

# Temperature dependences of phase and group birefringence in spun fibres

S.K. Morshnev, V.P. Gubin, Ya.V. Przhiyalkovsky, N.I. Starostin

**Abstract.** We consider the application of the spectral method for measuring the beat length of birefringence in spun fibre. We have found that the beat length measured in spun fibres is the geometric mean of phase and group birefringence beat lengths. Temperature measurements of beat lengths and shifts of the entire spectrum as a whole make it possible to separate types of birefringence and to determine their dispersion. We have performed an experiment on a spun fibre and a linear polarisation maintaining fibre, drawn from the same preform. The experimental results confirm the theory.

**Keywords:** spun fibre, HiBi fibre, birefringence, birefringent dispersion, helical structure model.

## 1. Introduction

Linear birefringence in spun fibres is often measured by analysing the transmission spectra of a system consisting of two linear fibre polarisers and a spun fibre in question placed between them [1] (similarly to measurements in linear polarisation maintaining fibres, i.e., high birefringence fibres or HiBi fibres [2–5]). Rashleigh [3,4] showed that the aim of such measurements in HiBi fibres is to determine the difference between the group velocities of orthogonally polarised light waves. Later, birefringence determined in such experiments was called group birefringence. The measurement error in these experiments can reach 20%, especially in fibres with a low beat length [3, 4].

In spun fibres, in addition to a different formula for determining the beat length [1], it was found that the dependence of the experimentally measured beat length on the phase and group birefringence is more complicated than in HiBi fibres. This paper considers the relations between the phase and group beat lengths in spun fibres.

We have studied experimentally the temperature dependences of the beat lengths of built-in linear birefringence for spun fibre of two types and for a HiBi fibre drawn from the same preform as the spun fibre, but without spinning. This made it possible to relate the purely group birefringence of HiBi fibre with mixed birefringence of spun fibre and to sepa-

rately determine the phase and group birefringence of these fibres. We have also carried out temperature measurements of the shifts of the beat spectrum as a whole. We have derived the relations of these shifts with the group and phase birefringence of spun fibres and calculated their values. We have estimated the dispersion of the built-in linear birefringence of spun fibre.

## 2. Theory

### 2.1. Determination of the spatial frequency of rotation of the polarisation plane in the spun fibres

The polarisation properties of a straight spun fibre with a helical structure of its built-in linear birefringence axes can be described according to [6] in a linear polarisation basis by the Jones matrix [7]:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} E_{x0} \\ E_{y0} \end{pmatrix} = \frac{2}{\Omega} \begin{pmatrix} a_{11} + ib_{11} & a_{12} + ib_{12} \\ -a_{12} + ib_{12} & a_{11} - ib_{11} \end{pmatrix} \begin{pmatrix} E_{x0} \\ E_{y0} \end{pmatrix}. \quad (1)$$

The matrix elements have the form

$$\begin{aligned} a_{11} &= \Omega \cos \xi z \cos \Omega z + \xi \sin \xi z \sin \Omega z, \\ a_{12} &= -\Omega \sin \xi z \cos \Omega z + \xi \cos \xi z \sin \Omega z, \\ b_{11} &= \frac{1}{2} \Delta \beta \cos \xi z \sin \Omega z, \\ b_{12} &= \frac{1}{2} \Delta \beta \sin \xi z \sin \Omega z, \end{aligned} \quad (2)$$

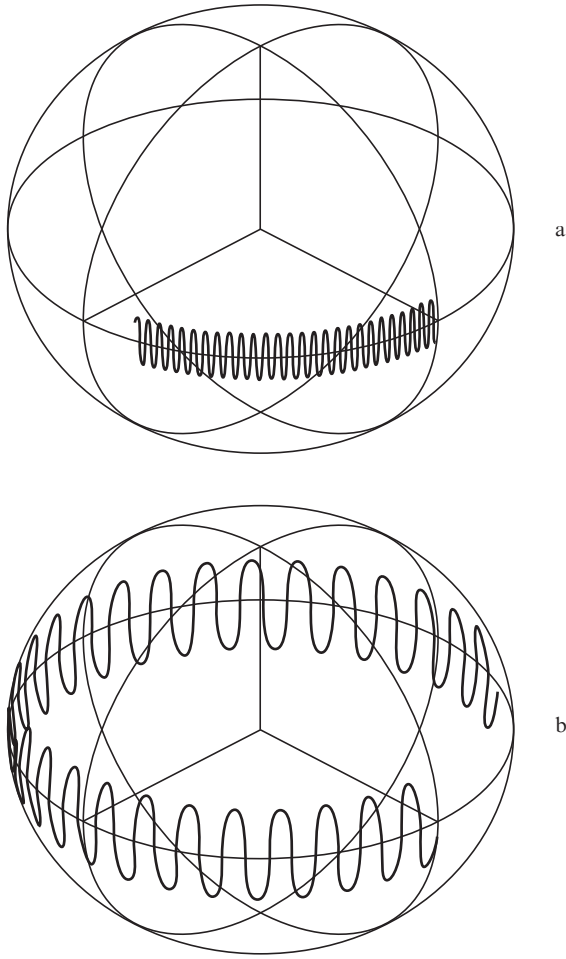
where  $\Delta \beta = 2\pi/L_b$  is the rate at which the phase delay between waves with orthogonal linear polarisations increases with fibre length,  $z$ , because of the built-in linear birefringence with a beat length  $L_b$ ;  $\xi = 2\pi/L_{tw}$  is the angular rotation rate of the helical structure of the built-in linear birefringence axes with a spin pitch  $L_{tw}$  during the propagation of light along the fibre; and the spatial frequency is defined as

$$\Omega = \sqrt{(\Delta \beta/2)^2 + \xi^2}. \quad (3)$$

Evolution of the polarisation states on the Poincare sphere for a fixed excitation wavelength, when the spun fibre is excited by linearly polarised light, is presented in Fig. 1. One can see that the polarisation states oscillate relative to the equator, deviating in elliptical state with the ellipticity that is the greater, the higher the built-in linear birefringence. Spun fibre also exhibits the rotation of the plane of radiation polarisation, i.e., there is an evolution of the polarisation state along the equator, although the model does not take into account

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**Figure 1.** Evolution of the polarisation states on the Poincare sphere in a spun fibre of length  $L = 40$  mm, having the parameters (a)  $L_{tw} = 3$  mm,  $L_b = 12$  mm and (b)  $L_{tw} = 3$  mm,  $L_b = 6$  mm.

the circular birefringence [8]. One can also see from Fig. 1 that the stronger the built-in linear birefringence (less  $L_b$ ), the greater the angle the plane of polarisation rotates in the spun fibre segments of equal length. The spatial period of rotation of the polarisation plane is determined by a complex number  $\chi$  at the fibre output:

$$\chi = \frac{E_y}{E_x} = \frac{(-a_{12}E_{x0} + a_{11}E_{y0}) + i(b_{12}E_{x0} - b_{11}E_{y0})}{(a_{11}E_{x0} + a_{12}E_{y0}) + i(b_{11}E_{x0} + b_{12}E_{y0})}. \quad (4)$$

As shown in [7], the rotation angle  $\theta$  of the plane of polarisation ( $2\theta$  is the azimuth angle on the Poincare sphere) is given by the expression

$$\begin{aligned} \tan 2\theta = \frac{2\text{Re}(\chi)}{1 - |\chi|^2} = & 2[(-a_{11}a_{12} + b_{11}b_{12})(E_{x0}^2 - E_{y0}^2) \\ & + (a_{11}^2 - a_{12}^2 - b_{11}^2 + b_{12}^2)E_{x0}E_{y0}][\{(a_{11}^2 - a_{12}^2 + b_{11}^2 - b_{12}^2) \\ & \times (E_{x0}^2 - E_{y0}^2) + 4(a_{11}a_{12} + b_{11}b_{12})E_{x0}E_{y0}\}]^{-1}. \end{aligned} \quad (5)$$

At the input to the fibre we make use of a linearly polarised state with the azimuth  $\alpha$ :

$$E_{x0} = E_0 \cos \alpha, \quad E_{y0} = E_0 \sin \alpha. \quad (6)$$

Substituting  $a_{11}$ ,  $a_{12}$ ,  $b_{11}$  and  $b_{12}$  from (2) and  $E_{x0}$  and  $E_{y0}$  from (6) into formula (5), we obtain the dependences of  $\theta(z)$  on the fibre length. Omitting complex trigonometric transformations and introducing new spatial frequencies  $\omega_- = 2(\Omega - \xi)$ ,  $\omega_+ = 2(\Omega + \xi)$ , and  $2\xi$ , we obtain

$$\begin{aligned} \tan 2\theta = & 2\{[-1/4\xi(\Omega + \xi)\sin\omega_-z - 1/4\xi(\Omega - \xi)\sin\omega_+z \\ & + 1/2(\Delta\beta^2/4)\sin 2\xi z]\cos 2\alpha + [1/2\Omega(\Omega + \xi) \\ & \times \sin(\omega_-z + \pi/2) + 1/2\Omega(\Omega - \xi)\cos\omega_+z]\sin 2\alpha\} \\ & \times \{[1/2\xi(\Omega + \xi)\cos\omega_-z - 1/2\xi(\Omega - \xi)\cos\omega_+z \\ & + (\Delta\beta^2/4)\cos 2\xi z]\cos 2\alpha + [-\Omega(\Omega + \xi)\cos(\omega_-z + \pi/2) \\ & + 1/2\Omega(\Omega - \xi)\sin\omega_+z]\sin 2\alpha\}^{-1}. \end{aligned} \quad (7)$$

At  $\alpha = 0$ , when the polarisation vector is parallel to the birefringence axis at the input to the fibre, the rotation angle of the polarisation plane at the output is

$$\begin{aligned} \tan 2\theta \Big|_{\alpha=0} = & -\tan \omega_-z \\ & \times \left[ 1 + \frac{\Omega - \xi}{\Omega + \xi} \frac{\sin \omega_+z}{\sin \omega_-z} - \frac{\Delta\beta^2/4}{\xi(\Omega + \xi)} \frac{\sin 2\xi z}{\sin \omega_-z} \right] \\ & \times \left[ 1 - \frac{\Omega - \xi}{\Omega + \xi} \frac{\cos \omega_+z}{\cos \omega_-z} + \frac{\Delta\beta^2/4}{\xi(\Omega + \xi)} \frac{\cos 2\xi z}{\cos \omega_-z} \right]^{-1}. \end{aligned} \quad (8)$$

At  $\alpha = \pi/4$ , when the polarisation vector is directed at an angle of  $45^\circ$  to the birefringence axes at the spun fibre input, the rotation angle of the polarisation plane at the output is

$$\begin{aligned} \tan 2\theta \Big|_{\alpha=\pi/4} = & -\tan(\omega_-z + \pi/2) \\ & \times \left[ 1 + \frac{\Omega - \xi}{\Omega + \xi} \frac{\cos \omega_+z}{\sin(\omega_-z + \pi/2)} \right] \\ & \times \left[ 1 + \frac{\Omega - \xi}{\Omega + \xi} \frac{\sin \omega_+z}{\cos(\omega_-z + \pi/2)} \right]^{-1}. \end{aligned} \quad (9)$$

We see from expressions (8) and (9) that the fundamental spatial frequency, which determines the rotation of the polarisation plane in spun fibres, is the ‘slowest’ frequency  $\omega_- = 2(\Omega - \xi)$ , the main contribution to which (in conventional spun fibres) is made by the built-in linear birefringence, represented by the value  $\Delta\beta$ . Corrections, apart from their smallness (the values of  $\Omega$  and  $\xi$  are close), also oscillate with high spatial frequencies  $\omega_+ = 2(\Omega + \xi)$  and  $2\xi$ .

**2.2. Measurement of the group birefringence in spun fibres by the spectral method**

The most popular method for measuring the built-in linear birefringence in HiBi fibres (for example, ‘Panda’, ‘bow-tie’, and ‘elliptical core’) is the spectral method [2–4], which takes the readings of the interference spectrum by scanning its wavelengths in a fibre of length  $z$ , placed between two linear polarisers. After measuring the spectrum one can determine the spectral period  $\Delta\lambda$  of interference beats in the vicinity of the working wavelength  $\lambda$  and calculate the beat length  $L_b$  of the built-in birefringence by the formula

$$L_b = (\Delta\lambda/\lambda)z. \quad (10)$$

In this case, the following problem arises. The spectrum is transmitted along a fibre at a group velocity, and an interference pattern with a period  $\Delta\lambda$  is formed by using the phase relations. Rashleigh was the first to notice it in his papers [3,4]. It turns out that by definition birefringence is the difference between the effective refractive indices that characterise the phase velocity of propagation of two orthogonally polarised waves, whereas the difference in the refractive indices is measured, which characterises the group velocity of propagation of these waves. Therefore, the measured characteristic was called ‘group birefringence’ and can only be used to compare similar HiBi fibres, yielding an error of up to 20% [4].

In HiBi fibres group birefringence is indeed measured. Let  $\beta_x$  and  $\beta_y$  be the propagation constants of two orthogonally polarised modes. Their corresponding effective refractive indices are  $n_x = \beta_x/k$  and  $n_y = \beta_y/k$ , where  $k$  is the wave vector in vacuum. Passing a fibre segment of length  $z$ , these modes will acquire the phase difference  $k(n_x - n_y)z$ , and as a result of their interference we obtain periodic beats at the wavelengths  $\lambda_m$ :

$$\frac{2\pi}{\lambda_m} (n_x - n_y)_m z = 2\pi m, \quad (11)$$

wherein the beat length is

$$L_b = \lambda/(n_x - n_y). \quad (12)$$

The phase difference  $2\pi$  (on the Poincare sphere) between the beats at  $\lambda_m$  and  $\lambda_{m+1}$  leads to the equation

$$\frac{(n_x - n_y)_{m+1}}{\lambda_{m+1}} - \frac{(n_x - n_y)_m}{\lambda_m} = \frac{1}{z}. \quad (13)$$

The spectral interval  $\Delta\lambda = \lambda_m - \lambda_{m+1} \ll \lambda_m$ ; therefore, we can write

$$(n_x - n_y)_{m+1} = (n_x - n_y)_m - \frac{d(n_x - n_y)}{d\lambda} \Delta\lambda, \quad (14)$$

$$\lambda_{m+1} = \lambda_m \left(1 - \frac{\Delta\lambda}{\lambda}\right).$$

Substituting (14) into (13) and neglecting the terms quadratic in  $\Delta\lambda/\lambda$ , we have

$$\frac{\Delta\lambda}{\lambda_m^2} z \left[ (n_x - n_y) - \lambda \frac{d(n_x - n_y)}{d\lambda} \right] = 1. \quad (15)$$

The expression in square brackets in (15) is the group birefringence. Assuming that the group beat length  $L_{b,gr} = \lambda/(n_x - n_y)_{gr}$ , we obtain expression (10).

Spectral measurements of the built-in linear birefringence in spun fibres are performed similarly to measurements in polarisation maintaining fibres; however, the processing of the results should be carried out according to the formulas obtained in [1]. The behaviour of light propagation in spun fibres is also different. Linearly polarised light in the course of propagation in spun fibre experiences the rotation of the polarisation plane, while in HiBi fibres the phase delay changes between the orthogonally polarised modes. The spatial frequency of rotation according to expressions (8) and (9) is equal to  $2(\Omega - \xi)$ . The condition for determining the period of

polarisation beats for spun fibres [similar to expression (11) for HiBi fibres] has the form

$$2(\Omega_{m+1} - \xi)z - 2(\Omega_m - \xi)z = 2\pi. \quad (16)$$

Let us substitute in this equation the values of  $\Omega$  from (3), transfer  $2(\Omega_m - \xi)z$  in the right-hand side and square it, then in terms of (13), equation (16) can be written as

$$\frac{(n_x - n_y)_{m+1}^2}{\lambda_{m+1}^2} - \frac{(n_x - n_y)_m^2}{\lambda_m^2} = \frac{1}{z^2} + \frac{4}{z} \left[ \frac{(n_x - n_y)_m^2}{\lambda_m^2} + \frac{1}{L_{tw}^2} \right]^{1/2}. \quad (17)$$

The difference between the squares in the left-hand side of (17) can be represented as the product of the difference and the sum. Neglecting the terms quadratic with respect to  $\Delta\lambda/\lambda$ , we express this part in the form

$$L = 2 \frac{\Delta\lambda}{\lambda_m^3} (n_x - n_y)_m \left[ (n_x - n_y)_m - \lambda_m \frac{d(n_x - n_y)}{d\lambda} \right] + o\left(\frac{\Delta\lambda}{\lambda}\right). \quad (18)$$

Here, the square brackets are used for the group birefringence and parenthesis – for the phase birefringence. We will denote below the group birefringence as  $(n_x - n_y)_{gr}$ , and the phase birefringence as  $(n_x - n_y)_{ph}$ . We will also assume  $\lambda_m$  to be the working wavelength and denote it as  $\lambda$ . Unfortunately, we have failed to express the left-hand side of (17) only through the phase or only through the group birefringence. We will neglect  $1/z^2$  in connection with a long fibre sample (a few meters), substitute (18) into (17), square it again and obtain

$$\left( \frac{\Delta\lambda}{2\lambda} z L_{tw} \right)^2 \frac{(n_x - n_y)_{ph}^2 (n_x - n_y)_{gr}^2}{\lambda^2} = 1 + \frac{L_{tw}^2 (n_x - n_y)_{ph}^2}{4\lambda^2}. \quad (19)$$

Thus, the equation for calculating the built-in linear birefringence in spun fibres equally includes the phase and the group birefringence. A ‘mixed’ value of birefringence is obtained experimentally. In the case of negligibly small dispersion of the phase birefringence [ $d(n_x - n_y)/d\lambda \approx 0$ ], they coincide, and from expression (19) we obtain a biquadratic equation for a relatively unknown beat length:

$$L_b^4 + \frac{L_{tw}^2}{4} L_b^2 - \left( \frac{\Delta\lambda}{2\lambda} z L_{tw} \right)^2 = 0. \quad (20)$$

The solution of equation (20) takes the form

$$L_b^2 = -\frac{L_{tw}^2}{8} + \left[ \left( \frac{\Delta\lambda}{2\lambda} z L_{tw} \right)^2 + \frac{L_{tw}^4}{64} \right]^{1/2}. \quad (21)$$

If the second term under the square root can be neglected, we have a relatively simple formula obtained earlier in [1]:

$$L_b^2 \approx \frac{\Delta\lambda}{2\lambda} z L_{tw}. \quad (22)$$

Let us return to equation (19). If the spin pitch  $L_{tw}$  is so small that the second term in the right-hand side of expression (19) can be neglected, then

$$L_{b,exp}^2 = L_{b,ph} L_{b,gr} \approx \frac{\Delta\lambda}{2\lambda} z L_{tw}. \quad (23)$$

Thus, the experimentally determined value of the beat length  $L_{b,exp}$  in this approximation is equal to the geometric mean of

the phase  $L_{b,ph}$  and group  $L_{b,gr}$  beat lengths. Note that formulas (22) and (23) were obtained in the same approximations.

The spectral method for measuring the linear birefringence in spun fibres (and in HiBi fibres) can be used only for comparison of birefringence of similar fibres. To accurately measure the built-in linear birefringence in spun fibres we should use a narrowband light source, for example, in a fibre twisted around its axis [6].

### 2.3. Temperature dependences of the phase and group birefringence

Temperature investigations of the built-in linear birefringence in HiBi fibres exhibit two types of changes in the observed spectrum. Besides changes in the spectral period of  $\Delta\lambda$  measured in the vicinity of the working wavelength, the whole spectrum is shifted monotonically to longer wavelengths upon cooling, and to shorter wavelengths upon heating (see Section 3). The temperature coefficient obtained from changes in the spectral beat period  $\Delta\lambda$  with temperature is attributed to a change in the group birefringence, and the coefficient calculated from the temperature shift of the entire spectrum as a whole – to a change in the phase birefringence [9].

The latter is true for HiBi fibres; however, for spun fibre, as shown above [see (23)] the group and phase birefringence make the same contribution to the measured value of the beat length  $L_{b,exp}$ . Let  $\alpha_{gr}$ ,  $\alpha_{ph}$  be the temperature coefficients of the group and phase birefringence, respectively; then,

$$\sqrt{(n_x - n_y)_{gr}(n_x - n_y)_{ph}} \approx \frac{\lambda}{L_{b,exp}} \left[ 1 + \frac{1}{2}(\alpha_{gr} + \alpha_{ph})\Delta T \right]. \quad (24)$$

The coefficient  $1/2$  at  $\alpha_{gr} + \alpha_{ph}$  was the result of extracting the root. In the case, when the birefringence dispersion is low,  $(n_x - n_y)_{gr} \approx (n_x - n_y)_{ph}$ .

Consider now the dependence of the wavelength of a specific  $m$ th minimum of the interference spectrum on the fibre temperature. The shift of this minimum can be regarded as a phase delay  $\Delta\phi$ . In fact, the experimental scheme records the azimuthal rotation  $\Delta\theta$  relative to the polarisation plane specified by the analyser. These quantities are related as

$$\Delta\theta = 2\Delta\phi. \quad (25)$$

For HiBi fibres we will use formula (11), whereas for spun fibres – the expression

$$2(\Omega_m - \xi)z = 2\pi m. \quad (26)$$

Both formulas yield the same result:

$$\frac{(n_x - n_y)_m}{\lambda_m} = \frac{(n_x - n_y)_0}{\lambda_0}, \quad (27)$$

where  $\lambda_m$  is the wavelength of the  $m$ th minimum; all quantities with the subscript ‘0’ are taken at a temperature  $T_0$ . The minimum is shifted along the wavelength and therefore the birefringence dispersion cannot be neglected:

$$(n_x - n_y)_m = (n_x - n_y)_0 + \frac{d(n_x - n_y)}{d\lambda}(\lambda_m - \lambda_0) + \alpha_{ph}(n_x - n_y)_0(T - T_0). \quad (28)$$

Substituting (28) into (27) and using (25) we obtain the wavelength shift caused by the phase shift:

$$2\frac{\lambda_m - \lambda_0}{\lambda_0} = \frac{\alpha_{ph}(T - T_0)}{1 - L_{b,ph}[d(n_x - n_y)/d\lambda]} = \frac{(n_x - n_y)_{ph}}{(n_x - n_y)_{gr}} \frac{\alpha_{ph}(T - T_0)}{1 + \alpha_{ph}(T - T_0)}. \quad (29)$$

In the experiment, we obtain

$$\lambda_m = \lambda_0[1 + \alpha_m(T - T_0)]. \quad (30)$$

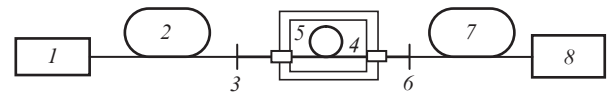
Then, the temperature coefficient of the wavelength of the  $m$ th minimum is

$$2\alpha_m = \alpha_{ph} \left[ 1 - L_{b,ph} \frac{d(n_x - n_y)}{d\lambda} \right]^{-1}. \quad (31)$$

The coefficient 2 in formulas (29) and (31) appears from (25). From formula (31), if the temperature coefficients  $\alpha_m$  and  $\alpha_{ph}$  are experimentally determined, we can estimate the birefringence dispersion  $d(n_x - n_y)/d\lambda$ .

## 3. Experiment

The experimental setup is shown in Fig. 2. As a broadband optical radiation source ( $I$ ) we use a superluminescent erbium fibre emitter ESS-30-M-01 (IRE-Polus) with a spectral width of  $\sim 100$  nm at the  $-60$  dBm level. Light is polarised by a linear fibre polariser (2) with 40 dB extinction. The planes of light polarisation in the region of splices (3) and (6) are inclined at the angle of  $\sim 45^\circ$  to the birefringence axes of the tested fibre (4), which is placed in a heat chamber (5), providing the desired temperature with an accuracy of  $\pm 0.2^\circ$  in the range of  $-60$  to  $+60^\circ\text{C}$ . As an analyzer (7) we use the second fibre polariser with 40 dB extinction. The interference spectrum is obtained with a Yokogawa AQ6370C optical spectrum analyser (8).



**Figure 2.** Scheme of the setup for measuring the temperature dependence of birefringence in optical fibres of HiBi and Spun type: (1) broadband light source; (2) fibre polariser; (3) and (6) splices; (4) optical fibre under study; (5) heat chamber; (7) fibre analyser; (8) spectrum analyser.

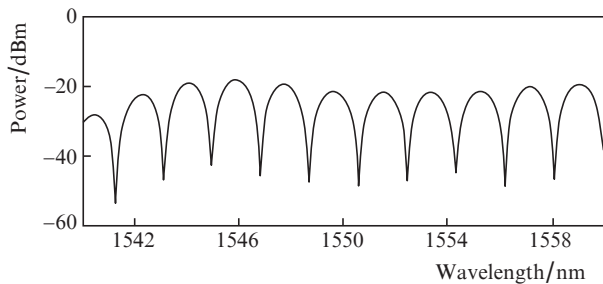
The typical spectrum of radiation transmitted through the optical fibre in question is shown in Fig. 3. The beat spectrum is used to measure the wavelengths  $\lambda_1$  and  $\lambda_2$  of two neighbouring minima in the vicinity of the working wavelength  $\lambda = 1550$  nm. Then, taking  $\Delta\lambda = \lambda_2 - \lambda_1$  and  $\lambda = (\lambda_2 + \lambda_1)/2$  into account, the formula [see (10)]

$$L_{b,gr} = 2\frac{\lambda_2 - \lambda_1}{\lambda_2 + \lambda_1}z \quad (32)$$

helps to find the group beat length of the HiBi fibre. In studying the spun fibre use is made of formula (19) and formula (23)

**Table 1.** Basic parameters of birefringence in spun- and HiBi fibres.

Fibre	$L_{b,exp}/mm$ $T = 0^\circ C$	$L_{tw}/mm$	$\alpha_{ph}/deg^{-1}$	$\alpha_m/deg^{-1}$	Calculation by formula (23)		Calculation by formulas (23) and (25)		
					$L_{b,gr}^{(HiBi)} = L_{b,gr}^{(Spun)}$		$d(n_x - n_y)/d\lambda/nm^{-1}$		
					$L_{b,ph}/mm$	$L_{b,gr}/mm$	$L_{b,ph}/mm$	$L_{b,gr}/mm$	
Spun I	$7.96 \pm 0.02$	3.0	$(6.3 \pm 0.1) \times 10^{-4}$	$(2.8 \pm 0.1) \times 10^{-4}$	8.94	7.09	8.44	7.50	$-2.9 \times 10^{-8}$ $-1.5 \times 10^{-8}$
Spun II	$9.06 \pm 0.02$	3.0	$(7.3 \pm 0.1) \times 10^{-4}$	$(2.9 \pm 0.1) \times 10^{-4}$	–	–	10.16	8.08	$-2.5 \times 10^{-8}$
HiBi I	$7.09 \pm 0.02$	–	$(6.4 \pm 0.1) \times 10^{-4}$	$(2.6 \pm 0.1) \times 10^{-4}$	–	–	8.73	7.09	$-2.6 \times 10^{-8}$


**Figure 3.** Typical spectrum of interference beats ( $\Delta\lambda = 2$  nm in the vicinity of the working wavelength  $\lambda = 1550$  nm).

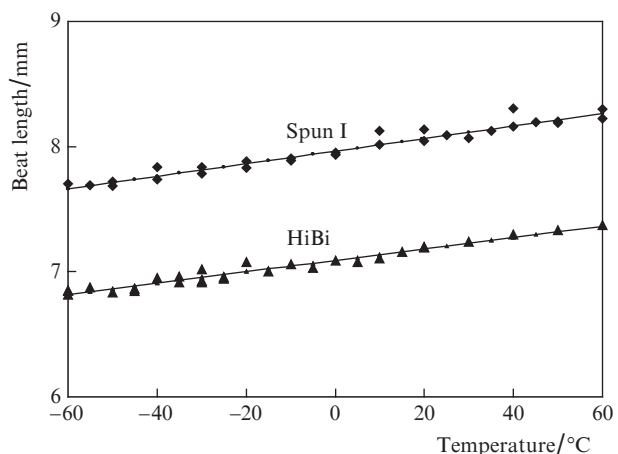
determines the geometric mean of the phase and group beat lengths.

During all temperature measurements virtually the entire fibre is placed in a quartz tube, coiled into a ring 20 cm in diameter. The splices are located outside the heat chamber. We studied spun fibres fabricated by the ‘elliptical core’ technology, with beat lengths of the built-in linear birefringence,  $L_b \sim 8$  mm, and HiBi fibres drawn from the same preform, but without spinning. The birefringence parameters for the fibres are listed in Table 1.

In the experiment for each value of the temperature stabilised by the heat chamber, we measured the beat period  $\Delta\lambda$  in the vicinity of the working wavelength, and then by using formula (32) we calculated the beat length of the HiBi fibre and by using formula (22) – the beat length  $L_{b,exp}$  of the spun fibre.

The results of measuring the dependences  $L_b(T)$  of the Spun I and HiBi I fibres are shown in Fig. 4. It can be seen that the plots of the temperature dependences are virtually parallel ( $\alpha_{spun} = 6.3 \times 10^{-4} deg^{-1}$ ,  $\alpha_{HiBi} = 6.4 \times 10^{-4} deg^{-1}$ ). Based on formula (24), we can conclude about the equality of the group and phase temperature coefficients:  $\alpha_{gr} = \alpha_{ph}$ . The beat length increases with temperature, i.e., the built-in linear birefringence decreases. In the spun fibre the linear birefringence is 12% less than in the HiBi fibre drawn from the same preform ( $L_{b,exp}^{(Spun)} > L_{b,exp}^{(HiBi)}$ , Table 1). The shape of the dependences in Fig. 4 confirms the correctness of formula (23). If calculations for the spun fibre are performed by formula (10), we obtain  $L_b = 42$  mm, which is six times greater than the measured beat length. A decrease in the built-in linear birefringence by 12% can be attributed, for example, to a small deformation of the elliptical spun fibre core due to rotation of the preform.

Moreover, the difference between the values  $L_{b,exp}$  and  $L_{b,gr}$  of the spun and HiBi fibres, respectively, can be explained using equation (23) if we assume that the values of the group birefringence in both fibres are identical [ $L_{b,gr}^{(Spun)} = L_{b,gr}^{(HiBi)} = 7.09$  mm] because the fibres are drawn from the same preform. Then, the phase beat length in the spun fibre is


**Figure 4.** Experimental temperature dependences of the beat length  $L_{b,exp}$  for Spun I and HiBi fibres. The temperature coefficients for Spun I and HiBi,  $\alpha_{ph} = 6.3 \times 10^{-4} deg^{-1}$  and  $\alpha_{ph} = 6.4 \times 10^{-4} deg^{-1}$ , were obtained at minimal mean-square deviation from the experimental points.

$$L_{b,ph} = L_{b,exp}^2 / L_{b,gr} = 8.94 \text{ mm}, \quad (33)$$

and the birefringence dispersion is

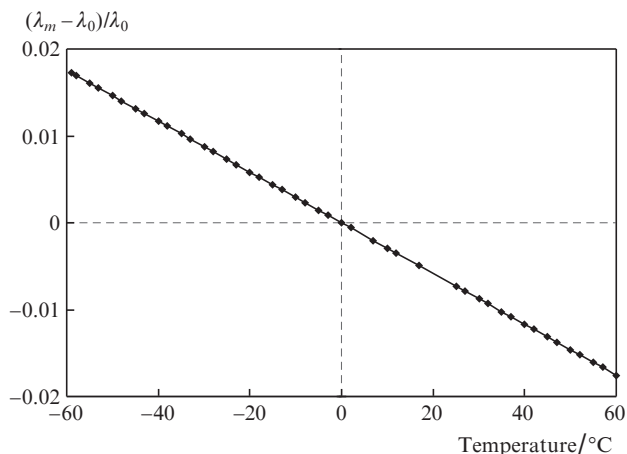
$$\frac{d(n_x - n_y)}{d\lambda} = \frac{1}{L_{b,ph}} - \frac{1}{L_{b,gr}} = -2.9 \times 10^{-8} \text{ nm}^{-1}. \quad (34)$$

The difference between the values  $L_{b,exp}$  and  $L_{b,gr}$  in the spun fibre is determined most likely by two factors: the drawing technology and the influence of the phase birefringence.

To measure the temperature shift of the entire beat spectrum as a whole, at each temperature the minimum wavelength  $\lambda_m$  was fixed. Next, we calculated the value of  $(\lambda_m - \lambda_0)/\lambda_0$  [see (29)], and determined the temperature coefficient  $\alpha_m$  from the slope of the temperature dependences of this value (Fig. 5). The obtained values are listed in Table 1. It is seen that  $\lambda_m$  is in fact approximately twice lower than the corresponding temperature coefficient  $\alpha_{ph}$ . If the expression  $\alpha_{ph}(T - T_0)$  is neglected in the denominator of formula (29) with respect to unity, then

$$L_{b,ph} = (\alpha_{ph}/2\alpha_m)L_{b,gr}, \quad (35)$$

and expression (23) can be used as the second equation to determine the unknown  $L_{b,ph}$  and  $L_{b,gr}$ . Thus, for the HiBi I fibre the experimentally determined beat length was  $L_{b,gr} = 7.09$  mm, and then using (35) we obtain  $L_{b,ph} = 8.73$  mm,  $d(n_x - n_y)/d\lambda = -2.64 \times 10^{-8} \text{ nm}^{-1}$ . Similarly, for the spun I fibre, using (35) and (23) we have  $L_{b,gr} = 7.5$  mm,  $L_{b,ph} = 8.44$  mm and  $d(n_x - n_y)/d\lambda = -1.5 \times 10^{-8} \text{ nm}^{-1}$ . It follows from



**Figure 5.** Temperature dependence  $(\lambda_m - \lambda_0)/\lambda_0$  of the Spun II sample. The temperature coefficient is  $\alpha_m = 2.8 \times 10^{-4} \text{ deg}^{-1}$ .

Table 1 that the birefringence dispersion is low and lies in the ranges  $(1.5-2.9) \times 10^{-8} \text{ nm}^{-1}$ .

In conclusion, we emphasise that the measurement of the temperature dependence of the beat length of the built-in linear birefringence spun fibre makes it possible to determine the phase and group birefringence, as well as to evaluate the dispersion of birefringence. Parallel temperature dependences for HiBi fibres and spun fibres drawn from the same preform confirms the validity of formula (22) derived based on the model of the helical structure of the linear birefringence axes in the absence of the built-in circular birefringence.

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