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Bistability of self-modulation oscillations in an autonomous solid-state ring laser

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Abstract. **Bistable self-modulation regimes of generation for a ring YAG : Nd chip laser with the counterpropagating waves asymmetrically coupled via backward scattering are simulated numerically. Two branches of bistable self-modulation regimes of generation are found in the domain of the parametric resonance between the self-modulation and relaxation oscillations. The self-modulation regimes observed in earlier experiments pertain to only one of the branches. Possible reasons for such a discrepancy are considered, related to the influence of technical and natural noise on the dynamics of solid-state ring lasers.**

Keywords: *solid-state ring laser, self-modulation regime of generation, dynamic chaos, amplitude and frequency nonreciprocity of ring cavity, bistability.*

1. Introduction

Solid-state ring lasers (SRLs) with a homogeneous gain are characterised by a complicated generation dynamics. Various operation regimes are observed in such lasers, which differ in temporal, spectral and polarisation characteristics of radiation. The nonlinear dynamics of SRL radiation was studied in many papers (see, for example, reviews [1–5] and references therein). The investigations conducted show that in non-autonomous SRLs [4] (in particular, SRLs with periodically modulated parameters) a noticeably greater number of operation regimes are observed as compared to autonomous SRLs [2, 3, 5].

Monolithic (monoblock) SRLs (ring chip lasers) are interesting from scientific and practical points of view. As compared to the SRLs comprised of discrete elements, the chip lasers possess higher temporal, frequency and polarisation stabilities of output radiation, low sensitivity to external actions and high efficiency.

The generation regime of a ring chip laser can be effectively controlled by an external magnetic field, which produces the optical nonreciprocity of a ring cavity [3, 6–11]. In theoretical and experimental studies performed earlier it was shown that the amplitude and frequency nonreciprocities of a ring laser strongly affect the dynamics of the ring chip laser. In particular, an external magnetic field applied to the active element makes it possible to realise the unidirectional generation regime (running wave regime) [8, 9] and a number of self-modulation regimes [7, 10, 11].

An important problem in studying the nonlinear dynamics of ring chip lasers is to find out the conditions in which bistable generation occurs. Bistable states are important in studying nonlinear stochastic phenomena in SRLs [12], in particular, the stochastic resonance in ring lasers [13–15].

It was found [12] that bistable self-modulation oscillations occur in a ring chip laser in the range of the parametric resonance between the self-modulation and relaxation oscillations. In this case, bistable are the self-modulation regime of the first kind and the quasi-periodic self-modulation regime. However, the authors of [12] did not study the influence of the amplitude nonreciprocity of the cavity on the bistability and the analysis was only performed in a limited, rather than the whole, range of parametric resonance.

In the present work, in the frameworks of the vector model of the SRL [16, 17] the detailed analysis is performed with the numerical simulation of bistable self-modulation generation regimes occurring in the range of the parametric resonance. The wide ranges of bistability are found with various self-modulation regimes of generation (the periodic and quasi-periodic self-modulation regimes and regime of dynamic chaos). The results obtained are compared with the experiments performed earlier. Based on this comparison, the adequacy is discussed of the SRL model employed in describing the dynamics of a ring chip laser in the range of the parametric resonance between the self-modulation and relaxation oscillations.

2. Theoretical model and laser parameters

In studying the dynamics of a ring chip laser we used the vector model of the SRL [16, 17]. In this model, the polarisation of radiation for the counterpropagating waves is assumed prescribed and determined by the unit vectors $\boldsymbol{e}_{1,2}$ for the counterpropagating directions. The system of equations in the vector model has the form

$$\frac{\mathrm{d}\tilde{E}_{1,2}}{\mathrm{d}t} = -\frac{\omega}{2Q_{1,2}}\tilde{E}_{1,2} \pm \mathrm{i}\frac{\Omega}{2}\tilde{E}_{1,2} + \frac{\mathrm{i}}{2}\tilde{m}_{1,2}\tilde{E}_{2,1}$$

$$+\frac{\sigma l}{2T}(N_0\tilde{E}_{1,2} + N_{\pm}\tilde{E}_{2,1}),$$

$$T_1\frac{\mathrm{d}N_0}{\mathrm{d}t} = N_{\mathrm{th}}(1+\eta) - N_0 - N_0 a(|\tilde{E}_1|^2 + |\tilde{E}_2|^2) \qquad (1)$$

$$-N_+ a\tilde{E}_1\tilde{E}_2^* - N_- a\tilde{E}_1^*\tilde{E}_2,$$

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$$T_1 \frac{dN_+}{dt} = - N_+ - N_+ a(|\tilde{E}_1|^2 + |\tilde{E}_2|^2) - \beta N_0 a \tilde{E}_1^* \tilde{E}_2.$$

Here, $\tilde{E}_{1,2}(t) = E_{1,2}\exp(i\varphi_{1,2})$ are the complex amplitudes of the counterpropagating waves; and N_0 and $N_\pm$ are the spatial harmonics of inverse population N, determined by the expressions

$$N_0 = \frac{1}{L}\int_0^L N dz, \quad N_\pm = \frac{1}{L}\int_0^L \boldsymbol{e}_1^* \boldsymbol{e}_2 N \exp(\pm i2kz) dz. \quad (2)$$

The system of equations (1) differs from the standard model equations [1] only by the polarisation factor that is present in the last equation

$$\beta = (\boldsymbol{e}_1 \boldsymbol{e}_2)^2 = \cos^2\gamma, \quad (3)$$

where γ is the angle between the unit vectors $\boldsymbol{e}_{1,2}$. In equations (1), the following notations were used: $\omega_c/Q_{1,2}$ is the cavity band widths; $Q_{1,2}$ is the cavity Q-factors for the counterpropagating waves; $T = L/c$ is the time needed for light to pass the cavity of length L; T_1 is the time of longitudinal relaxation; l is the length of the active element; $a = T_1 c\sigma/(8\hbar\omega_c\pi)$ is the saturation parameter; σ is the cross section of lasing transition; $\Omega = \omega_1 - \omega_2$ is the frequency nonreciprocity of the cavity; and ω_1 and ω_2 are the cavity eigenfrequencies for the counterpropagating waves. The pump rate is presented in the form $N_{th}(1 + \eta)/T_1$, where N_{th} is the threshold inverse population and $\eta = P/P_{th} - 1$ is the excess of the pump power over the threshold value. The linear coupling between the counterpropagating waves is determined by the phenomenological complex coupling coefficients

$$\tilde{m}_1 = m_1\exp(i\vartheta_1), \quad \tilde{m}_2 = m_2\exp(-i\vartheta_2), \quad (4)$$

where $m_{1,2}$ are the moduli of coupling coefficients and $\vartheta_{1,2}$ are the corresponding phases.

In the calculations, some parameters were taken equal to experimentally measured values for the laser under study. For a YAG : Nd chip laser the relaxation time is $T_1 = 240$ μs. The cavity bandwidth was determined by the relaxation frequency $\omega_r = \sqrt{\eta\omega_c/(QT_l)}$ ($Q \approx Q_1 \approx Q_2$). At the excess $\eta = 0.218$ in the laser under study the main relaxation frequency is $\omega_r/2\pi = 98.5$ kHz, which gives $\omega_c/Q = 4.37 \times 10^8$ s^{-1}. The value of the polarisation parameter $\beta = 0.75$ was found (as in [17]) by the experimentally measured dependence of the additional relaxation frequency ω_{r1} on the frequency nonreciprocity of the cavity Ω.

As was shown [18], in a ring chip laser the ratio of moduli of coupling coefficients m_1/m_2 can be varied by changing the temperature of the monoblock. In the present work it is assumed that the modulus of one of the coupling coefficients is $m_1/2\pi = 129.4$ kHz and the ratio is $m_1/m_2 = 0.41$. It is difficult to estimate the phase difference of the complex coefficients $\tilde{m}_{1,2}$ by the characteristics of self-modulated oscillations. For simplicity, the phase difference $\vartheta_1 - \vartheta_2$ was taken zero. The amplitude nonreciprocity of the ring cavity $\Delta = \omega_c/Q_2 - \omega_c/Q_1$ varied in the numerical simulation, and the frequency nonreciprocity of the cavity was taken zero.

3. Results

In an autonomous ring chip laser, the self-modulation regime of the first kind arises in a wide range of variations of laser parameters, which is specific in the opposite phase modulation of the intensity of counterpropagating waves. If the self-modulation frequency ω_m is close to the doubled fundamental relaxation frequency, then a number of the nonlinear effects arise related to the parametric interaction between the self-modulation and relaxation oscillations (parametric resonance). In this range instability of the self-modulation generation regime of the first kind arises and more complicated self-modulation regimes are excited (including that of dynamic chaos) [12, 19–23]. If the amplitude nonreciprocity of a ring cavity related to unequal moduli of the coupling coefficients m_1, m_2 is present and the cavity Q-factors Q_1 and Q_2 are not equal for the counterpropagating waves then, as was first shown in [12], the self-modulated generation regimes may become bistable in the considered range.

3.1. First branch of bistable self-modulation regimes

In varying the parameter η from 0.17 to 0.44, two branches of self-modulation generation regimes were observed depending on initial conditions. We denote them as branch 1 and branch 2. First, consider branch 1. In this case, at $\eta = 0.17$ and $\Delta = 0$ the ring chip laser operated in the self-modulation regime of the first kind. Then the value of η increased successively with a step $\delta\eta = 0.01$. After passing through the whole interval of η values, the amplitude nonreciprocity was changed and the calculation repeated again with η varying in the limits mentioned. The amplitude nonreciprocity changed from –500 to 3000 s^{-1} with a step of 250 s^{-1}.

In branch 1 at $0.17 < \eta < 0.22$ the self-modulation regime of the first kind (periodic regime – PR) was observed. The time dependence of the wave intensity I_1 and the spectrum of self-modulation oscillations are shown in Fig. 1.

In the range $0.22 < \eta < 0.33$, the periodic regime arises with the period twice that of self-modulation oscillations (PR2) (Fig. 2a). This regime is specific in that an additional component arises at the half-frequency of self-modulation oscillations (Fig. 2b).

In the range $0.33 < \eta < 0.39$, the periodic self-modulation regime with the doubled period of self-modulation oscillations is changed by the quasi-periodic regime with the period equal to the doubled period of the self-modulation oscillations (quasi-periodic regime – QPR2). In this regime, an envelope of self-modulation oscillations arises (Fig. 3a), and the spectrum of output radiation comprises, in addition to the spectral components at the self-modulation and relaxation frequencies, the components at half the self-modulation frequency. A characteristic dependence of the output radiation on time and the spectrum of self-modulation oscillations in this regime are shown in Fig. 3. At $\eta > 0.39$ this regime changes again to the self-modulation regime of the first kind.

Figure 4a presents the domains of existence for the self-modulation generation regimes in branch 1 in the plane of the laser parameters η, Δ.

3.2. Second branch of bistable self-modulation regimes

For branch 2, the evolution of the self-modulation generation regimes is shown in Fig. 4b under the varying parameters η and Δ. As was earlier established [12, 18], in the ring chip laser under study with the asymmetrical coupling ($m_1/m_2 = 0.4$) at $\eta > 0.19$, in addition to the periodic self-modulation regime of the first kind (PR), also arises the quasi-periodic self-mod-

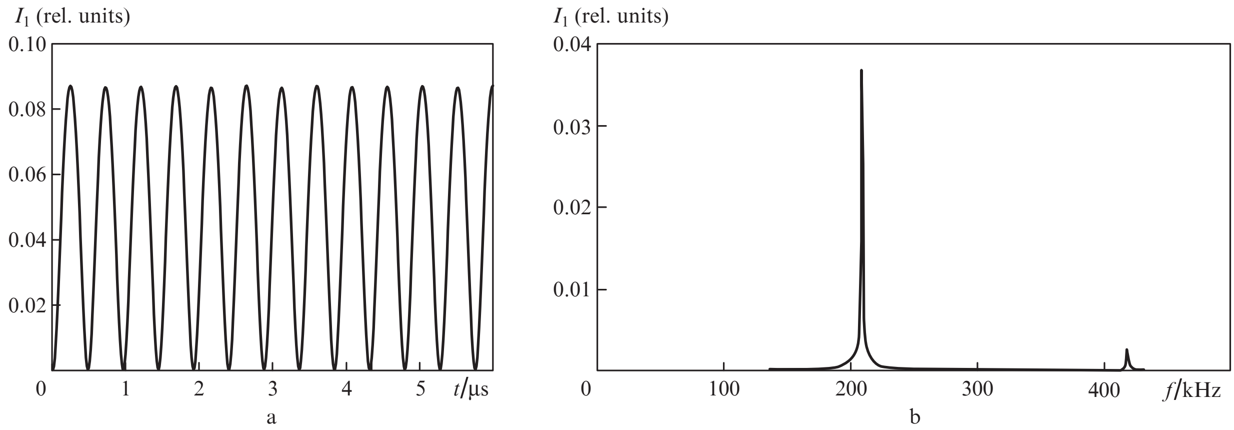

Figure 1. (a) Time dependence of the wave intensity $I_1 = aE_1^2$ in the self-modulation regime of the first kind and (b) the spectrum of intensity at η = 0.19 and Δ = 500 s^{-1}.

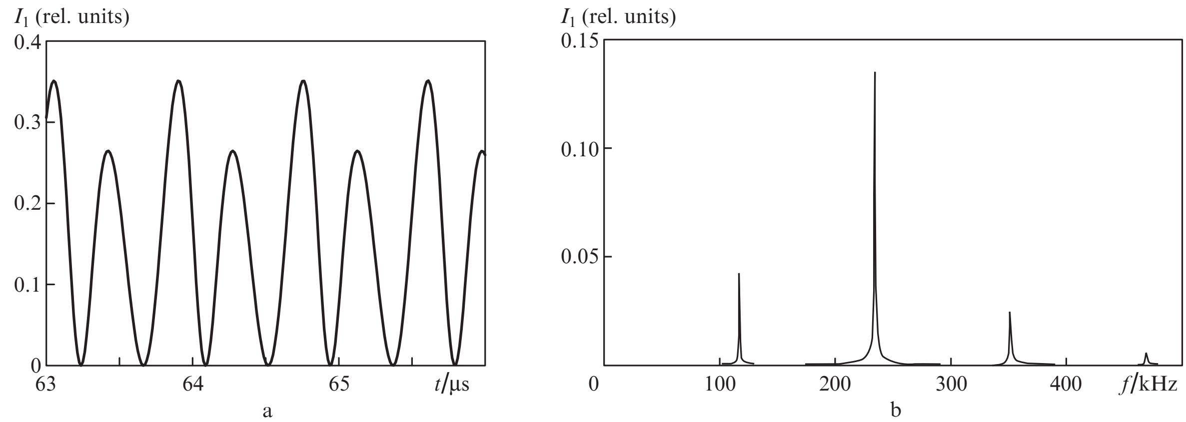

Figure 2. (a) Time dependence of the wave intensity $I_1 = aE_1^2$ in the periodic regime with the doubled period of the self-modulation oscillations and (b) the spectrum of intensity at η = 0.28 and Δ = 500 s^{-1}.

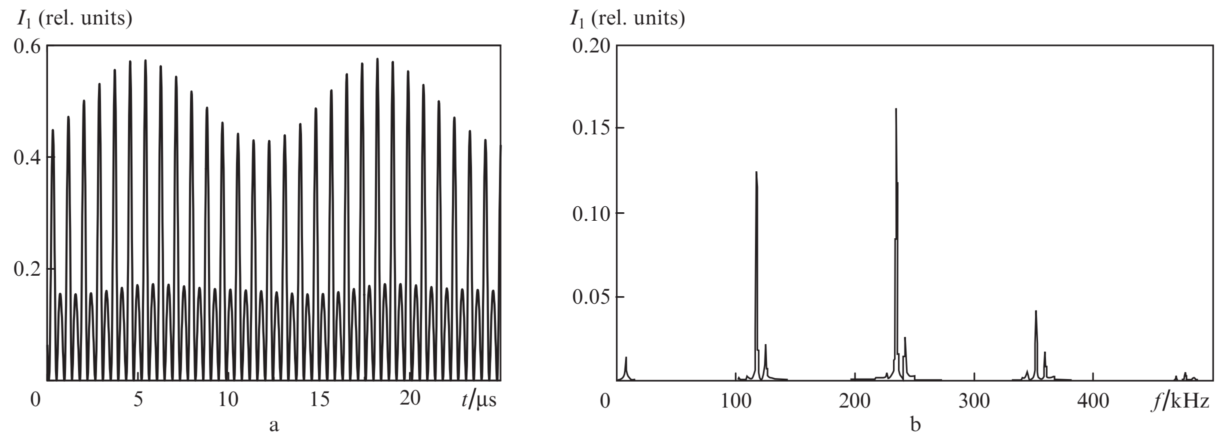

Figure 3. (a) Time dependence of the wave intensity $I_1 = aE_1^2$ in the quasi-periodic regime with the doubled period of the self-modulation oscillations and (b) the spectrum of intensity at η = 0.35 and Δ = 500 s^{-1}.

ulation regime (QPR). In view of this fact, in branch 2 at η = 0.19 and Δ = 0 the initial conditions were chosen corresponding to the QPR. Then the value of η was successively increased with a step $\delta\eta$ = 0.01. After passing over the whole interval of η variation, similarly to the case of branch 1, the amplitude nonreciprocity was changed and the calculation repeated with a new value of Δ.

From Fig. 4b one can see that the quasi-periodic regime QPR at Δ = 0 exists in the range $0.19 < \eta < 0.26$. At greater amplitude nonreciprocity Δ, the domain of existence for this regime converges. The characteristic shape of the self-modulation oscillations of the radiation intensity and the spectrum of one of the waves are shown in Fig. 5. In this regime, in addition to the frequency of self-modulation oscillations f_m,

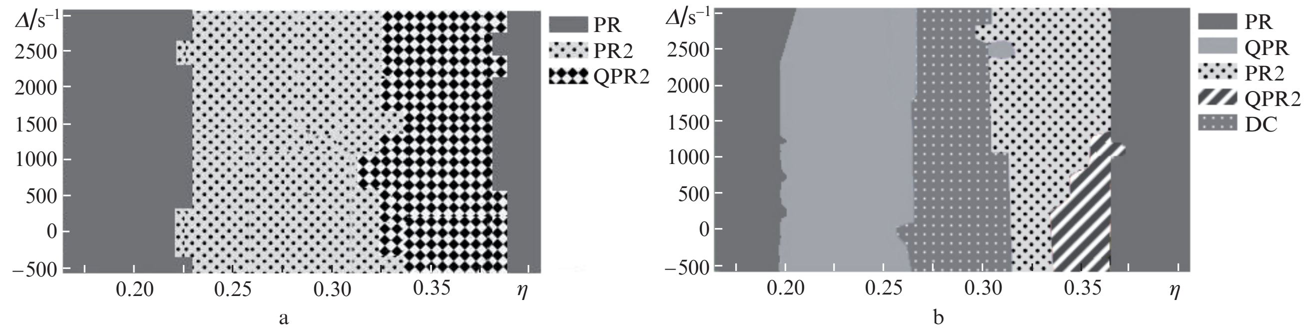

Figure 4. Domains of existence for the self-modulation regimes in branches (a) 1 and (b) 2 in the plane of laser parameters η and Δ.

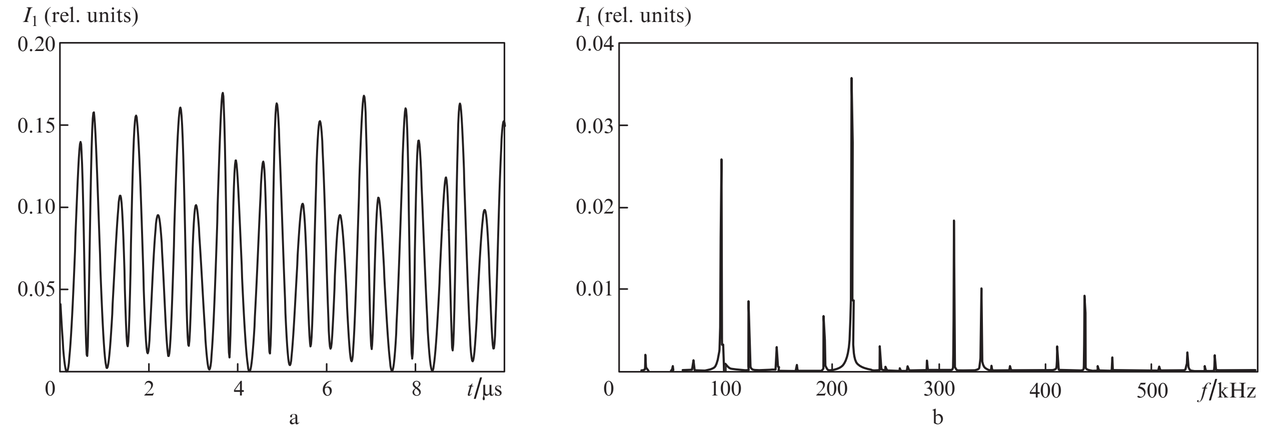

Figure 5. (a) Time dependence of the wave intensity $I_1 = aE_1^2$ in the quasi-periodic self-modulation regime and (b) the spectrum of intensity at η = 0.22 and $\Delta = 250$ s^{-1}.

the fundamental relaxation frequency f_r is excited, the rest spectral components being combination frequencies of these two fundamental frequencies.

In increasing the parameter η, the regime QPR in branch 2 transfers to the regime of dynamic chaos (DC), which at $\Delta = 0$ exists in the range $0.26 < \eta < 0.32$. One can see in Fig. 4b that the amplitude nonreciprocity weakly affects the width of the domain of existence for the DC regime. Time dependences of the counterpropagating wave intensities are shown in Figs 6a, b for the DC regime for two different time intervals (short and long). The spectrum of intensity of one of the waves is shown in Fig. 6c. One can see that this regime has the range of a continuous spectrum, specific for regimes of dynamic chaos, in which a number of discrete frequencies are separated corresponding to the self-modulation and relaxation oscillations.

In branch 2 in the range $0.32 < \eta < 0.37$, the periodic self-modulation regime arises with the doubled period (PR2) and the quasi-periodic self-modulation regime (QPR2). These regimes are similar to those arising in branch 1. In the range $\eta > 0.37$, the bistability vanishes and the self-modulation regime of the first kind arises (PR).

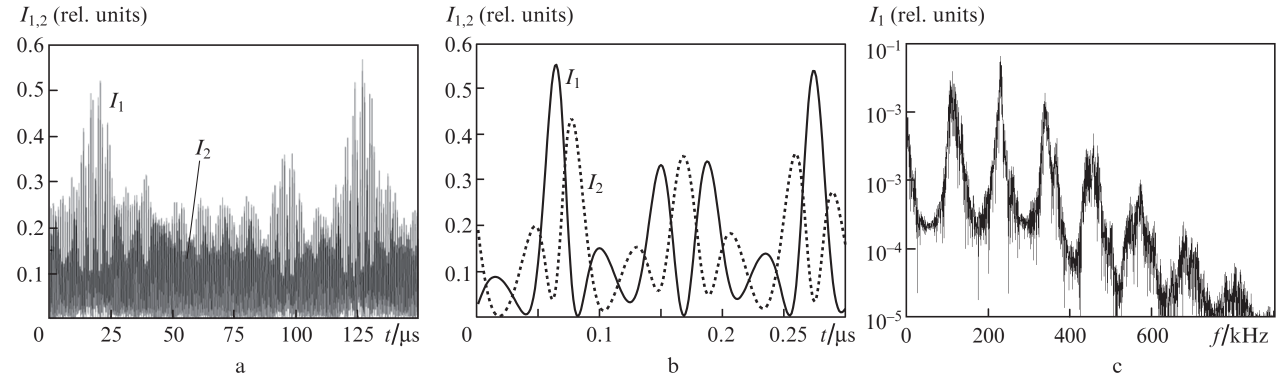

Figure 6. Time dependences of the intensities of counterpropagating waves $I_{1,2} = aE_{1,2}^2$ in the regime of dynamic chaos for (a) long and (b) short time intervals and (c) the spectrum of the intensity I_1 of the wave in this regime at $\eta = 0.28$ and $\Delta = 250$ s^{-1}.

4. Comparison with experiment

Self-modulation regimes arising in the range of the parametric resonance were studied experimentally earlier in [22, 23]. In those works, only the self-modulation regimes of oscillations arising in branch 1 were discovered: the periodic regime with the doubled period of self-modulation oscillations and the quasi-periodic regime with the doubled period of self-modulation. The experimental results well agree with the results of the numerical simulation for branch 1. Branch 2 studied in the present work has not been observed experimentally yet. The only exception is the quasi-periodic self-modulation regime, which has been observed indirectly under the noise modulation of pumping [12]. The investigations conducted in [12, 18] have shown that the bistability of self-modulation oscillations occurring under the asymmetrical coupling of the counterpropagating waves is strongly affected by fluctuations of the pump power. Even under a weak noise of the pumping the quasi-periodic self-modulation regime, as shown in [12], cannot be observed. Based on these results one may assume that the self-modulation regimes pertaining to branch 2 also cannot be observed experimentally due to technical fluctuations of the pump power. To observe instability in this case it is necessary to perform further experimental investigations under thorough stabilisation of the pump radiation and control of technical noise.

One more reason for the discrepancies between the results of numerical simulation and experiments may be the imperfect SRL model employed in the present work. The reason is that in equations (1) of the vector model, the noise of spontaneous emission is usually neglected while describing the generation dynamics. Possibly, in the frameworks of more exact model, the conclusions concerning bistability of self-modulation oscillations in the range of the parametric resonance may change if natural fluctuations of laser radiation parameters would be taken into account.

Thus, for revealing the reasons of the discrepancies between the numerical simulation of the present work and experimental results, additional theoretical and experimental investigations are needed.

5. Conclusions

The numerical simulation of the generation dynamics for a ring chip laser on the basis of the SRL model predicts existence of two bistable branches of self-modulation oscillations. In the experimental studies presented in [22, 23], only the generation regimes pertaining to branch 1 of the present work were observed (regimes PR2 and QPR2). This discrepancy may be related to the influence of technical and natural noise on the dynamics of SRL generation in the range of the parametric resonance between the self-modulation and relaxation oscillations.

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