary to consider it in the expression relating the diameter of the central lobe of the Bessel beam with geometrical parameters of the optical system:

$$d_{M^2} = \frac{2.4\lambda}{\pi \sin[\alpha/2 - \pi/2 + \arcsin(n\cos\alpha/2) - x(z)]}, \tag{1}$$

where λ is the wavelength; n is the refractive index of the axicon material; α is the apex angle of the axicon; and $x(z)$ is the beam divergence angle, which depends on the longitudinal coordinate z. In this paper, leaving the effects of misalignment [7, 14, 15] for further research, we will assume that the quasi-Gaussian beam is coaxial to the axicon and collimated in a plane passing through its apex. Therefore, after determining (Fig. 1) the angle of divergence as the arctangent of the ratio of transverse coordinate of the forming beam ray on the axicon to the wavefront curvature of the beam $R(z)$ when the beam reaches the symmetry axis,

$$x(z) = \arctan\left(\frac{z\tan\gamma}{R(z)}\right), \tag{2}$$

and using the known expression for the wavefront curvature of the quasi-Gaussian beam [16]

$$R(z) = z\left[1 + \left(\frac{\pi\omega_0^2}{M^2\lambda z}\right)^2\right] \tag{3}$$

(ω_0 is the beam aperture), after obvious transformations and taking into account Snell's law $\gamma = (n-1)\beta$ in the paraxial approximation, we can write:

$$x(z) = \frac{(n-1)\beta}{1 + [\pi\omega_0^2/(M^2\lambda z)]^2}, \tag{4}$$

where $\beta = 90° - \alpha/2$ is an additional apex angle of the axicon. Substituting (4) into (1), after a simple trigonometric transformations we finally obtain

$$d_{M^2}(z) = \frac{2.4}{\pi}\lambda\left\{(n-1)\sin\left[\beta + \frac{n-1}{1+\left(\pi\omega_0^2/(M^2\lambda z)\right)^2}\beta\right] - n\sin\left[\frac{n-1}{1+\left(\pi\omega_0^2/(M^2\lambda z)\right)^2}\beta\right]\right\}^{-1}. \tag{5}$$

This cumbersome expression can be greatly simplified in the paraxial approximation,

$$d_{M^2}(z) \approx \frac{2.4\lambda}{\pi(n-1)\sin\beta}\left[1 + \left(\frac{M^2\lambda z}{\pi\omega_0^2}\right)^2\right], \tag{6}$$

wherein the difference between the results of calculations using expressions (5) and (6) does not exceed 5% in the entire range of practically important apex angles of the axicon. Equation (6) also provides a simple calculation of the propagation distance of the Bessel beam, defined by the divergence of the forming beam [7]. Having defined the propagation distance z_B of the Bessel beam as the distance at which the transverse size of its central lobe increases by $\sqrt{2}$ times (similarly to the definition of the Rayleigh length), we, using (6), can write:

$$\left(\frac{M^2\lambda z_B}{\pi\omega_0^2}\right)^2 + 1 = \sqrt{2}. \tag{7}$$

After completing the obvious transformations, we obtain the expression

$$z_B = \sqrt{\sqrt{2}-1}\,\frac{\pi\omega_0^2}{M^2\lambda} \approx \frac{2\omega_0^2}{M^2\lambda}, \tag{8}$$

which is fully consistent with the result obtained directly from the analysis of the divergence of the forming beam [7].

It should be noted that when the forming beam divergence is not taken into account, in considering only the geometric parameters of the optical scheme, the Bessel beam propagation distance is found from the known equation [17]

$$z_{B0} = \frac{\pi\omega_0 d_0}{\kappa\lambda} \tag{9}$$

(κ is a numerical factor), which in view of (6) in the geometrical optics approximation takes the form

$$z_{B0} \approx \frac{\omega_0}{(n-1)\sin\beta}, \tag{10}$$

where d_0 is the central lobe diameter of the Bessel beam without divergence.

From the comparison of expressions (8) and (10) one can easily see that the reduction of the beam forming aperture ω_0 in comparison with

$$\omega_0' = \frac{M^2\lambda}{2(n-1)\sin\beta} \tag{11}$$

leads to restriction of the propagation distance of the Bessel beam because of the divergence of the forming beam, while the geometric parameters of the optical scheme have no effect. Obviously, this effect is particularly important in the formation of Bessel beams from quasi-Gaussian beams with large values of M^2.

Figure 2 shows the calculated dependence of the diameter of the central lobe of the Bessel beam on the longitudinal coordinate z for different values of the propagation parameter M^2 of the forming beam and its aperture ω_0.

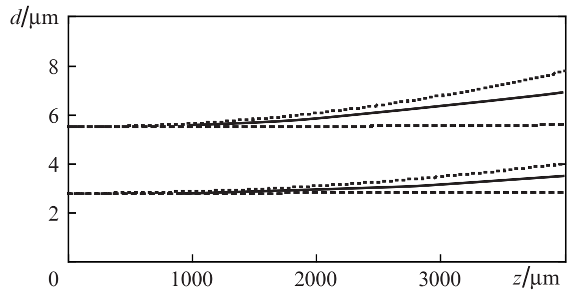

Figure 2. Dependences of the diameter of the Bessel beam central lobe on the longitudinal coordinate z at $\omega_0 = 100$ μm, $M^2 = 1$ (dashed lines), $\omega_0 = 50$ μm, $M^2 = 1$ (solid lines), $\omega_0 = 100$ μm, $M^2 = 5$ (dotted lines). The calculation was performed for the axicon with an apex angle of $\alpha = 160°$ (the upper family of curves) and $\alpha = 140°$ (the lower family of curves).

3. Effect of the rounded tip of the axicon on the transverse dimension of the central lobe of the Bessel beam

Rounding of the axicon tip is a highly undesirable defect occurring during the axicon manufacture, which is explained by inevitable technological difficulties in the final polishing of the conical surface of the axicon. The effect of the rounded tip of the axicon can be neglected in the study of the formation of Bessel beams with a large beam-forming aperture and significant propagation distance. However, when the beam-forming aperture is reduced to hundreds of micrometers and the Bessel beam propagation distance is correspondingly reduced, the account for the effect of the axicon tip rounding on the transverse dimension of the central lobe of the Bessel beam is absolutely necessary.

Consider the axicon with a tip rounded to a radius R and the transverse size H of the rounded region (Fig. 3). The radius of rounding is related to the size of the rounded area by the obvious relationship

$$H = R\sin\beta. \tag{12}$$

It follows from expression (12) that the region of the Bessel beam formation by the rounded tip of the axicon is shifted with respect to the 'ideally' sharp axicon tip by a distance $z_0 = R/n$, and with respect to the round tip – by a distance $z_0 + \delta$ (Fig. 3), where

$$\delta = R\frac{1-\cos\beta}{\cos\beta}. \tag{13}$$

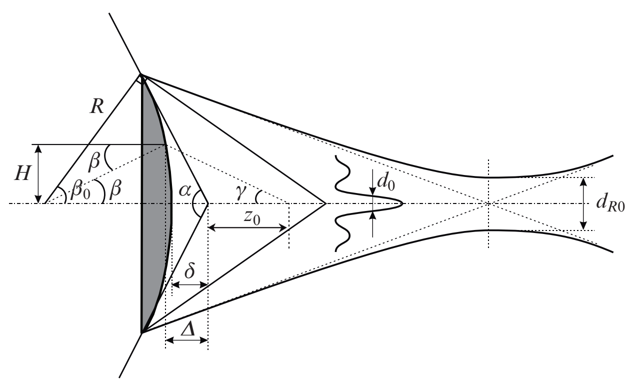

Figure 3. Calculation of the effect of the axicon tip rounding on the formation of a Bessel beam.

As shown schematically in Fig. 3, the rounded tip of the axicon acts on the forming beam as a plano-convex lens with a focal length f, described by the matrix [16]

$$\begin{bmatrix} 1 & 0 \\ \dfrac{1}{f} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{n-1}{R} & 1 \end{bmatrix}. \tag{14}$$

Therefore, in the region of 'geometric shadow', $z_0 + \delta$, the central part of the forming beam is focused, and in the region of the Bessel beam propagation focused radiation may interfere with the conically converging rays, distorting the transverse profile of the Bessel beam. Note that as follows from (14), the focal length of the 'extra lens' is

$$f = \frac{R}{n-1}, \tag{15}$$

and at $n \approx 1.5$ it is several times the size of the 'geometric shadows', $z_0 + \delta \approx R/n$.

The numerical aperture of the lens of radius R and aperture H with the small focal length f taken into account is given by

$$\mathrm{NA}_R^2 = \frac{(n-1)^2}{2}\left\{\sin^2\beta \pm \sqrt{\sin^4\beta - 4\left[\frac{M_c^2\lambda}{\pi R(n-1)}\right]^2}\right\}, \tag{16}$$

that in the paraxial approximation using (12) and (15) can be written in the form

$$\mathrm{NA}_R \approx \sqrt{(n-1)^2\sin^2\beta - 4\left(\frac{M_c^2\lambda}{\pi R\sin\beta}\right)^2}, \tag{17}$$

where M_c^2 is the propagation parameter of the central part of the forming beam incident on the rounded tip of the axicon. It should be noted that in most cases we can approximately assume $M_c^2 \approx 1$. The diameter of the focal spot of the forming beam with the given parameter M_c^2 is

$$d_R \approx \frac{2M_c^2\lambda}{\pi(n-1)\sin\beta}, \tag{18}$$

and the diameter of the focused beam as a function of z is determined by the expression

$$d_R(z) \approx 2\sqrt{\left[\frac{M_c^2\lambda}{\pi(n-1)\sin\beta}\right]^2 + [z(n-1)-R]^2\sin^2\beta}. \tag{19}$$

Comparison of (6) and (18) shows that the minimum transverse dimension of the beam focused by the lens, resulting from the rounding of the axicon tip, corresponds with good accuracy to the transverse dimension of the central lobe of the Bessel beam. From this it is evident that in the experiment one should expect a relatively drastic visual increase in the transverse size of the central lobe of the Bessel beam in approaching the rounded tip of the axicon at a distance less than the axicon focus (15). The effect of the axicon tip rounding on the increase in the diameter of the propagating central

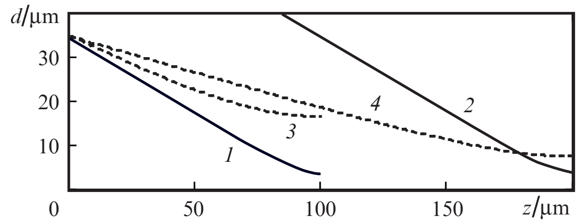

Figure 4. Calculated dependences of the Bessel beam central lobe diameter on z. The parameters used in the calculation are: (*1*) $\alpha = 140°$, $R = 50$ μm, $M^2 = 1$; (*2*) $\alpha = 140°$, $R = 100$ μm, $M^2 = 1$; (*3*) $\alpha = 140°$, $R = 50$ μm, $M^2 = 4$; (*4*) $\alpha = 160°$, $R = 100$ μm, $M^2 = 1$.

lobe of the Bessel beam at different radii of rounding is shown in Fig. 4. It is clearly seen that the round tip of the axicon can lead to a significant increase in the visible size of the central lobe of the Bessel beam, even when it is formed by an ideal Gaussian beam.

4. Experiment

For experimental verification of expressions (5) and (19) determining the change in the transverse dimension of the central lobe of the Bessel beam generated by an axicon with a tip rounded to a radius R from a quasi-Gaussian beam with the propagation parameter $M^2 > 1$, we used an optically pumped vertical external cavity surface-emitting laser (VECSEL) with a wide active area and wavelength of 1040 nm [18]. The laser structure utilised the active region based on InGaAs quantum dots and GaAs/AlGaAs distributed Bragg mirrors. The active semiconductor element was mounted on the intracavity diamond heat sink with a copper base, ensuring effective heat removal to the holder with water cooling. The pump source was a semiconductor laser with a fibre pigtail for radiation with a wavelength of 808 nm. The pump radiation was focused to a spot of 120 μm in diameter. The external V-shaped cavity was formed by a distributed Bragg mirror of an active semiconductor element, by a concave spherical mirror 75 mm in radius, and by a flat input mirror with 0.6% transmission. The propagation parameter of the output beam in our experiments was $M^2 = 2$.

Bessel beams were formed by the axicon with an apex angle of 140° ($\beta = 20°$) and recorded using a telescopic projection system and a CCD detector. The intensity distribution of the Bessel beams at different distances from the axicon was measured by moving the detection system on a micropositioner stage. The parameters of the detection system were chosen so as to ensure the width of the field of view of 100 μm.

Figures 5a–c present several transverse intensity distribution of the Bessel beam, obtained at different distances from the axicon tip with a beam-forming aperture of $\omega_0 = 60$ μm. Quite clear is a tendency to a decrease in the transverse dimension of the central lobe of the Bessel beam with the distance from the axicon tip (i.e., with a decrease in the influence of the tip rounding), and to its increase due to the beam divergence with increasing markedly distance from the axicon. Figure 5d shows the diameter of the central lobe of the Bessel beam versus the distance from the axicon tip z. One can see good agreement of the experimental data with theoretical predictions.

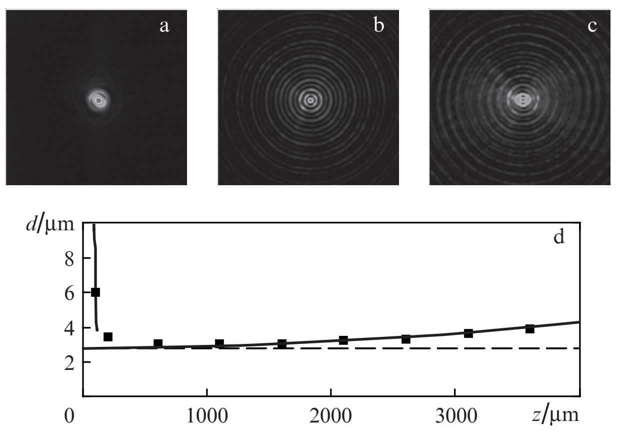

Figure 5. Transverse intensity distributions of the Bessel beam at the beam-forming aperture of the VECSEL $\omega_0 = 60$ μm, $M^2 = 2$ and $\alpha = 140°$, obtained at a distance z = (a) 100 μm, (b) 1100 μm and (c) 3100 μm from the axicon, and (d) the dependence of the Bessel beam central lobe diameter on the longitudinal coordinate z. The experimental values are indicated by black squares. The radius of the axicon tip rounding, R, in calculations is assumed to be 60 μm; the dashed line is the dependence $d(z)$ in the absence of the forming beam divergence.

5. Conclusions

Thus, we have studied the characteristics of the Bessel beam formation using radiation of semiconductor lasers with a high beam propagation parameter M^2. It is shown that the distance of the Bessel beam propagation is determined by the divergence of the quasi-Gaussian beam with high M^2 rather than the geometric parameters of the optical system. We have demonstrated that the rounding of the axicon tip leads to a significant increase in the transverse dimension of the central lobe of the Bessel beam near the axicon.

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References

1. Durnin J. *J. Opt. Soc. Am. A*, **4**, 651 (1987).
2. Zel'dovich B.Ya., Pilipetskii N.F. *Izv. Vyssh. Uchebn. Zaved., Ser. Radiofiz.*, **9**, 95 (1966).
3. McLeod J.H. *J. Opt. Soc. Am.*, **44**, 592 (1954).
4. Garces-Chavez V., McGloin D., Meiville H., Sibbett W., Dholakia K. *Nature*, **419**, 145 (2002).
5. Dholakia K., Cizmar T. *Nature Photon.*, **5**, 335 (2011).
6. Sokolovskii G.S., Dyudelev V.V., Losev S.N., Zolotovskaya S.A., Deryagin A.G., Kuchinskii V.I., Rafailov E.U., Sibbett W. *Pis'ma Zh. Tekh. Fiz.*, **34**, 75 (2008) [*Tech. Phys. Lett.*, **34**, 1075 (2008)].
7. Sokolovskii G.S., Butkus M., Losev S.N., Dudelev V.V., Deryagin A.G., Kuchinskii V.I., Sibbett W., Rafailov E.U. *Proc. SPIE Int. Soc. Opt. Eng.*, **8242**, 82420T (2012).
8. Sokolovskii G.S., Dyudelev V.V., Losev S.N., Deryagin A.G., Vinokurov D.A., Lyutetskii A.V., Pikhtin N.A., Slipchenko S.O., Tarasov I.S., Zolotovskaya S.A., Rafailov E.U., Kuchinskii V.I., Sibbett W. *Pis'ma Zh. Tekh. Fiz.*, **36**, 22 (2010) [*Tech. Phys. Lett.*, **36**, 9 (2010)].
9. Yanson D.A., Rafailov E.U., Sokolovskii G.S., Kuchinskii V.I., Bryce A.C., Marsh J.H., Sibbett W. *J. Appl. Phys.*, **95**, 1502 (2004).
10. Sokolovskii G.S., Dyudelev V.V., Losev S.N., Deryagin A.G., Kuchinskii V.I., Sibbett W., Rafailov E.U. *Pis'ma Zh. Tekh. Fiz.*, **38** (9), 8 (2012) [*Tech. Phys. Lett.*, **38**, 402 (2012)].
11. Losev S.N., Dyudelev V.V., Deryagin A.G., Kuchinskii V.I., Sibbett W., Rafailov E.U., Sokolovskii G.S., in *Tez. 3-go Rossiiskogo simpoziuma 'Poluprovodnikovye lazery: fizika i tekhnologiya'* (Abstracts of the 3rd Russian Symposium

'Semiconductor Lasers: Physics and Technology' (St. Petersburg: Izd. STU, 2012).
12. Siegman A.E. *OSA Annual Meeting* (1998).
13. ISO 11146.
14. Bin Zh., Zhu Li. *Appl. Opt.*, **37**, 2563 (1998).
15. Thaning A., Jaroszewicz Z., Friberg A.T. *Appl. Opt.*, **42**, 9 (2003).
16. Gerard A., Burch J.M. *Introduction to Matrix Methods in Optics* (New York: Wiley, 1975).
17. Sokolovskii G.S., Zolotovskaya S.A., Losev S.N., Dudelev V.V., Deryagin A.G., Kuchinskii V.I., Sibbett W., Rafailov E.U. *Proc. SPIE Int. Soc. Opt. Eng.*, **7919**, 79190J (2011).
18. Butkus M., Wilcox K.G., Rautiainen J., Okhotnikov O.G., Mikhrin S.S., Krestnikov I.L., Kovsh A.R., Hoffmann M., Sudmeyer T., Keller U., Rafailov E.U. *Opt. Lett.*, **34**, 1672 (2009).