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Nonlinear waves in an array of zigzag waveguides with alternating positive and negative refractive indices

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Abstract. **Interaction of coupled waves propagating in a system of waveguides with alternating positive and negative refractive indices is studied theoretically. The zigzag configuration of the waveguides in the array allows communication not only between the nearest neighbours, but also with the waveguides beyond them. It is shown that the spectrum of linear waves in such a waveguide system has a bandgap. Partial solutions are found to the system of coupled waves corresponding to a stationary electromagnetic field pulse that propagates along the array of tunnel-coupled waveguides as a whole. Investigation of the interaction of nonlinear solitary waves has demonstrated numerically the stability of their relatively weak disturbances and collisions with each other.**

Keywords: *tunnel-coupled waveguides, negative refraction, forward and backward waves, optical solitons.*

1. Introduction

A new branch has recently appeared in applied optics, termed 'transformation optics' [1–6]. In many ways, transformation optics owes its origin to the construction of artificial media, i.e. metamaterials, which feature unusual electrodynamic properties [7–10]. In particular, metamaterials are characterised by 'negative refraction'. It is sometimes said of a negative refractive index for electromagnetic waves in such media. Media with negative refraction, designed for the optical (more precisely, IR) range [11], allow one to control the light flux in both micro- and macroscopic scales [4, 5]. It is predicted that the methods of transformation optics can make it possible to curve rays arbitrarily, thereby forming any desired electromagnetic field distribution in space. However, even in traditional fields of optics, metamaterials can be useful. For example, the properties of the negative index media can be used in the construction of new optical components for the integrated or fibre optics.

It is known that the unusual properties of negative index materials manifest themselves upon refraction or localisation of electromagnetic waves near the ordinary medium–negative index medium interface [12–14]. The authors of papers [15, 16] considered a similar case when the waves propagate in the closely spaced waveguides, one of which is made of a nonlinear positive index material, and the other – of linear or nonlinear negative index material, and their coupling is due to frustrated total internal reflection. It was found that in such an extended nonlinear anti-directional coupler (NADC), a slit soliton – a stationary electromagnetic field momentum that runs in both waveguides as a single solitary wave – can propagate. In addition to a double waveguide NADC, the authors of [17, 18] discussed the array of tunnel-coupled waveguides with alternating signs of refractive indices and showed that the spectrum of linear waves has under certain conditions a bandgap.

In a system of tunnel-coupled waveguides, when the fields are strongly localised in the waveguides themselves, only the interaction between nearest neighbours is essential [19]. Efremidis and Christodoulides [20] proposed a configuration of a waveguide array, in which the interaction between the next-to-nearest neighbours can be just as strong as the interaction between the nearest neighbours. To do this, above an array of waveguides periodically arranged in the same direction a second array of similar waveguides is placed, which is shifted by half a period with respect to the first one (Fig. 1a). The numbering of the waveguides can be chosen so that, for example, even numbers indicate the lower row of the waveguides, and the odd numbers – the upper row.

In this paper we investigate the waveguide configuration, similar to that described in [20], but with the difference that the refractive indices of the first and second waveguide arrays differ in sign (Fig. 1b). We determine the spectrum of linear waves in this system and consider the formation of a stationary solitary wave in an extended asymmetric (i.e., only a conventional optical waveguide has nonlinear properties) zigzag array. The numerical solution of the corresponding equations demonstrates the stability of the solitary waves.

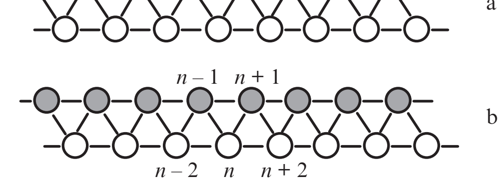

Figure 1. Schematics illustrating the system of the waveguides: (a) an array of identical waveguides [20] and (b) an array of alternating positive and negative index waveguides.

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2. Basic equations of the model

Assuming that the waveguide with the number n has a positive refractive index and neighbouring waveguides with numbers $n-1$ and $n+1$ have a negative refractive index for the spectral region in question, we can write the system of equations for the fields in the waveguides:

$$\mathrm{i}\left(\frac{\partial}{\partial\zeta}+\frac{\partial}{\partial\tau}\right)e_n + c_1(e_{n+1}+e_{n-1})$$

$$+c_2(e_{n+2}+e_{n-2}) + r|e_n|^2 e_n = 0, \quad (1)$$

$$\mathrm{i}\left(-\frac{\partial}{\partial\zeta}+\frac{\partial}{\partial\tau}\right)e_{n+1} + c_1(e_{n+2}+e_n) + c_3(e_{n+3}+e_{n-1}) = 0, \quad (2)$$

where e_n is the normalised electric field strength of a quasi-harmonic wave localised in the nth waveguide [19]. The details of the derivation of these equations with the nonlinearity taken into account can be found in [17, 21]. Single-mode, polarisation-maintaining waveguides are considered as a simple common example of discrete optical systems [22–24].

In equations (1) and (2), nonlinearity is taken into account only for waves propagating in positive index waveguides and is characterised by the coefficient r. The relationship between the fields in neighbouring positive index waveguides is determined by the constant c_2, the coupling constant c_3 characterises the interaction between neighbouring negative index waveguides and c_1 – between positive and negative index waveguides. This configuration is called an asymmetric nonlinear optical waveguide zigzag array (ANOWZA).

3. Linear properties of the waveguide system

In the linear approximation, the equations describing the propagation of the waves in an asymmetric waveguide zigzag array have the form:

$$\mathrm{i}\left(\frac{\partial}{\partial\zeta}+\frac{\partial}{\partial\tau}\right)e_n + c_1(e_{n+1}+e_{n-1}) + c_2(e_{n+2}+e_{n-2}) = 0, \quad (3)$$

$$\mathrm{i}\left(\frac{\partial}{\partial\zeta}-\frac{\partial}{\partial\tau}\right)e_{n+1} - c_1(e_{n+2}+e_n) - c_3(e_{n+3}+e_{n-1}) = 0. \quad (4)$$

To find the spectrum of the linear waves, we must search, as usual, for a solution to this system in the form of a travelling wave with constant amplitudes of its components

$$e_n = A\exp(-\mathrm{i}\omega\tau + \mathrm{i}q\zeta + \mathrm{i}\phi n),$$

$$e_{n+1} = B\exp[-\mathrm{i}\omega\tau + \mathrm{i}q\zeta + \mathrm{i}\phi(n+1)].$$

Substitution of these expressions into (3) and (4) leads to a system of uniform linear equations for the amplitudes A and B, a non-trivial solution of which is possible only if its determinant

$$\det\begin{pmatrix} q-\omega-\gamma_2 & -\gamma_1 \\ \gamma_1 & q+\omega+\gamma_3 \end{pmatrix}$$

is equal to zero. Here we introduced the following parameters:

$$\gamma_1 = 2c_1\cos\phi, \quad \gamma_2 = 2c_2\cos 2\phi, \quad \gamma_3 = 2c_3\cos 2\phi.$$

The requirement that the determinant is zero leads to the equation

$$(\omega+\omega_0)^2 = \gamma_1^2 + (q-q_0)^2,$$

where

$$2\omega_0 = \gamma_2+\gamma_3, \quad 2q_0 = \gamma_2-\gamma_3.$$

Thus, we obtain a spectrum of linear waves in the ANOWZA with the alternating sign of the refractive index:

$$\omega^{(\pm)}(q) = -\omega_0 \pm \sqrt{\gamma_1^2+(q-q_0)^2}. \quad (5)$$

One can see that the spectrum of the linear waves has a gap of width $\Delta\omega = 2\gamma_1$ and that the spectrum is shifted with respect to the frequency and wavenumber scale. The rest of the spectrum is the same as in the waveguide array [17, 18]. The gapless spectrum is possible only under the condition $\phi = \pi/2$. In this case, the radiation propagates along the waveguides with the same sign of the refractive index and the neighbouring waveguides do not exchange energy.

4. Nonlinear waves in the ANOWZA

The simplest approximate solution, which allows us to consider analytically the propagation of the waves in the ANOWZA, described by equations (1) and (2) has the form:

$$e_n(\zeta,\tau) = A(\zeta,\tau)\exp(\mathrm{i}\phi n), \quad e_{n+1}(\zeta,\tau) = B(\zeta,\tau)\exp[\mathrm{i}\phi(n+1)].$$

In the linear approximation, A and B were the amplitudes of the quasi-harmonic waves. Here, these quantities are the envelopes of the quasi-harmonic waves. The system of equations (1) and (2) yields the equations for these envelopes:

$$\mathrm{i}\left(\frac{\partial}{\partial\zeta}+\frac{\partial}{\partial\tau}\right)A + \gamma_1 B + \gamma_2 A + r|A|^2 A = 0, \quad (6)$$

$$\mathrm{i}\left(\frac{\partial}{\partial\zeta}-\frac{\partial}{\partial\tau}\right)B - \gamma_1 A - \gamma_3 B = 0. \quad (7)$$

When $\phi = \pi/2$, the system is uncoupled into two independent equations, one of which is linear and the other – nonlinear. When $\phi = \pi/4$, the system of equations (6) and (7) reduces to that discussed previously in [15, 16]. Both of these cases will not be considered here.

The system of equations (6) and (7) can be represented in a real form by setting $A = a_1\exp(\mathrm{i}\varphi_1)$ and $B = a_2\exp(\mathrm{i}\varphi_2)$:

$$\left(\frac{\partial}{\partial\zeta}+\frac{\partial}{\partial\tau}\right)a_1 = \gamma_1 a_2\sin\Phi, \quad \left(\frac{\partial}{\partial\zeta}-\frac{\partial}{\partial\tau}\right)a_2 = \gamma_1 a_1\sin\Phi,$$

$$\left(\frac{\partial}{\partial\zeta}+\frac{\partial}{\partial\tau}\right)\varphi_1 = \gamma_1\frac{a_2}{a_1}\cos\Phi + \gamma_2 + ra_1^2,$$

$$\left(\frac{\partial}{\partial\zeta} - \frac{\partial}{\partial\tau}\right)\varphi_2 = -\gamma_1\frac{a_1}{a_2}\cos\Phi - \gamma_3,$$

where $\Phi = \varphi_1 - \varphi_2$.

4.1. Stationary waves

Among the variety of the waves we can single out the stationary waves (sometimes referred to as travelling waves). The solutions of the equations, which depend only on one variable, correspond to these waves. Let the variable in this case have the form

$$\xi = \gamma_1\frac{\zeta + \beta\tau}{\sqrt{1-\beta^2}},$$

where β is the parameter determining the speed of the wave's propagation. If we introduce new envelopes u_1 and u_2, such that $u_1 = \sqrt{1+\beta}\,a_1$, $u_2 = \sqrt{1-\beta}\,a_2$, then u_1, u_2 and Φ will be determined by the system of equations:

$$\frac{\partial u_1}{\partial\xi} = u_2\sin\Phi, \tag{8}$$

$$\frac{\partial u_2}{\partial\xi} = u_1\sin\Phi, \tag{9}$$

$$\frac{\partial\Phi}{\partial\xi} = \delta + \left(\frac{u_1}{u_2} + \frac{u_2}{u_1}\right)\cos\Phi + \vartheta u_1^2, \tag{10}$$

where we have introduced two parameters

$$\vartheta = \frac{r}{\gamma_1(1+\beta)}\sqrt{\frac{1-\beta}{1+\beta}},\quad \delta = \left(\frac{\gamma_3}{\gamma_1}\sqrt{\frac{1+\beta}{1-\beta}} + \frac{\gamma_2}{\gamma_1}\sqrt{\frac{1-\beta}{1+\beta}}\right).$$

From equations (8) and (9) it follows that the first integral of the system is

$$u_1^2 - u_2^2 = C_1 = \text{const}. \tag{11}$$

If we multiply both sides of equation (10) by $u_1u_2\cos\Phi$ and use again (8) and (9), we can obtain the second integral

$$u_1u_2\cos\Phi + \frac{\delta}{2}u_1^2 + \frac{\vartheta}{4}u_1^4 = C_2 = \text{const}. \tag{12}$$

Depending on boundary conditions there can exist different solutions to equations (8)–(10) describing stationary waves.

4.2. Solitary waves

Stationary solitary waves correspond to solutions of equations (8)–(10), provided that the $u_{1,2} \to 0$ for $|\xi| \to \infty$. In this case, the values of the constants C_1 and C_2 in (11) and (12) can be calculated, so that

$$u_1^2 - u_2^2 = 0,\quad \alpha\cos\Phi + \frac{\delta}{2} + \frac{\vartheta}{4}u_1^2 = 0,\quad \alpha = \pm 1.$$

The system of equations (8)–(10) using the equality $u_2 = \alpha u_1$ reduces to the system:

$$\frac{\partial u_1}{\partial\xi} = \alpha u_1\sin\Phi,$$

$$\frac{\partial\Phi}{\partial\xi} = \delta + 2\alpha\cos\Phi + \vartheta u_1^2.$$

Taking into account the second integral of motion, the equation for u_1 can be written as

$$\left(\frac{\partial u_1}{\partial\xi}\right)^2 = u_1^2\left[1 - \left(\frac{\delta}{2} + \frac{\vartheta}{4}u_1^2\right)^2\right].$$

The substitution $u_1 = w^{-1/2}$ and some simple transformations lead this equation to the form

$$\left(\frac{\partial w}{\partial\xi}\right)^2 = 4\Delta^2(w - w_1)(w + w_2),$$

where

$$\Delta^2 = 1 - \frac{\delta^2}{4};\quad w_1 = \frac{\vartheta}{4(1-\delta/2)};\quad w_2 = \frac{\vartheta}{4(1+\delta/2)}.$$

The equation for w can be integrated (as in [16]); as a result, we obtain

$$w = w_0 + w_3\cosh[2\Delta(\xi - \xi_0)],$$

where ξ_0 is the integration constant;

$$w_0 = \frac{\delta\vartheta}{4(1-\delta^2/4)};\quad w_3 = \frac{\vartheta}{4(1-\delta^2/4)}.$$

Returning to the original variables, we can write the relation

$$u_1^2(\xi) = u_2^2(\xi) = \frac{4\Delta^2/|\vartheta|}{\cosh[2\Delta(\xi-\xi_0)] + \delta/2}. \tag{13}$$

Thus, the real envelopes of a solitary wave propagating in the waveguide system under study are given by the expressions:

$$a_1^2(\xi) = \frac{4\Delta^2}{|\vartheta|(1+\beta)\{\cosh[2\Delta(\xi-\xi_0)] + \delta/2\}}, \tag{14}$$

$$a_2^2(\xi) = \frac{4\Delta^2}{|\vartheta|(1-\beta)\{\cosh[2\Delta(\xi-\xi_0)] + \delta/2\}}. \tag{15}$$

The solutions found are a generalisation of the solutions obtained in [16]. The phase difference Φ varies according to the expression

$$\Phi(\xi) = \Phi(-\infty) + \text{sgn}(\vartheta)S(\delta/2, 2\Delta(\xi - \xi_0)),$$

where we used the function

$$S(\rho, y) = \arctan\frac{(1-\rho^2)^{1/2}e^y}{1+\rho e^y}.$$

The value of $\Phi(-\infty)$ should be chosen so that when ξ tends to $-\infty$, the derivative $\partial u_1/\partial\xi$ is positive.

4.3. Solitary waves of algebraic type

The above-found solutions to equations (8)–(10) are characterised by exponentially falling edges; however, sometimes there arise the situations when the edges of the solitary waves decay to zero more slowly, such as $\sim 1/\xi^2$.

The solutions to (14) and (15) contain the parameter Δ, which vanishes in the limit $|\delta| \to 2$. Therefore, the solutions make sense when $-2 < \delta < 2$, but on the boundaries of this interval, the behaviour of a solitary wave can be quite different. To clarify the possible solutions we can consider the behaviour of u^2 for small values of Δ. From (13) it follows that if $\Delta \ll 1$

$$u^2(\xi) \approx \frac{4\Delta^2/\vartheta}{1+2\Delta^2(\xi-\xi_0)^2+\delta/2}$$
$$= \frac{4}{\vartheta}\left[\frac{(1-\delta^2/4)}{1+\delta/2+2(1-\delta^2/4)(\xi-\xi_0)^2}\right]$$
$$= \frac{4}{\vartheta}\left[\frac{(1-\delta/2)(1+\delta/2)}{(1+\delta/2)+2(1-\delta/2)(1+\delta/2)(\xi-\xi_0)^2}\right]$$
$$= \frac{4}{\vartheta}\left[\frac{1-\delta/2}{1+2(1-\delta/2)(\xi-\xi_0)^2}\right].$$

Near the left boundary of the interval of admissible values of the parameter δ, we can find the function u^2 by setting $\delta = -2 + \varepsilon$, where $\varepsilon \ll 1$*. Then

$$u^2(\xi) = \lim_{\varepsilon\to 0}\frac{4}{\vartheta}\left[\frac{2-\varepsilon/2}{1+2(2-\varepsilon/2)(\xi-\xi_0)^2}\right] = \frac{8}{\vartheta[1+4(\xi-\xi_0)^2]}.$$

Near the right boundary of the interval of admissible values of the parameter δ, we can find the function u^2 by setting $\delta = 2-\varepsilon$. Then

$$u^2(\xi) = \lim_{\varepsilon\to 0}\frac{4}{\vartheta}\left[\frac{\varepsilon/2}{1+\varepsilon(\xi-\xi_0)^2}\right] = 0.$$

Thus, in approximating the parameter δ to the value –2, the solutions to equations (8)–(10) transform into an algebraic soliton:

$$a_1^2(\xi) = \frac{8}{\vartheta(1+\beta_1)[1+4(\xi-\xi_0)^2]}, \tag{16}$$

$$a_2^2(\xi) = \frac{8}{\vartheta(1-\beta_1)[1+4(\xi-\xi_0)^2]}. \tag{17}$$

Here, β_1 corresponds to $\delta = -2$. If δ tends to +2, the amplitude of the solutions to equations (8)–(10) vanish.

4.4. Restrictions on the velocity parameter β

Since the parameter δ is limited, the associated velocity parameter β is also limited. Given the definition of the parameter δ, we can introduce the function

$$F(\mu) = \frac{\gamma_3}{\gamma_1}\mu + \frac{\gamma_2}{\gamma_1}\frac{1}{\mu},$$

whose argument is

$$\mu = \sqrt{\frac{1+\beta}{1-\beta}}.$$

This is an odd function of μ. Therefore, the condition $|\delta| \leqslant 2$ will be performed at intervals $\mu_- \leqslant \mu \leqslant \mu_+$ and $-\mu_- \geqslant \mu \geqslant -\mu_+$, where μ_- and μ_+ are the positive real roots of the equation $|F(\mu)| = 2$. Real roots exist if $\min F(\mu) \leqslant 2$ for positive μ. It can be shown that this function reaches a minimum* at $\mu = +\sqrt{\gamma_2/\gamma_3}$, the value of the function itself at a given point being equal to $2\sqrt{\gamma_2\gamma_3}$. Thus, the condition for the existence of solitary waves is expressed by the inequality

$$\sqrt{\gamma_2\gamma_3} \leqslant |\gamma_1|.$$

From the definition of the parameters $\gamma_{1,2,3}$ it follows that $\gamma_2\gamma_3 = 4c_2c_3\cos^2 2\phi$. In the coupled-wave model the coupling coefficients $c_{1,2,3}$ can always be chosen positive. Therefore, the product $\gamma_2\gamma_3$ is also positive or zero at $\phi = \pi/4$.

The expression for the parameter δ can be rewritten as

$$\delta = \left(\frac{c_2}{c_1}\sqrt{\frac{1+\beta}{1-\beta}} + \frac{c_3}{c_1}\sqrt{\frac{1-\beta}{1+\beta}}\right)\frac{\cos 2\phi}{\cos\phi}.$$

It follows that the sign of this parameter is determined by the phase shift of the waves in the neighbouring waveguides. Consequently, δ is negative in the intervals $\pi/2 < \phi < \pi$ and $3\pi/2 < \phi < 2\pi$.

The roots of the equation $F(\mu) = 2$ in the region of positive μ are given by the expressions:

$$\mu_- = \frac{|\gamma_1|}{|\gamma_2|}\left(1 - \sqrt{1-\frac{\gamma_2\gamma_3}{\gamma_1^2}}\right),$$

$$\mu_+ = \frac{|\gamma_1|}{|\gamma_2|}\left(1 + \sqrt{1-\frac{\gamma_2\gamma_3}{\gamma_1^2}}\right).$$

The parameters of the velocity of a stationary solitary wave $\beta_\pm$, which are the boundaries of admissible values, are defined as

$$\beta_\pm = \frac{\mu_\pm^2 - 1}{\mu_\pm^2 + 1}.$$

For example, suppose that $\gamma_2 = \gamma_3 = 0.5\gamma_1$. Then

$$\mu_+ \approx 3.73,\ \ \beta_+ \approx 0.866,\ \ \mu_- \approx 0.266,\ \ \beta_- \approx -0.865.$$

In the absence of interaction with next-to-nearest neighbouring waveguides the stationary solitary waves exist at $|\beta| < 1$ [16].

5. Numerical simulation of the interaction of solitary waves

To investigate the stability of solitary waves, which are the solutions to the system of equations (6) and (7), we simulated the collision of these waves with each other and with the localised harmonic wave packet. As initial pulses we used those corresponding to the solutions in the form of solitary waves whose amplitudes are determined by expressions (14) and (15); the phase component of the waves in positive and negative index waveguides were chosen as follows:

$$\varphi_1(\zeta_j,\tau) = \frac{3}{2}\mathrm{sgn}(\vartheta)S(0,2\xi_j),$$

$$\varphi_2(\zeta_j,\tau) = -\frac{\pi}{2} + \frac{1}{2}\mathrm{sgn}(\vartheta)S(0,2\xi_j).$$

*The direct solution of equations (8)–(10) for $\delta = -2$ gives the same result.

*Extremes $F(\mu)$ exist at points $\mu = \pm\sqrt{\gamma_2/\gamma_3}$, but at positive μ this is the minimum, and at negative – the maximum.

Here, $\xi_j = \gamma_1[\zeta_j - \beta(\tau - \tau_j)](1-\beta^2)^{1/2}$, and the parameters ζ_j and τ_j define the initial location of input pulses.

In studying the stability with respect to the collisions, we used stationary pulses characterised by the parameters $\beta = 0.4$ and -0.4. The pulse with $\beta = 0.4$ is set at the output from the waveguide system (for $\zeta = 50$) and the pulse with $\beta = -0.4$ is set at the input of the waveguide system (for $\zeta = 0$). The value of the coupling γ_1, determining the interaction between the nearest positively and negatively refracting waveguides is equal to unity, and the interaction between identical waveguides (positively or negatively refracting), characterised by constant γ_2 and γ_3, is assumed the same, i.e., $\gamma_2 = \gamma_3$.

In the absence of coupling between identical waveguides ($\gamma_2 = \gamma_3 = 0$) the collision between pulses occurs elastically, as illustrated in Fig. 2 (see also [16]). By varying the magnitude of the coupling between identical waveguides we found that in the range $\gamma_2 = \gamma_3 = 0.001-0.0075$, the collision between the counterpropagating pulses is also almost elastic.

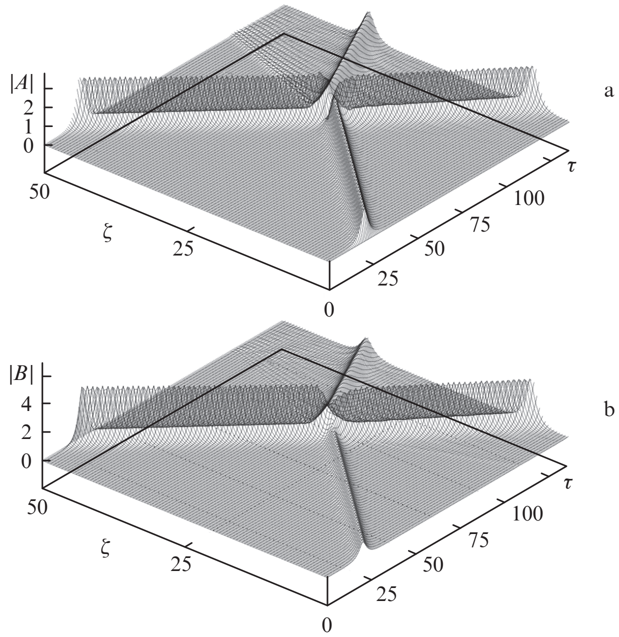

Figure 2. Collision between pulses at $\gamma_2 = \gamma_3 = 0$ for (a) positive index and (b) negative index waveguides.

In the range 0.0075 to 0.01 the collision between the pulses is accompanied by the formation of a weak harmonic wave (Fig. 3), and in the range 0.01 to 0.021, this linear wave increases with increasing coupling between identical waveguides.

A further increase in the coupling between identical waveguides (range $\gamma_2 = \gamma_3 = 0.0215-0.03$) leads to the fact that the propagated linear wave decays, merging with the transmitted fundamental wave (Fig. 4) and starting with $\gamma_{2,3} = 0.02$ there appears a reflected wave which emerges as a result of the reflection of the initial pulse with $\beta = -0.4$ from the pulse with $\beta = 0.4$, moving from the opposite side of the waveguide system.

The reflected wave is first amplified in the range 0.025 to 0.07, then decreases (Fig. 5) and nearly disappears at $\gamma_{2,3} = 0.075$. In the range 0.08 to 0.135, pulses interact without formation of a reflected and a transmitted linear waves; only the nature of the collision changes, which leads to a significant

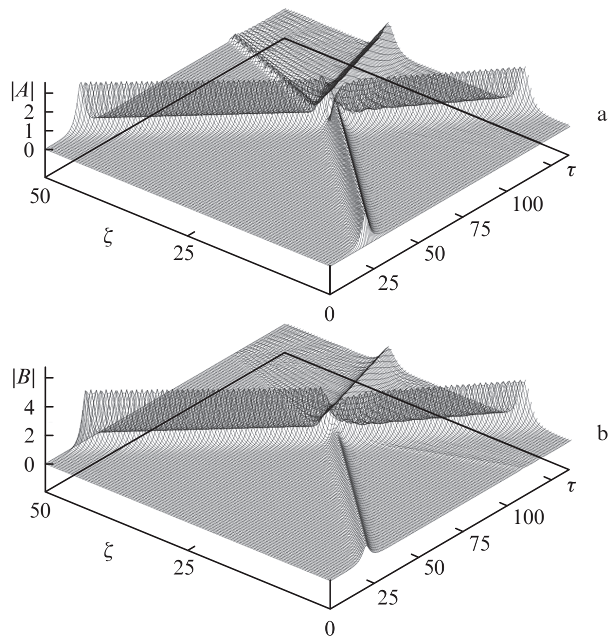

Figure 3. Collision between pulses at $\gamma_2 = \gamma_3 = 0.01$ for (a) positive index and (b) negative index waveguides.

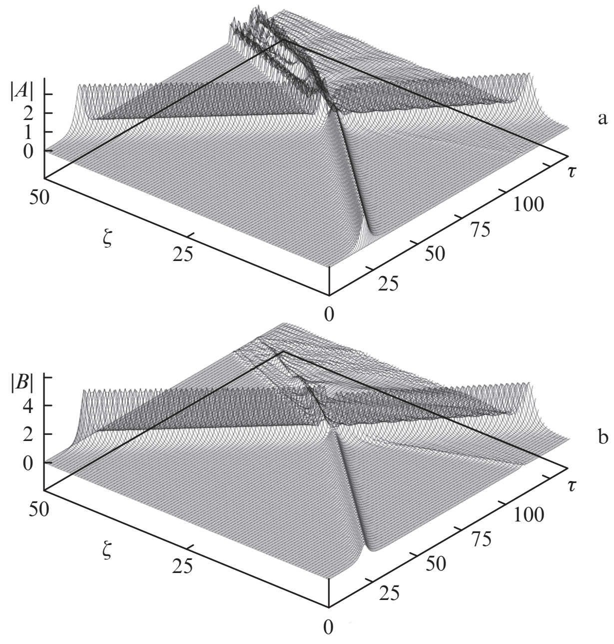

Figure 4. Collision between pulses at $\gamma_2 = \gamma_3 = 0.022$ for (a) positive index and (b) negative index waveguides.

variation in the velocity of pulses emerging due to collisions, as compared with the initial velocity (cf. Fig. 6).

In the range 0.14 to 0.15 a linear transmitted wave reappears, decaying in the range 0.151–0.16 (this linear wave merges with the transmitted wave) (Fig. 7). In the ranges 0.17 to 0.2 and 0.31 to 0.34, part of the pulse with $\beta = -0.4$ is reflected in a collision with a counterpropagating pulse ($\beta = 0.4$) and a reflected wave appears (Fig. 8). In the range $\gamma_2 = \gamma_3 = 0.21-0.3$, collisions between pulses do not produce linear waves, but the nature of these collisions is different. Thus, if $\gamma_2 = \gamma_3 \approx 0.22$, pulses move along trajectories that are close to initial, i.e., the exchange of pulses in a collision is small, and if

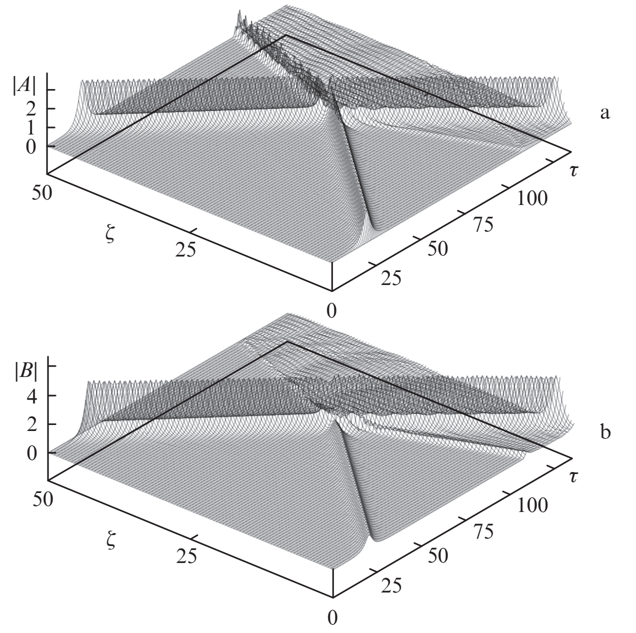

Figure 5. Collision between pulses at $\gamma_2 = \gamma_3 = 0.06$ for (a) positive index and (b) negative index waveguides.

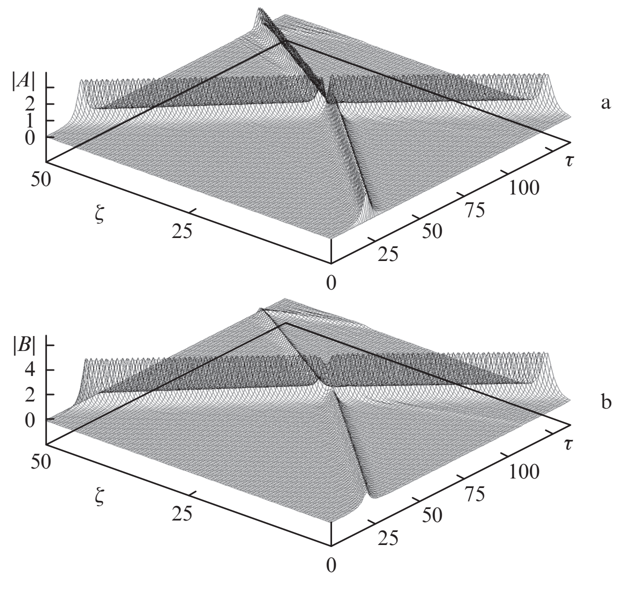

Figure 6. Collision between pulses at $\gamma_2 = \gamma_3 = 0.08$ for (a) positive index and (b) negative index waveguides.

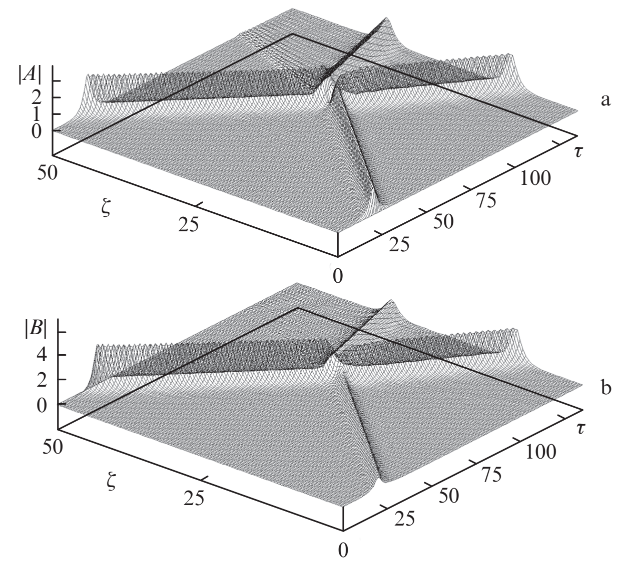

Figure 7. Collision between pulses at $\gamma_2 = \gamma_3 = 0.13$ for (a) positive index and (b) negative index waveguides.

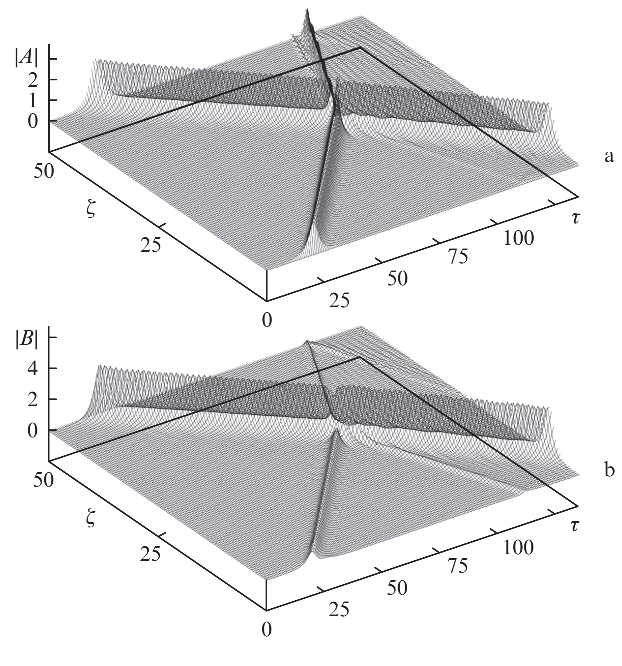

Figure 8. Collision between pulses at $\gamma_2 = \gamma_3 = 0.2$ for (a) positive index and (b) negative index waveguides.

$\gamma_2 = \gamma_3 \approx 0.25$, the trajectories of the pulses after the collision are very different from those before the collision.

In simulating the collision of a pulse with $\beta = 0.4$, moving from the output ($\zeta = 50$) of the waveguide system, with a counterpropagating localised harmonic perturbation

$$f(0,\tau) = (A_{\text{mod}}/2)[\tanh(\tau - 40) - \tanh(\tau - 70)]\sin\omega_{\text{mod}}\tau,$$

specified at the input to the waveguide system at $\zeta = 0$, we found that the result of collisions for small values of $\gamma_2 = \gamma_3$ lying in the range 0.001–0.05 does not differ much from that obtained in the case when the coupling between identical waveguides is absent ($\gamma_2 = \gamma_3 = 0$). The simulation was performed for the case when the perturbation amplitude A_{mod} was taken from the range 0.1 to 3, and the frequency of the periodic modulation ω_{mod} was chosen to be 0.7 or 3. Interaction of a solitary wave with this perturbation leads to the fact that the pulse is refracted in a space filled with a modulated wave packet. This refraction the stronger, the greater the modulation amplitude and frequency (for frequencies lying in the range 0.7–3). At the output from the modulated region, the pulse propagated in the form of a solitary wave at a velocity different from the initial one.

6. Conclusions

Localisation of the fields in the waveguides leads to an increase in the strength of electric and magnetic fields. This makes inevitable both self-action and nonlinear interaction between the spectral and spatial components of the localised fields. In this paper we have analysed the propagation of an electromagnetic pulse in the array of the waveguides with sign-alternating refractive indices. In contrast to previous studies, we have taken into account the interaction with the next-to-nearest neighbours. The zigzag configuration allows for this type of coupling between the waveguides. Because of the alternating sign of the refractive index, the spectrum of linear waves has a bandgap. This property of the considered model makes it different from that studied in [20].

Nonlinear wave have been studied in our model for the case when only positive index waveguides have nonlinear properties. The assumption of a linear dependence of the phase of the coupled waves on the waveguide number has allowed us to find a particular solution to the corresponding coupled wave equations. We have shown that in addition to solitary waves with exponentially falling edges there is a wave the rising and falling edges of which fall slower. This is a solitary wave of algebraic type that exists only for certain values of the parameters of the waveguide system.

It should be emphasised that in the case of an array of the waveguides coupled only with their nearest neighbours, the known nonlinear solitary waves are gap solitons of standard form, i.e., their envelope is described by a hyperbolic secant, like solitons in optical fibres with Kerr nonlinearity.

For an undeformed waveguide array the angle ϑ_b between the lines connecting the neighbouring waveguides is equal to π. In this case, the coupling with the next-to-nearest waveguides is negligible. (It is assumed that the radiation is localised in the waveguide.) If the array is deformed, so that the angle ϑ_b is reduced, we obtain a zigzag waveguide array. Therefore, by changing the angle ϑ_b, one can control the value of the coupling between the next-to-nearest neighbours of the waveguides [20].

The numerical solution of nonlinear coupled-wave equations has shown that interaction of solitary waves is inelastic. After the collision, the velocity of solitary wave propagation changes, and one of them can be even destroyed and turned into a wave packet of linear waves. Changing the velocity, as shown by numerical calculations, depends on the angle ϑ_b of deformation of the waveguide array, which allows the angle to be treated as a control parameter.

If the ANOWZA is formed of two arrays of waveguides (Fig. 1b), the displacement of the upper array relative to the lower one makes it possible, within certain limits, to control the coupling parameter c_1. Thus, displacement in the vertical direction changes the value of this parameter, and hence magnitude of the gap in the spectrum. The displacement in the horizontal direction breaks the symmetry with respect to the replacement of the right waveguide by the left one. There appears a nonzero energy flow along the waveguide array either from right to left or from left to right.

Thus, the ANOWZA has great potential to control the distribution of the electromagnetic field at the output from the array as compared with the conventional linear array.

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