Quantum Electronics **43** (7) 646–650 (2013)

OPTICAL IMAGE PROCESSING

PACS numbers: 42.30.–d
DOI: 10.1070/QE2013v043n07ABEH015189

New method of contour image processing based on the formalism of spiral light beams

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Abstract. **The possibility of applying the mathematical formalism of spiral light beams to the problems of contour image recognition is theoretically studied. The advantages and disadvantages of the proposed approach are evaluated; the results of numerical modelling are presented.**

Keywords: *spiral light beams, recognition of contour images.*

1. Introduction

The problem of image recognition is rather wide [1] and the first approaches to its solution were proposed as long ago as in the middle of the 20th century. A traditional instrument in this field is the contour analysis [2], the essence of which is considering the image as a set of contours. The existing methods of storage, compression and recognition of contour images suffer from a number of essential drawbacks. In particular, they are based on the presentation of contour images in the form of reduced expansions over certain bases, which makes the expansion dependent on the choice of the reference point at the contour and thus introduce ambiguity into the method of its definition. This circumstance makes necessary the development of new approaches, which could provide reliable and unambiguous recognition of objects having a complex contour structure.

In the present paper we propose an essentially new method of contour image recognition, based on using the mathematical formalism of spiral Gaussian beams, which may remove a part of drawbacks inherent in the known methods of contour analysis.

The basis of the proposed approach are the so called spiral beams, i.e., the light fields, conserving their structure under focusing and propagation and being the subject of study in coherent optics.

At zeros of the complex amplitude of the light field, the distribution of phase possesses a number of specific features, or singularities. They are also referred to as wave front dislocations. Initially this term was introduced into use and considered from the geometrical point of view in the paper by J.F. Nye and M. Berry [3], where the attention was drawn to the distinction between the zeros of the wave field complex amplitude in 1D and 2D cases. The meaning of this difference consists in the following. While for a 1D field the geometrical set of zeros of real and imaginary parts of the field complex amplitude is a set of points, for a 2D field the zeros of real and imaginary parts are lines. In connection with this fact the behaviour of isolated zeros of the amplitude becomes different at small variations (‘stirring’) of the field. In a 1D case the zero points of the real and imaginary parts easily ‘lose’ each other, and a zero of amplitude vanishes. On the contrary, in a 2D field the zero lines of the real and imaginary parts are deformed, but the points of their crossing, i.e., the isolated zeros of amplitude remain stable. The phase of the field is indefinite at the zeros of its amplitude and possesses a helical structure is the vicinity of a zero, while the circulation of the phase gradient around each zero is an integer multiple of 2π. Such isolated points became to be referred to as wave front dislocations, or phase singularities. The sign of a wave front dislocation is determined by the sign of the phase increment during the path-tracing around it.

Among the Russian authors, the attention to these optical objects was paid by Zel’dovich et al. [4]. In their papers the density and sign of dislocations were studied in random light fields (speckle fields). The studies of light fields with phase singularities are carried out also by the research teams lead by Soskin, Volar and Bekshaev. They studied the topological properties of such fields and the methods of their synthesis using specific holograms [5–7]. It is worth noting that the spiral beams used in the present paper are a certain sub-family of singular light fields, since all singularities of the considered beams have the same sign.

The essence of the approach proposed in the present paper is that the operations are carried out not with a planar curve, but with the spiral beam, determined by it. This is possible because there is a one-to-one correspondence between curves and beams. However, it is more profitable to consider exactly a spiral beam as an object, more ‘rich’ from the mathematical point of view and possessing a number of useful properties.

2. Image, contour and curve

In the problem of image recognition the first and obligatory procedure is the selection of the boundaries (contours) of the object. However, in the present work it is implied that the contours have been already extracted using one of the existing methods. The next step is correct description of the obtained contours, i.e., their characteristics should be unambiguous

[*] Presented at the All-Russia Conference on Photonics and Information Optics, Moscow, Russia, January 2013.

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Received 28 March 2013; revision received 3 June 2013
Kvantovaya Elektronika **43** (7) 646–650 (2013)
Translated by V.L. Derbov

and invariant with respect to different factors (particularly, they should be independent of the reference point choice). Figure 1 presents the image of a cat, in which for simplicity only one contour, the image boundary, is selected. Undoubtedly, the result of recognition can (and should) be based on a variety of solutions corresponding to individual contours separated from the image, which is easily provided if a mechanism for comparison of two contours exists. Just the determination of qualitative characteristics of a contour and extracting from them the information about the similarity with the object is the goal of the present paper.

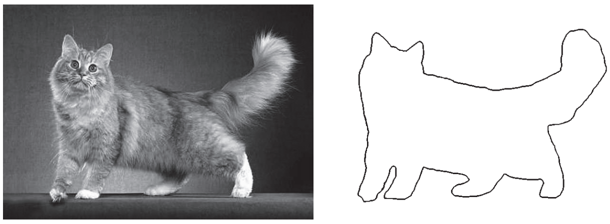

Figure 1. Original image (left) and object contour (right).

It is natural to consider as the mathematical representation of contours some closed planar curves, consisting of an ordered set of points:

$$\zeta(t) = x(t) + \mathrm{i}y(t), \quad t \in [0, T]. \tag{1}$$

Obviously, any closed curve is a periodic function with the period T.

It is clear that any contour can be presented in the form of an infinite expansion in a certain complete set of orthogonal functions. The problem of choosing a convenient basis is, of course, essential. The problem of expansion of the above functions is thoroughly considered in [8], where the classical bases, used in image recognition, are presented. The problem is, however, two-fold. First, to provide reasonable time limits for the analysis process one has to confine himself to a finite number of basis functions. Second, the dimension of the finite set of expansion coefficients with respect to the basis is radically dependent on the reference point, from which the curve 'begins' (i.e., on defining the corresponding function in the interval $[0,T]$ or $[a, a+T]$). Of course, from the point of view of curve description it is of no importance, but only in the case of a complete basis set, which is not implemented in practice because of the limited time and computation resources. All this stimulated us to seek for an alternative approach.

3. Spiral beam

In the process of analysing light fields of different types a new type of light beams referred to as spiral beams was discovered, analysed theoretically and implemented experimentally [9]. It appeared that a spiral beam represents a light field, conserving its structure to a scale factor and rotation in the course of propagation and focusing. Moreover, the structure of such a light field can be rather diverse; in particular, it can have the shape of an arbitrary planar curve, including a closed one.

It was found that the complex amplitude $S(z,z^*)$ of the field of such a beam is uniquely related to the corresponding curve $\zeta(t)$ and is described by the expression

$$S(z,z^* \,|\, \zeta(t),\, t \in [0,T]) = \exp\left(\frac{-zz^*}{\rho^2}\right) f(z)$$

$$= \exp\left(\frac{-zz^*}{\rho^2}\right) \int_0^T \exp\left\{-\frac{\zeta(t)\zeta^*(t)}{\rho^2} + \frac{2z\zeta^*(t)}{\rho^2}\right.$$

$$\left. + \frac{1}{\rho^2}\int_0^t \left[\zeta^*(\tau)\,\mathrm{d}\zeta - \zeta(\tau)\,\mathrm{d}\zeta^*\right]\right\} |\mathrm{d}\zeta/\mathrm{d}t|\,\mathrm{d}t, \tag{2}$$

where ρ is the Gaussian parameter of the beam, and the asterisk denotes complex conjugation. The example of a given curve and the corresponding spiral beam is presented in Fig. 2.

Rather essential is the property of 'quantisation' of spiral beams in the form of closed curves. If the condition (the quantisation condition) holds

$$S_{\mathrm{curve}} = \frac{1}{2}\pi\rho^2 N, \quad N = 0, 1, 2, \ldots, \tag{3}$$

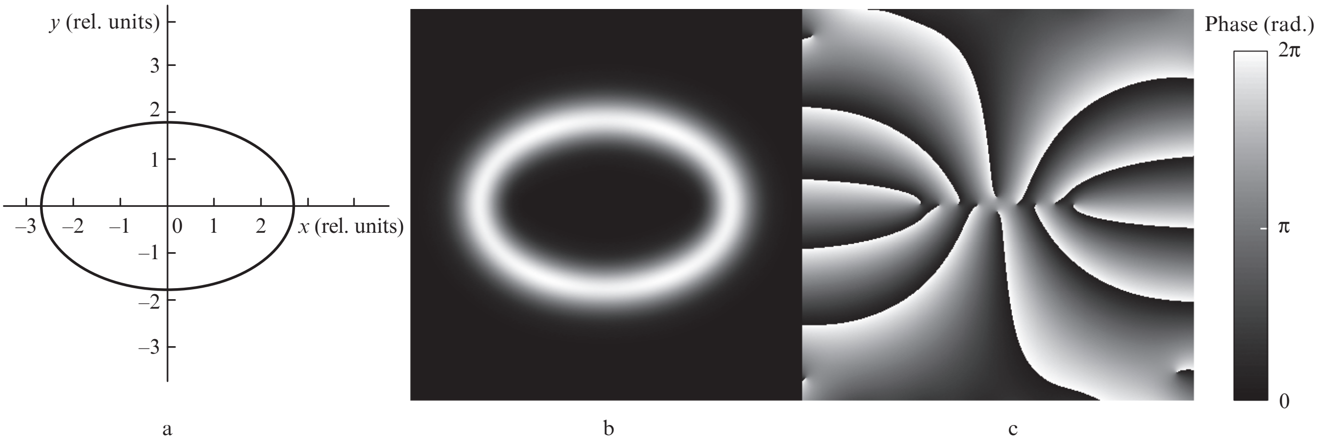

Figure 2. (a) 'Generating' curve; (b) intensity [squared modulus of the complex amplitude $S(z, z^*)$] distribution and (c) phase [argument of $S(z, z^*)$] distribution of the corresponding spiral beam.

where S_{curve} is the area under the curve, then the complex amplitude of the beam field does not depend on the choice of the reference point at the curve. In other words, the spiral beam is not determined by the reference point at the contour. Therefore, any finite sum of the series $S_N(z,z^*\,|\,\zeta(t)\,|,\ t\in[a,a+T])$ is also independent of this reference point to the mutual unimodular term, depending only on the parameter a. Therefore, the problem of choosing the reference point in the analysis and recognition of the input contour is removed. This means that with any required precision we can put into correspondence to the spiral beam $S(z,z^*\,|\,\zeta(t), t\in[a,a+T])$ a finite sum of the series

$$S_N(z,z^*\,|\,\zeta(t),t\in[a,a+T]) = \exp\left(-\frac{zz^*}{\rho^2}\right)\sum_{n=0}^{N} c_n z^n. \tag{4}$$

Since under the rotation of the analysed contour by the angle α the finite sum of the series changes as

$$S_N(z\exp(\mathrm{i}\alpha), z^*\exp(-\mathrm{i}\alpha)\,|\,\zeta(t),t\in[a,a+T]) = \exp\left(-\frac{zz^*}{\rho^2}\right)$$

$$\times\sum_{n=0}^{N}[c_n\exp(\mathrm{i}\alpha n)]z^n = \exp\left(-\frac{zz^*}{\rho^2}\right)\sum_{n=0}^{N} c'_n z^n\,, \tag{5}$$

the problem of the contour rotation is removed, too, and this proves once more that the expansion coefficients can characterise the rotation angles.

We should emphasise one more rather important aspect here. As shown in [10], the quantisation parameter determines the number of zeros of the complex amplitude within the contour and, in fact, the power of the polynomial, corresponding to the initial analytic function of the spiral beam. Apparently, if the analysed contour is complex, the quantisation parameter cannot be small: the complex object cannot be described simply. Nevertheless, the fact that the problem of dependence on the choice of the reference point and the rotation angle is removed is rather essential and makes the proposed method worthy of a detailed analysis. Note, though, that in the case of a complex analysed contour any other method will be also not free of the objective limitation related to the presence of the contour cumbersome configuration.

4. Comparison of contours

Now let us consider two contours, the input one and the reference one, stored in a database, and let us determine whether they correspond to each other or not. Let us construct spiral beams for both contours, keeping the necessary number of terms in the series. Using the above scheme let us put into correspondence to the contours two spiral beams or two sets of complex coefficients $\{c_n^{(1)}\}$ и $\{c_n^{(2)}\}$ $(n = 0, \ldots, N)$.

It is assumed that before the construction of beams the normalisation of the area, bounded by the contours, has already been carried out. The reduction to one area allows determination of the scale of the input object. In the case when the quantisation parameter is sufficient to provide distinguishing between two contours, the above sets of coefficients coincide to a rotation (naturally, within the framework of the fixed basis):

$$\forall n\in[0,N],\quad \frac{|c_n^{(1)}|}{|c_n^{(2)}|} = 1,\quad \varphi_n = \frac{1}{\mathrm{i}n}\ln\frac{c_n^{(1)}}{c_n^{(2)}}. \tag{6}$$

If $\varphi_n = \text{const}$ for all n, then φ_n is the angle α of relative rotation of the contours. This fact is easily obtained by deriving the expression for the ratio of two complex amplitudes from the representation of the spiral beams in the form of series (4) and (5). If condition (6) is not satisfied, one can conclude that the contours do not correspond to each other.

5. Brief description of the algorithm, its advantages and drawbacks

Based on the abovementioned considerations, one can briefly formulate the step-by-step sequence of operations aimed to establish the similarity of two given contours. First, it is necessary to present them as ordered sets of points in the plane, i.e., curves. Using the obtained curves one should calculate the corresponding spiral beams and then expand them in the orthogonal basis, taking the required number of expansion coefficients, which is empirically determined by the problem frameworks. Finally, based on the comparison of two sets of coefficients one can draw a conclusion whether the two contours (or rather there 'hypostases' in the form of spiral beams) are similar to a scale factor and a rotation. Such a sequence of operations can be presented in the form of the block diagram, shown in Fig. 3.

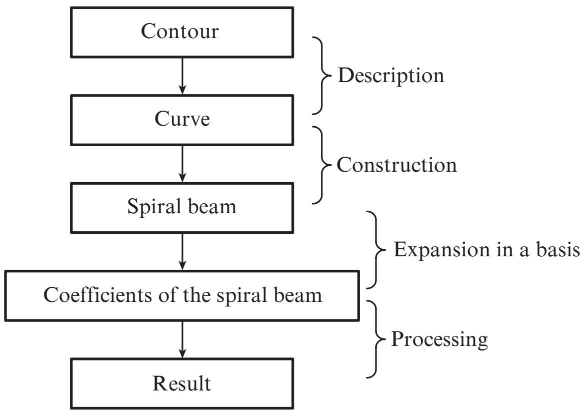

Figure 3. General block diagram of the method.

Thus, it is possible to implement the recognition algorithm, having the following specific features. The first one is the independence of the algorithm operation on the choice of the reference point at the contour and the scale of the contour image. The second one is the decreased recognition time due to reducing the problem of processing a 2D contour to the problem of processing a 1D one. This second feature is not specially reflected in the text, but it is implied that the specificity of spiral beams allows the reduction of 2D expansions in the basis of Laguerre–Gaussian functions [used in the representation of the beam in the form (4)] to 1D Hermite–Gaussian functions. Third, the contour object can be of arbitrary shape; its complexity is limited by the system resolution only, but not by the number of the contour sections, as in other methods. An attractive property of the proposed method is that the enumeration of possibilities, which is typical for such a generally accepted recognition method as the contour analysis using correlation functions, is not required.

As a drawback of the method one should mention the necessity of 2D calculations in the operations carried out at the intermediate stages. However, there are real reasons to believe that this drawback is completely removable using some additional properties of the spiral beams, as already mentioned above. The following unresolved problem is also of importance for real application of the method. Surely, the complete spiral bean unambiguously determines the curve 'generating' it. However, there is one rather nontrivial question that requires an answer, namely, what should be the minimal number of powers in the residual series for a spiral beam, i.e., what should be the quantisation parameter to provide reliable distinction between two objects?

6. Results of numerical modelling

Let us consider four contours: delta-shaped, delta-shaped rotated by the angle 15°, square, and square rotated by the angle 8°. The corresponding illustrations are presented in Figs 4 and 5, and the calculated sets of coefficients are summarised in Tables 1 and 2.

To construct the spiral beams we used the quantisation parameter $N = 10$. The result remains valid for $N > 10$, the precision of the calculations being even better. In the course of construction the mesh of the complex grid was automatically chosen such that the area under the curve S_{curve} was kept equal to $N\pi = 10\pi$ and satisfied relation (3), which yields the Gaussian parameter $\rho = \sqrt{2}$ and the scaling factor equal to 1. From Tables 1, 2 it is seen that there are pairs of contours, corresponding to each other, namely 1, 2 and 3, 4, since only for them condition (6) is satisfied. The conclusion that the contours from different pairs do not correspond to each other is made already at the step of comparing the moduli of the coefficients c_n, so that the calculation of the rotation angle is not necessary.

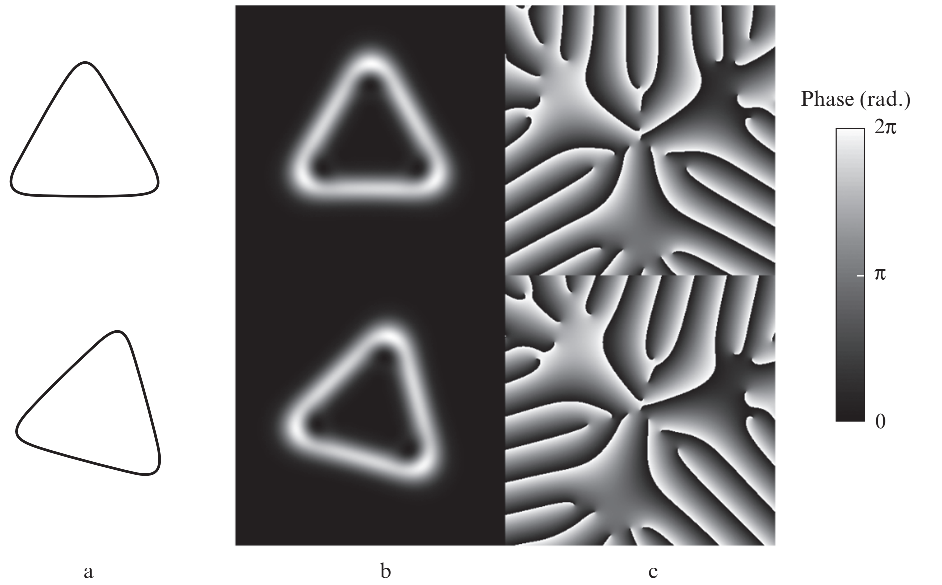

Figure 4. (a) Delta-shaped 'generating' curves and corresponding distributions of (b) intensity and (c) phase of the spiral beams.

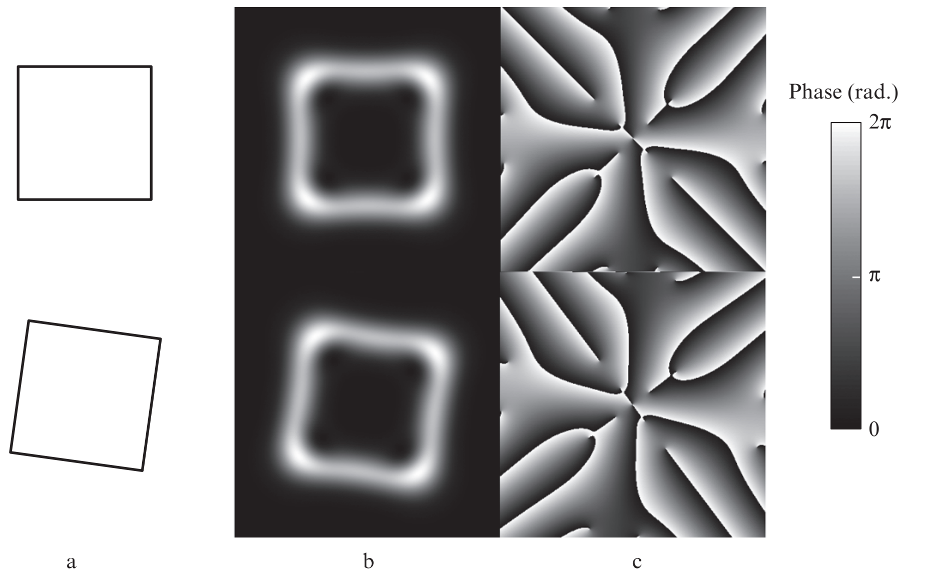

Figure 5. (a) Square-shaped 'generating' curves and corresponding distributions of (b) intensity and (c) phase of the spiral beams.

Table 1. Complex expansion coefficients of delta-shaped spiral beams, their moduli and rotation angles.

n	$c_n^{(1)}/10^{-6}$	$c_n^{(2)}/10^{-6}$	$\lvert c_n^{(1)}\rvert/10^{-6}$	$\lvert c_n^{(2)}\rvert/10^{-6}$	φ_n
0	–1026 – 334i	–1026 – 334i	1079	1079	–
1	14365 – 60664i	29577 – 54879i	62342	62342	15
2	2173 + 367i	1698 + 1404i	2203	2203	15
3	–120 – 256i	96 – 266i	283	283	15
4	–248374 – 64057i	–68712 – 247127i	256501	256501	15
5	457 – 1679i	1740 + 7i	1740	1740	15
6	–865 – 269i	269 – 865i	906	906	15
7	–18539 + 67022i	–59940 – 35254i	69539	69539	15
8	–46 – 17i	37 – 31i	49	49	15
9	–8 + 37i	21 – 32i	38	38	15
10	1039 + 265i	–1032 + 290i	1072	1072	15

Table 2. Complex expansion coefficients of square-shaped spiral beams, their moduli and rotation angles.

n	$c_n^{(3)}/10^{-6}$	$c_n^{(4)}/10^{-6}$	$\lvert c_n^{(3)}\rvert/10^{-6}$	$\lvert c_n^{(4)}\rvert/10^{-6}$	φ_n
0	21	21	21	21	–
1	1494 + 1499i	1271 + 1692i	2117	2117	8
2	–11 + 53909i	–14870 + 51817i	53909	53909	8
3	750 – 768i	997 – 396i	1073	1073	8
4	567 + 2i	480 + 302i	567	567	8
5	8310 + 8338i	1007 + 11729i	11772	11772	8
6	–20 + 99862i	–74226 + 66806i	99862	99862	8
7	1383 – 1395i	1930 + 367i	1965	1965	8
8	36	16 + 33i	36	36	8
9	444 + 446i	–287 + 560i	629	629	8
10	–1 + 3200i	–3151 + 555i	3200	3200	8

7. Conclusions

In the present paper a new approach is proposed within the framework of contour analysis, based on close cooperation of modern coherent optics, theory of functions, and numerical methods. An algorithm for comparing contours is presented and theoretically justified, which allows determination of whether two contours are similar or not to a scale factor and/or rotation. In addition, it was clarified that due to the choice of specific intermediate objects, the spiral beams, the answer to the above question is always unambiguous, which is achieved without enumeration of all possibilities. One should agree that the pay for this advantage is a large volume of necessary calculations, caused by the two-dimensionality of the approach; however, to the authors' opinion there are serious promises for significant reduction of the computation volume, as well as for attaining reasonable complexity of the algorithm.

Acknowledgements. The work was carried out under the support from the Educational and Research Complex of the P.N. Lebedev Physics Institute.

References

1. Vizil'ter Yu.V., Zheltov S.Yu., Bondarenko A.V., Ososkov M.V., Morzhin A.B. *Obrabotka i analiz izobrazhenii v zadachakh mashinnogo zreniya* (Image Processing and Analysis in the Problems of Machine Vision) (Moscow: Fizmatkniga, 2010).
2. Furman Ya.A. *Vvedeiye v konturnyi analiz i ego prilozheniya k obrabotke izobrazhenii i signalov* (Introduction to Contour Analysis and Its Applications to Image and Signal Processing) (Moscow: Fizmatlit, 2003).
3. Berry M.V., Nye J.F. *Proc. R. Soc. London, Ser. A*, **336**, 165 (1974).
4. Baranova N.B., Zel'dovich B.Ya. *Zh. Eksp. Teor. Fiz.*, **80**, 1789 (1981) [*Sov. Phys. JETP*, **53**, 925 (1981)].
5. Bazhenov V.Yu. et al. *J. Mod. Opt.*, **39**, 985 (1992).
6. Soskin M.S., Gorshkov V.N., Vasnetsov M.V., Malos J.T., Heckenberg N.R. *Phys. Rev. A*, **56** (5), 4064 (1997).
7. Soskin M.S., Vasnetsov M.V., in *Progress in Optics* (Amsterdam: Elsevier, 2001) Vol. 42, pp 219–276.
8. Dedus F.F., Kulikova L.I., Pankratov A.N., Tetuyev R.K. *Klassicheskiye ortogonal'nye bazisy v zadachakh analiticheskogo opisaniya i obrabotki signalov* (Classical Orthogonal Bases in the Problems of Analytical Description and Processing of Signals) (Moscow: Izd-vo MGU, 2004).
9. Abramochkin E.G., Volostnikov V.G. *Sovremennaya teoriya gaussovykh puchkov* (Modern Theory of Gaussian Beams) (Moscow: Fizmatlit, 2010).
10. Abramochkin E.G., Volostnikov V.G. *Usp. Fiz. Nauk*, **174**, 1273 (2004) [*Phys. Usp.*, **47** (12), 1177 (2004)].