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On the resolution of lenses made of a negative-index material

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Abstract. **Resolution of the lenses made of a negative-index material is considered. It is shown that the super-resolution concept is untenable and the possibility of obtaining a perfect image on its own eventually contradicts Maxwell's equations in vacuum. It is also shown that known limitations of the diffraction theory on resolution of optical instruments hold true for the resolution of lenses of a negative-index material, in particular, the resolution of a Veselago lens.**

Keywords: *resolution, negative-index material, diffraction theory.*

1. Introduction

In last decades, much attention has been paid to electrodynamics of materials characterised by a negative refractive index. Such materials were first mentioned by L.I. Mandelshtam (see the historical review in [1]), and a systematic study of their properties was started by V.G. Veselago [2]. Due to progress in composite material nanotechnology, the new media were designed possessing the properties that can be explained by a negative refractive index [3, 4]. The thesis was proposed [5, 6] that lenses made of such materials allow one to overcome the diffraction limit for optical instruments. This thesis encounters numerous objections [7], which are not absolutely unquestionable [8]. However, the concept of a superlens (in the form of a plane layer of a negative-index material) suggested in [5], which implies in the ideal case overcoming the diffraction limit and creation of an 'ideal image' has found numerous successors (see, for example, [9–14] and references therein). Nevertheless, the superlens concept itself seems quite strange. Indeed, since Fresnel times the dimension of a focal spot is known to be only determined by the angle between utmost converging rays in the focal plane and by the radiation wavelength (see, for example, [15], §55). The dimension of the focal spot is the property of converging wave regardless of the optical system producing the wave. In the case of a superlens, the converging wave is outside the layer with negative refraction and it seems reasonable that it should be focused to a domain of usual dimensions. It seems strange that the authors of the mentioned works on sulerlenses ignore this conflict with well established fundamental facts.

In [16–18], the strictly stated problem was considered on propagation of the electromagnetic wave emitted by the elementary Hertzian electric dipole, placed parallel to a layer boundary or, in a particular case, to a semi-space. The layer or semi-space is filled with a nergative-index material. The problem was solved based on the classical approach [19] to similar problems with ordinary dielectrics, which date back to the classical Sommerfeld approach [20, 21].

Calculations in [16, 17] were performed for the materials with negative refraction and weak absorption. The size of the focal spot was approximately equal to half-wavelength and no super-resolution was found. In the present work, we will discuss this result from the viewpoint of a strict consequence from a theory of electromagnetic field, namely, the equivalence theory.

2. Resolution of a Veselago lens

Pendry [5] considered the Veselago lens, which is a plane plate (Fig. 1) made of a material with dielectric and magnetic permeabilities $\varepsilon = -\varepsilon_0$ и $\mu = -\mu_0$, respectively, where ε_0 and μ_0 are the permeabilities in vacuum in the SI system. Waves initiated by a point source A undergo negative refraction at the plate boundary and are focused to point P (in the ray approximation). Pendry [5] comes to the conclusion that in the limiting case of infinitesimal losses the 'ideal image of the source' will be formed at point P, that is, a singularity of the focused field will arise or, at least, the spot with the size noticeably smaller then the characteristic problem dimension. Such a conclusion

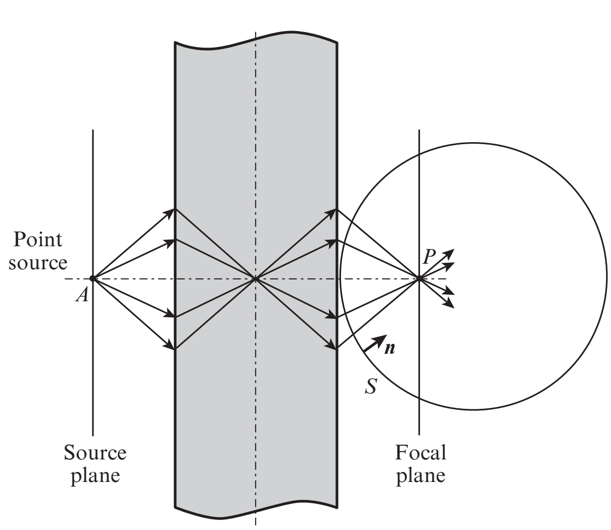

Figure 1. Veselago lens and the auxiliary surface surrounding the focal point P.

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is explained by amplification of evanescent waves in the plate that forms the negative-index lens. In the limiting case of zero losses this amplification would compensate, according to author [5], the damping evanescent waves outside the plate and ideally reproduce all evanescent harmonics in the image plane, thus forming the ideal image. However, the transfer to zero-loss limit in [5] was performed incorrectly. To make the transfer one should fix the absorption, find the field from the point source in the focal plane and then study the focal distribution variation under reduced absorption, which has not been made in [5]. In solving this problem under strict formulation [16, 17] it was found that for each particular value of material losses, the focal spot remains distributed in a finite space domain in accordance with known fundamental diffraction limitation. The followers of the super-resolution concept were not convinced of such an argument (see [22] and references therein). They hope that by reducing substantially the losses of the lens material or introducing into it an active medium one may attain the desirable. But the limitations of diffraction theory are of fundamental character. They cannot depend on the form of harmonic decomposition of the electromagnetic field, because the field can be decomposed on various complete basic wave sets (the field decomposition on plane and evanescent waves is a particular case of possible complete wave sets). Hence, in the present work we will consider the limitations on resolution of focusing optical instruments imposed by the electromagnetic field equations (ultimately by Maxwell's equations).

We will prove that super-resolution is impossible and find the real dimension of the focal spot in the image plane.

Let us surround the image point P by the closed surface S that resides completely in vacuum outside the negative-index material (see Fig. 1). The problem of monochromatic electromagnetic wave propagation in vacuum is well studied. Inside S one may write known and well studied electrodynamics equations. According to the Stratton–Chu formulae [23], the electric and magnetic fields at point P can be found by the known fields at the boundary S in the following way:

$$\boldsymbol{E}_P = \mathrm{i}\omega\mu\boldsymbol{A}_{\mathrm{e}} - \frac{1}{\mathrm{i}\omega\varepsilon}\,\mathrm{grad}\,\mathrm{div}\boldsymbol{A}_{\mathrm{e}} - \mathrm{rot}\,\boldsymbol{A}_{\mathrm{m}}, \qquad (1)$$

$$\boldsymbol{H}_P = \mathrm{i}\omega\varepsilon\boldsymbol{A}_{\mathrm{m}} - \frac{1}{\mathrm{i}\omega\mu}\,\mathrm{grad}\,\mathrm{div}\boldsymbol{A}_{\mathrm{m}} + \mathrm{rot}\,\boldsymbol{A}_{\mathrm{e}}, \qquad (2)$$

where

$$\boldsymbol{A}_{\mathrm{e}} = \frac{1}{4\pi}\int_S [\boldsymbol{n},\boldsymbol{H}]\frac{\exp(\mathrm{i}kr)}{r}\,\mathrm{d}S \ \text{и}\ \boldsymbol{A}_{\mathrm{m}} = \frac{1}{4\pi}\int_S [\boldsymbol{E},\boldsymbol{n}]\frac{\exp(\mathrm{i}kr)}{r}\,\mathrm{d}S$$

are the electric and magnetic vector potentials; $\boldsymbol{n}$ is the internal normal to surface S; $r = |\boldsymbol{r}_P - \boldsymbol{r}_S|$ is the distance from surface element dS to point P; $k = \omega/c$ is the wavenumber; $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$ are the dielectric and magnetic permeabilities inside S; ω is the wave frequency; c is the speed of light in vacuum; and the time dependence of complex representation for harmonic fields is taken in the form $\exp(-\mathrm{i}\omega t)$. Note that the differential operators in (1) and (2) affect the coordinates of point P.

The Stratton–Chu formulae for the domain of S lacking foreign sources of the electromagnetic field express the equivalence theory [24], which follows from Lorentz lemma and is thoroughly considered in the works by Love, Shchelkunov, Kotler, et al. [25].

Consider the physical meaning of Eqns (1), (2). If real electromagnetic fields $\boldsymbol{E}$ and $\boldsymbol{H}$ are known at the boundary of the domain S comprising no foreign sources, then the field at an arbitrary point inside S is a sum of the fields from electric and magnetic surface currents residing on the surface S and having the surface densities $\boldsymbol{j}_{\mathrm{e}} = [\boldsymbol{n},\boldsymbol{H}]$ and $\boldsymbol{j}_{\mathrm{m}} = [\boldsymbol{E},\boldsymbol{n}]$, respectively. The electromagnetic fields of these surface currents are completely equivalent to the fields of external (with respect to surface S) currents; this fact explains the name of the theory of equivalence.

Thus, the electromagnetic field at any point inside domain S is completely determined by the real electromagnetic field on the surface S. This conclusion is independent of the surface shape; hence, the surface S can be easily deformed and turned to a semi-sphere of large radius (Fig. 2).

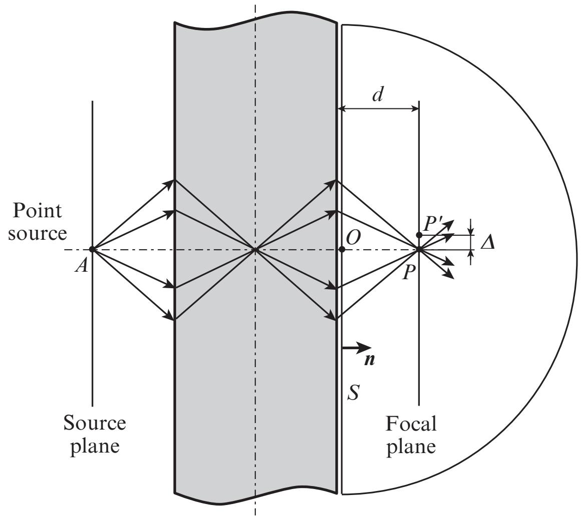

Figure 2. Deformed surface S transformed to the semi-sphere of infinitely large radius.

The integrals in (1), (2) over an infinitely remote semi-sphere surface turn to zero due to the Sommerfeld radiation condition. Finally we come to the integral over the only plane parallel to the plate and residing in vacuum at an infinitely short distance from it.

Note that the transformation of surface S to semi-sphere is not principal for further proof and is only used for clear and convenient consideration. The employment of the Sommerfeld radiation condition is not principal as well.

Further on, in view of the Stratton–Chu formulae, the electromagnetic field at any point P inside surface S is the sum of fields from the surface currents $\boldsymbol{j}_{\mathrm{e}}\mathrm{d}S = [\boldsymbol{n},\boldsymbol{H}]\mathrm{d}S$ and $\boldsymbol{j}_{\mathrm{m}}\mathrm{d}S = [\boldsymbol{E},\boldsymbol{n}]\mathrm{d}S$ distributed over surface S. The area dS with the surface currents written gives at the observation point P the fields

$$\mathrm{d}\boldsymbol{E}_P = \mathrm{i}\omega\mu\left(\frac{\exp(\mathrm{i}kr)}{4\pi r}\,\boldsymbol{j}_{\mathrm{e}}\mathrm{d}S\right) - \frac{1}{\mathrm{i}\omega\varepsilon}\,\mathrm{graddiv}\left(\frac{\exp(\mathrm{i}kr)}{4\pi r}\,\boldsymbol{j}_{\mathrm{e}}\mathrm{d}S\right) - \mathrm{rot}\left(\frac{\exp(\mathrm{i}kr)}{4\pi r}\,\boldsymbol{j}_{\mathrm{m}}\mathrm{d}S\right), \qquad (3)$$

$$\mathrm{d}\boldsymbol{H}_P = \mathrm{i}\omega\varepsilon\left(\frac{\exp(\mathrm{i}kr)}{4\pi r}\,\boldsymbol{j}_{\mathrm{m}}\mathrm{d}S\right) - \frac{1}{\mathrm{i}\omega\mu}\,\mathrm{graddiv}\left(\frac{\exp(\mathrm{i}kr)}{4\pi r}\,\boldsymbol{j}_{\mathrm{m}}\mathrm{d}S\right) + \mathrm{rot}\left(\frac{\exp(\mathrm{i}kr)}{4\pi r}\,\boldsymbol{j}_{\mathrm{e}}\mathrm{d}S\right). \qquad (4)$$

Differentiation of (3) results in the tensor form:

$$(\mathrm{d}\boldsymbol{E}_P)_q = \left(\frac{a_{qw}}{r} + \frac{b_{qw}}{r^2} + \frac{c_{qw}}{r^3}\right)\exp(\mathrm{i}kr)j_{\mathrm{e},w}\mathrm{d}S + \left(\frac{d_{qw}}{r} + \frac{g_{qw}}{r^2}\right)\exp(\mathrm{i}kr)j_{\mathrm{m},w}\mathrm{d}S, \qquad (5)$$

where a_{qw}, b_{qw}, c_{qw}, d_{qw}, and g_{qw} are tensors dependent only on direction cosines of the unity vector directed from the point of integration to observation point, which have the structure

$$\frac{C_1}{r^2}\begin{bmatrix} x^2 & xy & xz \\ xy & y^2 & yz \\ xz & yz & z^2 \end{bmatrix} + \frac{C_2}{r}\begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} + C_3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad (6)$$

where C_1, C_2, C_3 are constants (specific for each tensor): $x = x_P - x_S$; $y = y_P - y_S$; $z = z_P - z_S$; $r = |\boldsymbol{r}_P - \boldsymbol{r}_S|$. The tensors slowly vary versus the direction cosines and this dependence is not principal in further consideration. The field $(\mathrm{d}\boldsymbol{E}_P)_q$ in (5) is intentionally presented in this form in order to stress the dependence of its components on distance r. The expression $(\mathrm{d}\boldsymbol{E}_P)_q$ in (5) where q ranges over values 1, 2, 3 or x, y, z, presents the projection of vector $\mathrm{d}\boldsymbol{E}_P$ to qth coordinate axis (it is a complex value). In the right side of equation (5) we employ the Einstein summation convention with respect to index w.

In view of (5), formula (1) can be rewritten in the form

$$(\boldsymbol{E}_P)_q = \int_S \left[\left(\frac{a_{qw}}{r} + \frac{b_{qw}}{r^2} + \frac{c_{qw}}{r^3}\right)\exp(\mathrm{i}kr)j_{\mathrm{e},w} + \left(\frac{d_{qw}}{r} + \frac{g_{qw}}{r^2}\right)\exp(\mathrm{i}kr)j_{\mathrm{m},w}\right]\mathrm{d}S. \qquad (7)$$

Prior to determining the size of the focusing domain we have to answer the question: what does the ideal focusing mean from the viewpoint of the equivalence theory?

The answer is obvious: under ideal focusing the fields $(\mathrm{d}\boldsymbol{E}_P)_q$ from all areas $\mathrm{d}S$ are added in-phase. On the complex plane, the field $(\mathrm{d}\boldsymbol{E}_P)_q$ from each element $\mathrm{d}S$ is presented by a vector. Hence, under the ideal focusing, the vectors for all $\mathrm{d}S$ are parallel to each other and form the complex vector of the resulting field component $(\boldsymbol{E}_P)_q$, for which the vector diagram of each component $(\boldsymbol{E}_P)_q$ of the total field can be considered similarly to summing oscillations in optics ([26], p. 372). The vector diagram in Fig.3 conditionally shows the summation of fields from separate surface elements $(\mathrm{d}\boldsymbol{E}_P)_q$, which gives the vector of the resulting field component $(\boldsymbol{E}_P)_q$ at the focal point P (light arrows correspond to ideal focusing). The conditional character of this diagram implies that real integration in (7) requires summation of infinitesimal vectors $(\mathrm{d}\boldsymbol{E}_P)_q$ having different lengths. The angle of the resulting vector relative to the origin of coordinate system is not significant and is determined by the origin of time scale. We specially stress that the vectors shown in Fig. 3 in a complex plane present the complex numbers $(\boldsymbol{E}_P)_q$ and $(\mathrm{d}\boldsymbol{E}_P)_q$, which in the real space are not vectors, but the components of the complex electric field vector at the coordinate axes; for example, $(\boldsymbol{E}_P)_q$ may be a projection of the electric field vector onto one of the coordinate axes in the focal plane.

Let the observation point be shifted from position P to P' by the small distance Δ along the focal plane (see Fig. 2), such that $\Delta \ll d$ and $\Delta \ll \lambda$ (d is the distance from the lens to focal plane and λ is the wavelength inside S). The total electric field at the observation point in this case slightly changes. In the

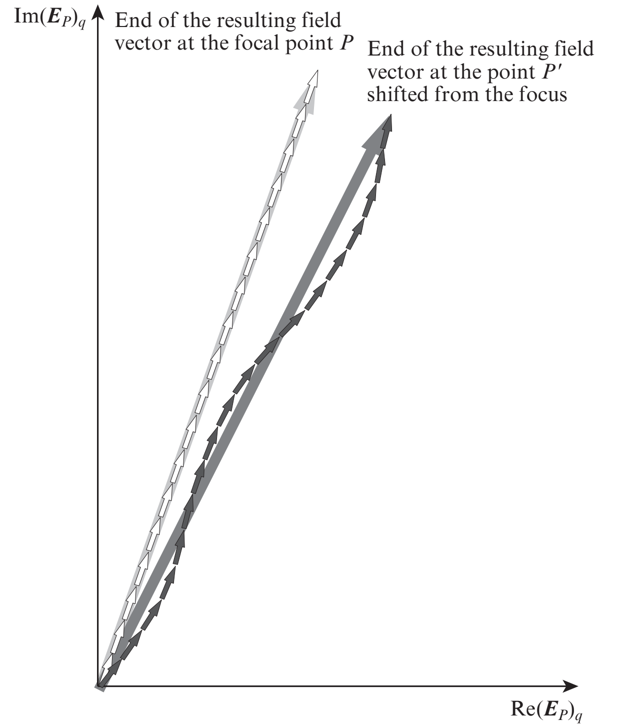

Figure 3. Schematic vector diagram showing summation of fields from separate surface elements $(\mathrm{d}\boldsymbol{E}_P)_q$ to the vector of the resulting field $(\boldsymbol{E}_P)_q$ at the focal point P (light arrows) and at shifted point P' (dark arrows).

shift, the contributions from separate surface elements $\mathrm{d}S$ vary according to (5) due to both length variations of vectors $(\mathrm{d}\boldsymbol{E}_P)_q$ and their small turning (in accordance with the phase changes in the waves arising in shifting the observation point). In Fig. 3, the result of summing vectors $(\mathrm{d}\boldsymbol{E}_P)_q$ at shifted point P' is shown by black arrows.

We may estimate the size of the domain in the focal plane where the field is close to that at the focal point P. The two limiting cases may be separated: $d \gg \lambda$ and $d \ll \lambda$. We will assume that in the case $d \gg \lambda$ point P resides in the far-field zone of the lens and in the case $d \ll \lambda$ it resides in the near-field zone.

First, consider the far-field case ($d \gg \lambda$). Under a small shift ($\Delta \ll d$) the variations of $(\mathrm{d}\boldsymbol{E}_P)_q$ in (5) are determined by the changes in the phase factor $\exp(\mathrm{i}kr)$ arising in the shift. The elementary vectors on the vector diagram would turn with a virtually constant amplitude. Hence, the resulting vector would reduce in amplitude and, generally speaking, it would turn as well. It is not difficult to understand that a noticeable change in the amplitude of the resulting vector occurs at $\Delta \sim \lambda/2$, where λ is the wavelength in vacuum. In this case, the phases of the waves determined by separate elements vary from zero for $\mathrm{d}S$ on the axis of symmetry (point O in Fig. 2) to $\pm k\Delta \sim \pi$ for infinitely far elements of the plane. Hence, the characteristic dimension of the focal spot in the vicinity of the focal point P is $\sim\lambda$. At small shifts (such that $\Delta \ll \lambda/2$), the resulting vector $(\boldsymbol{E}_P)_q$ cannot change noticeably because turns of the summed vectors $(\mathrm{d}\boldsymbol{E}_P)_q$ in this case are negligible and their length is not changed.

The last result directly entails the impossibility of Pendry super-resolution in the far-field zone. Indeed, if the field at point P is infinitely high then under a small but finite shift $\Delta \ll \lambda/2$ the relative variation of the field amplitude $(\boldsymbol{E}_P)_q$ is small; hence, the field will be infinitely high in a limited domain, which is not possible, because an infinite electromagnetic energy will be concentrated in the domain (the point source before the lens emits a finite energy). Moreover, successive (from the boundary of the infinite field zone obtained) finite shifts $\Delta \ll \lambda/2$ allow one to further expand this zone of infinite field, thus gradually filling the domain S.

In the near-field zone ($d \ll \lambda$) at small shift $\Delta \ll d$ the greatest changes in (5) will be contributed by the factors

$$\left(\frac{a_{qw}}{r} + \frac{b_{qw}}{r^2} + \frac{c_{qw}}{r^3}\right) \text{ and } \left(\frac{d_{qw}}{r} + \frac{g_{qw}}{r^2}\right),$$

whereas the contribution of the phase multiplier $\exp(ikr)$ will be negligible. In this case, the elementary vectors $(\mathrm{d}\boldsymbol{E}_P)_q$ on the vector diagram actually do not turn but only change in amplitude. On surface S, maximal equivalent surface currents $\boldsymbol{j}_e = [\boldsymbol{n}, \boldsymbol{H}]$ and $\boldsymbol{j}_m = [\boldsymbol{E}, \boldsymbol{n}]$ will be observed near the axis of symmetry (point O in Fig. 2) being in-phase [at $\Delta \ll d$ the variation of $\exp(ikr)$ is negligible]. It is obvious [see (7)] that a noticeable change in the resulting vector amplitude will occur at $\Delta \sim d/2$, where d is the characteristic problem dimension. Now we may conclude that the focal spot in the vicinity of point P in the near-field zone has the dimension of $\sim d$. At small shifts (such that $\Delta \ll d$) the resulting vector $(\boldsymbol{E}_P)_q$ cannot become noticeably smaller because the summands $(\mathrm{d}\boldsymbol{E}_P)_q$ negligibly change their lengths at invariable directions.

The proof of impossibility of Pendry super-resolution in the near-field zone is similar to that for the far-field zone. Indeed, if the field is infinitely high at point P then a small, however, finite shift $\Delta \ll d$ will cause a small relative change in the amplitude of the field $(\boldsymbol{E}_P)_q$. Hence, the field will be infinitely high in a finite domain in the vicinity of focus which is impossible because an infinitely high energy will be concentrated in this domain (a point source emits a finite energy). Similar consideration leads to the conclusion that super-resolution is impossible in the intermediate zone as well.

The conclusions derived above are completely confirmed by the calculations of focusing properties of the Veselago lens with weak material absorption performed in the frameworks of theory in [16, 17].

It is appropriate to answer the criticism of [16, 17] expressed in [22]. The authors contend that the integration limits in [16, 17] are not sufficiently large to correctly take into account all evanescent waves contributing into 'super-resolution'. The fact that analytical solutions in [16, 17] are correct and strict is not subjected to question in [22]. However, the criticism of the numerical calculations [16, 17] is not grounded by the following reasons.

1. In [16], the focus inside the Veselago lens (which is by some reason missed in [22] and which should also comprise the perfect Pendry image) is rigorously investigated. The upper integration limit in [16] is approximately $10^2 k_0$, where k_0 is the wavenumber in vacuum at the considered frequency. In this case, the details of focal distribution of an approximately 100-times smaller scale than the wavelength λ in vacuum could be resolved; however, the calculations resulted in that the focal field distribution has an ordinary dimension (on the order of λ). The calculation details are given in [16].

2. In [17], the rigorous solution is presented in the form of an integral for the problem on finding the focal energy distribution for the Veselago lens. As the absorption in the lens tends to zero, the numerical integration fails at some stage due to large exponents arising in the integrand followed by overflow of numerical notation. In order to avoid this difficulty and solve the problem, the small absorption was added to the lens material $\varepsilon = \varepsilon_0(-1 + 0.001i)$ and $\mu = \mu_0(-1 + 0.001i)$. One can ascertain that the evanescence waves will have noticeable amplitudes only for the wavenumbers slightly larger than k_0; then an increase in the wavenumber over which the integral is taken, the amplitudes sharply fall. Hence, to obtain a correct result (at the mentioned absorption inside the lens) it is sufficient to consider the wavenumber range limited by $2k_0 - 3k_0$, which was done in [17]. The calculation in [17] might reveal the details of focal distribution with the dimensions of $\sim\lambda/3$; however, the distribution with the ordinary dimension $\sim\lambda$ was obtained. Issuing from the conclusions of the present work we can assert that this is not accidental. Hence, the answer to criticism in [22] is as follows: the limits in [17] were chosen sufficient for the integral to converge and for observing the hypothetical super-resolution that was not eventually found.

3. Much more precise numerical calculations were performed in [16], which prove that the spot in the focus inside the Veselago lens has a dimension of $\sim\lambda$. The Veselago lens can be presented as two such lenses (one of them gives the image of the point source inside the Veselago lens and the second transfers this image to the external focus of the lens). Hence, in the focus of the Veselago lens, the dimension of a minimal spot should also be approximately λ! A finite-size spot inside the lens cannot be converted to a point in the external focus!

Finally, note that mathematically incorrect calculations were made by Pendry in [5], which finally resulted in all those misunderstandings with super-resolution. The author first takes a sum of geometric series and then the denominator in the obtained formula tends to infinity, which is not correct mathematically.

3. Conclusions

Thus, in the present work we prove that a super-resolution and, hence, a system producing the ideal image is impossible. The field focused in free space cannot have a singularity. The dimension of a minimal focal spot from a point source in the image plane is the value $\sim\lambda$ in the far-field zone and on the order of the characteristic size of problem d in the near-field zone. These conclusions are independent of the construction of focusing system and materials from which the system is created including constructions with active and superconducting media.

To avoid oppressive search for super-resolution in future, the impossibility of super-resolution is suggested to be termed second principle of electrodynamics by analogy with second principle of thermodynamics (impossibility of perpetuum mobile).

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