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Determination of refractive index, extinction coefficient and thickness of thin films by the method of waveguide mode excitation

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Abstract. **We propose a method for measuring simultaneously the refractive index n_f, extinction coefficient m_f and thickness H_f of thin films. The method is based on the resonant excitation of waveguide modes in the film by a TE- or a TM-polarised laser beam in the geometry of frustrated total internal reflection. The values of n_f, m_f and H_f are found by minimising the functional $\phi = [N^{-1}\Sigma_{i=1}^{N}(R_{exp}(\theta_i) - R_{thr}(\theta_i))^2]^{1/2}$, where $R_{exp}(\theta_i)$ and $R_{thr}(\theta_i)$ are the experimental and theoretical coefficients of reflection of the light beam from the interface between the measuring prism and the film at an angle of incidence θ_i. The errors in determining n_f, m_f and H_f by this method are $\pm 2\times10^{-4}$, $\pm 1\times10^{-3}$ and $\pm 0.5\,\%$, respectively.**

Keywords: *thin films, method of excitation of waveguide modes.*

1. Introduction

Measurement of the refractive index n_f, extinction coefficient m_f and thickness H_f of thin films is important in the formation of optical coatings and the fabrication of various integrated optics devices. Traditionally, ellipsometry is used in measurements [1]. However, the error in determining n_f and m_f by the ellipsometric method is rarely less than $\pm 5\times10^{-3}$. On the other hand, the method of resonant excitation of waveguide modes in a film under conditions of frustrated total internal reflection (FTIR) yields a measurement error in n_f at a level of $\pm 2\times10^{-4}$ and in H_f – at a level of $\pm 1\,\%$ [2–13]; however, m_f cannot be found by this method.

The method of resonant excitation of waveguide modes was proposed in the 1970s and is currently conventionally accepted in the study of thin-film structures [14]. The method consists in the fact that the film under study is in optical contact with the working face of the measuring prism having a high refractive index N_p and is illuminated by a monochromatic light beam from the prism's side. For beams incident at the prism–film interface at angles θ_i, for which the synchronism condition $N_p\sin\theta_i = \beta_i$ ($i = 0, 1, 2, \dots$; β_i is the effective refractive index of the mode with index i) is fulfilled, the condition for total internal reflection is violated and light can penetrate into the film, exciting a corresponding mode. In this case, the angular dependence of the reflection coefficient $R(\theta)$ of the beam from the working face of the prism features sharp and narrow minima. If two mode angles θ_i are known, then knowing N_p, one can calculate β_i, and solving the system of nonlinear dispersion equations for waveguide modes, one can determine the two unknown parameters: refractive index n_f and thickness H_f of the film. This method was successfully used for studying polymer [15] and metal [16, 17] films, and transparent conductive [18] and dielectric [19] layers. A number of recent publications that extend the capabilities of this method show the possibility of measuring ultrathin films by using immersion liquids [20], of determining the refractive index gradient across the film thickness [21], and of measuring the dispersion of the film material in the visible and near-IR wavelength regions [19].

In this paper we propose a method for measuring simultaneously the refractive index n_f, extinction coefficient m_f and thickness H_f of thin films. The method is based on the resonant excitation of waveguide TE or TM modes in the film in the FTIR geometry. Errors in determining n_f, m_f and H_f by this method are $\pm 2\times10^{-4}$, $\pm 1\times10^{-3}$ and $\pm 0.5\,\%$, respectively. The proposed method is conceptually similar to the approach shown in [22] by the example of leaky and TE waveguide modes and is its extension.

2. Calculation of the reflection coefficient of plane TE and TM waves from an absorbing film in the FTIR geometry

The scheme of waveguide mode excitation in a film having a refractive index n_f in the FTIR geometry is shown in Fig. 1. A plane electromagnetic wave E_{in} is incident on the prism–film interface from the measuring prism's side having a refractive index N_p at an angle θ. Below, we assume that absorption in the prism material is absent, while the permittivity of the prism is $\varepsilon_p = N_p^2$. The film is located on a substrate with a refractive index n_s. The gap between the prism and the film has a thickness H_{im} and can be filled with air or an immersion liquid with a refractive index n_{im}. The permittivities of the immersion layer, film and substrate are, generally speaking, complex quantities and are detemined by the expressions $\varepsilon_{im} = (n_{im} + \mathrm{i}m_{im})^2$, $\varepsilon_f = (n_f + \mathrm{i}m_f)^2$ and $\varepsilon_s = (n_s + \mathrm{i}m_s)^2$, where m_{im}, m_f and m_s are the extinction coefficients of respective media.

Monochromatic electromagnetic fields

$$\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}(\boldsymbol{r})\exp(-\mathrm{i}\omega t) + \mathrm{c.c.},$$

$$\boldsymbol{H}(\boldsymbol{r}, t) = \boldsymbol{H}(\boldsymbol{r})\exp(-\mathrm{i}\omega t) + \mathrm{c.c.}$$

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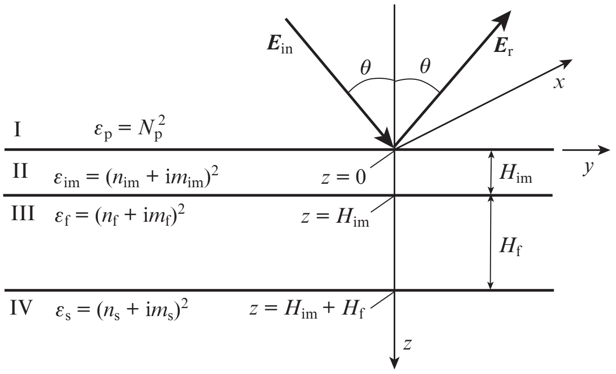

Figure 1. Scheme of excitation of the waveguide modes in the film in the FTIR geometry ($\varepsilon_{\rm p}$, $\varepsilon_{\rm im}$, $\varepsilon_{\rm f}$ and $\varepsilon_{\rm s}$ are the permittivities of the measuring prism, the immersion liquid (or air), the film and the substrate, respectively; $H_{\rm im}$ is the thickness of the gap between the film and the prism; $H_{\rm f}$ is the film thickness; and θ is the angle of incidence).

[$\boldsymbol{r} = (x, y, z)$, ω is the frequency, and t is the time] in the case of an incident TE wave (s polarisation) can be written in the form.

In medium I with a permittivity $\varepsilon_{\rm p} = N_{\rm p}^2$

$$\boldsymbol{E}^{\rm I}(y,z) = \boldsymbol{x}[E_{\rm in}^{\rm s}\exp({\rm i}k_y y + {\rm i}k_z z) + R_{\rm a}^{\rm s}\exp({\rm i}k_y y - {\rm i}k_z z)],$$

$$\boldsymbol{H}^{\rm I}(y,z) = \boldsymbol{y}\frac{k_z}{k}[E_{\rm in}^{\rm s}\exp({\rm i}k_y y + {\rm i}k_z z) - R_{\rm a}^{\rm s}\exp({\rm i}k_y y - {\rm i}k_z z)]$$
$$-\boldsymbol{z}\frac{k_y}{k}[E_{\rm in}^{\rm s}\exp({\rm i}k_y y + {\rm i}k_z z) + R_{\rm a}^{\rm s}\exp({\rm i}k_y y - {\rm i}k_z z)], \tag{1}$$

where $E_{\rm in}^{\rm s}$ is the amplitude of the incident s-polarised wave; $R_{\rm a}^{\rm s}$ is the reflected wave amplitude; $k = 2\pi/\lambda$; λ is the wavelength of light in vacuum; $k_y = kN_{\rm p}\sin\theta$; and $k_z = kN_{\rm p}\cos\theta$.

In medium II with a permittivity $\varepsilon_{\rm im} = (n_{\rm im} + {\rm i}m_{\rm im})^2$

$$\boldsymbol{E}^{\rm II}(y,z) = \boldsymbol{x}[E^{\rm IIdown}\exp({\rm i}k_y y - \gamma^{\rm II}z) + E^{\rm IIup}\exp({\rm i}k_y y + \gamma^{\rm II}z)],$$

$$\boldsymbol{H}^{\rm II}(y,z) = \boldsymbol{y}\frac{{\rm i}\gamma^{\rm II}}{k}[E^{\rm IIdown}\exp({\rm i}k_y y - \gamma^{\rm II}z)$$
$$-E^{\rm IIup}\exp({\rm i}k_y y + \gamma^{\rm II}z)]$$
$$-\boldsymbol{z}\frac{k_y}{k}[E^{\rm IIdown}\exp({\rm i}k_y y - \gamma^{\rm II}z) + E^{\rm IIup}\exp({\rm i}k_y y + \gamma^{\rm II}z)], \tag{2}$$

where $E^{\rm IIdown}$ and $E^{\rm IIup}$ are the amplitudes of the waves propagating in the gap in the positive and negative directions along the z axis, respectively; $\gamma^{\rm II} = (k_y^2 - k^2\varepsilon_{\rm im})^{1/2}$; $\operatorname{Re}\gamma^{\rm II} \geqslant 0$; and $\operatorname{Im}\gamma^{\rm II} \leqslant 0$.

In medium III with a permittivity $\varepsilon_{\rm f} = (n_{\rm f} + {\rm i}m_{\rm f})^2$

$$\boldsymbol{E}^{\rm III}(y,z) = \boldsymbol{x}[E^{\rm IIIdown}\exp({\rm i}k_y y - \gamma^{\rm III}z)$$
$$+E^{\rm IIIup}\exp({\rm i}k_y y + \gamma^{\rm III}z)], \tag{3}$$

$$\boldsymbol{H}^{\rm III}(y,z) = \boldsymbol{y}\frac{{\rm i}\gamma^{\rm III}}{k}[E^{\rm IIIdown}\exp({\rm i}k_y y - \gamma^{\rm III}z) -$$
$$-E^{\rm IIIup}\exp({\rm i}k_y y + \gamma^{\rm III}z)]$$
$$-\boldsymbol{z}\frac{k_y}{k}[E^{\rm IIIdown}\exp({\rm i}k_y y - \gamma^{\rm III}z) + E^{\rm IIIup}\exp({\rm i}k_y y + \gamma^{\rm III}z)],$$

where $E^{\rm IIIdown}$ and $E^{\rm IIIup}$ are the amplitudes of the waves in the film; $\gamma^{\rm III} = (k_y^2 - k^2\varepsilon_{\rm f})^{1/2}$; $\operatorname{Re}\gamma^{\rm III} \geqslant 0$; and $\operatorname{Im}\gamma^{\rm III} \leqslant 0$.

In medium IV with a permittivity $\varepsilon_{\rm s} = (n_{\rm s} + {\rm i}m_{\rm s})^2$

$$\boldsymbol{E}^{\rm IV}(y,z) = \boldsymbol{x}E^{\rm IV}\exp({\rm i}k_y y - \gamma^{\rm IV}z),$$

$$\boldsymbol{H}^{\rm IV}(y,z) = \boldsymbol{y}\frac{{\rm i}\gamma^{\rm IV}}{k}E^{\rm IV}\exp({\rm i}k_y y - \gamma^{\rm IV}z)$$
$$-\boldsymbol{z}\frac{k_y}{k}E^{\rm IV}\exp({\rm i}k_y y - \gamma^{\rm IV}z), \tag{4}$$

where $E^{\rm IV}$ is the amplitude of the wave in the substrate; $\gamma^{\rm IV} = (k_y^2 - k^2\varepsilon_{\rm s})^{1/2}$; $\operatorname{Re}\gamma^{\rm IV} \geqslant 0$; and $\operatorname{Im}\gamma^{\rm IV} \leqslant 0$.

Taking into account the form of electromagnetic fields (1)–(4) and sewing these fields at the interfaces $z = 0$, $z = H_{\rm im}$ and $z = H_{\rm im} + H_{\rm f}$ (see Fig. 1), for the amplitude of the s-polarised reflected wave, $R_{\rm a}^{\rm s}$, we obtain

$$\frac{R_{\rm a}^{\rm s}}{E_{\rm in}^{\rm s}} = \left[\frac{k_z - {\rm i}\gamma^{\rm II}}{k_z + {\rm i}\gamma^{\rm II}} + A_{\rm s}\exp(-2\gamma^{\rm II}H_{\rm im})\right]$$
$$\times\left[1 + \frac{k_z - {\rm i}\gamma^{\rm II}}{k_z + {\rm i}\gamma^{\rm II}}A_{\rm s}\exp(-2\gamma^{\rm II}H_{\rm im})\right]^{-1},$$
$$A_s = \left[\frac{\gamma^{\rm II} - \gamma^{\rm III}}{\gamma^{\rm II} + \gamma^{\rm III}} + \frac{\gamma^{\rm III} - \gamma^{\rm IV}}{\gamma^{\rm III} + \gamma^{\rm IV}}\exp(-2\gamma^{\rm III}H_{\rm f})\right]$$
$$\times\left[1 + \frac{\gamma^{\rm II} - \gamma^{\rm III}}{\gamma^{\rm II} + \gamma^{\rm III}}\frac{\gamma^{\rm III} - \gamma^{\rm IV}}{\gamma^{\rm III} + \gamma^{\rm IV}}\exp(-2\gamma^{\rm III}H_{\rm f})\right]^{-1}. \tag{5}$$

Similarly, in the case of the incident TM wave (p polarisation) the electromagnetic fields in media I–IV can be represented as follows:

in medium I

$$\boldsymbol{H}^{\rm I}(y,z) = \boldsymbol{x}[H_{\rm in}^{\rm p}\exp({\rm i}k_y y + {\rm i}k_z z) + R_{\rm a}^{\rm p}\exp({\rm i}k_y y - {\rm i}k_z z)],$$

$$\boldsymbol{E}^{\rm I}(y,z) = -\boldsymbol{y}\frac{k_z}{k\varepsilon_{\rm p}}[H_{\rm in}^{\rm p}\exp({\rm i}k_y y + {\rm i}k_z z)$$
$$-R_{\rm a}^{\rm p}\exp({\rm i}k_y y - {\rm i}k_z z)]$$
$$+\boldsymbol{z}\frac{k_y}{k\varepsilon_{\rm p}}[H_{\rm in}^{\rm p}\exp({\rm i}k_y y + {\rm i}k_z z) + R_{\rm a}^{\rm p}\exp({\rm i}k_y y - {\rm i}k_z z)]; \tag{6}$$

in medium II

$$\boldsymbol{H}^{\rm II}(y,z) = \boldsymbol{x}[H^{\rm IIdown}\exp({\rm i}k_y y - \gamma^{\rm II}z)$$
$$+H^{\rm IIup}\exp({\rm i}k_y y + \gamma^{\rm II}z)], \tag{7}$$

$$\boldsymbol{E}^{\rm II}(y,z) = -\boldsymbol{y}\frac{{\rm i}\gamma^{\rm II}}{k\varepsilon_{\rm im}}[H^{\rm IIdown}\exp({\rm i}k_y y - \gamma^{\rm II}z)$$

$$- H^{\rm IIup}\exp({\rm i}k_y y + \gamma^{\rm II}z)]$$

$$+ \boldsymbol{z}\frac{k_y}{k\varepsilon_{\rm im}}[H^{\rm IIdown}\exp({\rm i}k_y y - \gamma^{\rm II}z) + H^{\rm IIup}\exp({\rm i}k_y y + \gamma^{\rm II}z)];$$

in medium III

$$\boldsymbol{H}^{\rm III}(y,z) = \boldsymbol{x}[H^{\rm IIIdown}\exp({\rm i}k_y y - \gamma^{\rm III}z)$$

$$+ H^{\rm IIIup}\exp({\rm i}k_y y + \gamma^{\rm III}z)],$$

$$\boldsymbol{E}^{\rm III}(y,z) = -\boldsymbol{y}\frac{{\rm i}\gamma^{\rm III}}{k\varepsilon_{\rm f}}[H^{\rm IIIdown}\exp({\rm i}k_y y - \gamma^{\rm III}z) \qquad (8)$$

$$- H^{\rm IIIup}\exp({\rm i}k_y y + \gamma^{\rm III}z)]$$

$$+ \boldsymbol{z}\frac{k_y}{k\varepsilon_{\rm f}}[H^{\rm IIIdown}\exp({\rm i}k_y y - \gamma^{\rm III}z) + H^{\rm IIIup}\exp({\rm i}k_y y + \gamma^{\rm III}z)];$$

in medium IV

$$\boldsymbol{H}^{\rm IV}(y,z) = \boldsymbol{x}H^{\rm IV}\exp({\rm i}k_y y - \gamma^{\rm IV}z),$$

$$\boldsymbol{E}^{\rm IV}(y,z) = -\boldsymbol{y}\frac{{\rm i}\gamma^{\rm IV}}{k\varepsilon_{\rm s}}H^{\rm IV}\exp({\rm i}k_y y - \gamma^{\rm IV}z) \qquad (9)$$

$$+ \boldsymbol{z}\frac{k_y}{k\varepsilon_{\rm s}}H^{\rm IV}\exp({\rm i}k_y y - \gamma^{\rm IV}z).$$

In expressions (6)–(9) $H_{\rm in}^{\rm p}$ is the amplitude of the incident p-polarised wave; $R_{\rm a}^{\rm p}$ is the amplitude of the reflected wave; and $H^{\rm IIdown}$, $H^{\rm IIup}$, $H^{\rm IIIdown}$, $H^{\rm IIIup}$ and $H^{\rm IV}$ are the amplitudes of the waves in media II, III and IV, respectively. By sewing the fields (6)–(9) at the interfaces $z = 0$, $z = H_{\rm im}$ and $z = H_{\rm im} + H_{\rm f}$, for the amplitude of the reflected wave $R_{\rm a}^{\rm p}$ we derive

$$\frac{R_{\rm a}^{\rm p}}{H_{\rm in}^{\rm p}} = \left[\frac{\varepsilon_{\rm im}k_z - {\rm i}\gamma^{\rm II}\varepsilon_{\rm p}}{\varepsilon_{\rm im}k_z + {\rm i}\gamma^{\rm II}\varepsilon_{\rm p}} + A_p\exp(-2\gamma^{\rm II}H_{\rm im})\right]$$

$$\times\left[1 + \frac{\varepsilon_{\rm im}k_z - {\rm i}\gamma^{\rm II}\varepsilon_{\rm p}}{\varepsilon_{\rm im}k_z + {\rm i}\gamma^{\rm II}\varepsilon_{\rm p}}A_{\rm p}\exp(-2\gamma^{\rm II}H_{\rm im})\right]^{-1},$$

$$A_{\rm p} = \left[\frac{\gamma^{\rm II}\varepsilon_{\rm f} - \gamma^{\rm III}\varepsilon_{\rm im}}{\gamma^{\rm II}\varepsilon_{\rm f} + \gamma^{\rm III}\varepsilon_{\rm im}} + \frac{\gamma^{\rm III}\varepsilon_{\rm s} - \gamma^{\rm IV}\varepsilon_{\rm f}}{\gamma^{\rm III}\varepsilon_{\rm s} + \gamma^{\rm IV}\varepsilon_{\rm f}}\exp(-2\gamma^{\rm III}H_{\rm f})\right] \qquad (10)$$

$$\times\left[1 + \frac{\gamma^{\rm II}\varepsilon_{\rm f} - \gamma^{\rm III}\varepsilon_{\rm im}}{\gamma^{\rm II}\varepsilon_{\rm f} + \gamma^{\rm III}\varepsilon_{\rm im}}\,\frac{\gamma^{\rm III}\varepsilon_{\rm s} - \gamma^{\rm IV}\varepsilon_{\rm f}}{\gamma^{\rm III}\varepsilon_{\rm s} + \gamma^{\rm IV}\varepsilon_{\rm f}}\exp(-2\gamma^{\rm III}H_{\rm f})\right]^{-1}.$$

Figure 2 presents the dependences [calculated by expressions (5) and (10)] of the specular reflection coefficients $R_{\rm s}(\theta) = |R_{\rm a}^{\rm s}/E_{\rm in}^{\rm s}|^2$ and $R_{\rm p}(\theta) = |R_{\rm a}^{\rm p}/H_{\rm in}^{\rm p}|^2$ on the angle θ of the beam incidence at the prism–film interface. The parameters of the prism, immersion layer, film, substrate and wavelength of light are given in the legend to Fig. 2. One can see that the dependences $R_{\rm s}(\theta)$ and $R_{\rm p}(\theta)$ exhibit sharp minima (*m* lines) due to excitation of waveguide TE and TM modes in the film. An increase in the extinction coefficient in the film material leads to a broadening of the width of *m* lines, their angular position remaining virtually unchanged. Note the break in the curves $R_{\rm s}(\theta)$ and $R_{\rm p}(\theta)$ at $\theta = 42.9°$ (the so-called knee), arising due to the tunnelling of light in the substrate, when the condition $N_{\rm p}\sin\theta = n_{\rm s}$ is met.

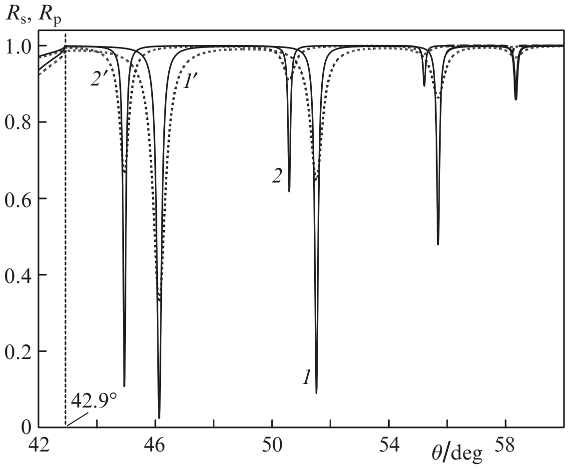

Figure 2. Dependences of the reflection coefficients $R_{\rm s}(\theta)$ and $R_{\rm p}(\theta)$ under illumination of the prism–film interface by a plane electromagnetic wave with $\lambda = 632.8$ nm in the case of TE (*1*, *1′*) and TM (*2*, *2′*) polarisations; $N_{\rm p} = 2.14044$, $\varepsilon_{\rm im} = 1$, $H_{\rm im} = 150$ nm, $H_{\rm f} = 1082$ nm, $n_{\rm s} = 1.45705$ and $m_{\rm s} = 0$. Curves (*1*) and (*2*) refer to the case $\varepsilon_{\rm f} = (1.840 + {\rm i}0.001)^2$, curves (*1′*) and (*2′*) – to the case $\varepsilon_{\rm f} = (1.840 + {\rm i}0.005)^2$.

The idea of the proposed method for measuring the extinction coefficient $m_{\rm f}$ of the film by the angular dependence of the specular reflection coefficient is based on the fact that the depth and width of the resonance associated with the excitation of the waveguide modes in the film are very sensitive to the value of $m_{\rm f}$ [22, 23]. The method is based on the numerical simulation for determining the parameters $n_{\rm f}$, $m_{\rm f}$, $H_{\rm f}$ and $H_{\rm im}$, at which the reflection coefficients calculated by expressions (5) and (10) are fitted best with the experimentally measured dependences of $R_{\rm s}(\theta)$ and $R_{\rm p}(\theta)$ in a wide range of angles of incidence.

3. Experiment and method for determining the film parameters

A film of silicon oxide $\rm SiO_x$ ($x \approx 1$), having a thickness $H_{\rm f} \approx 1$ μm, was fabricated by thermal evaporation of SiO monoxide followed by vacuum deposition on a $\rm SiO_2$ substrate. This structure will be denoted below as $\rm SiO_{\it x}/SiO_2$. The reflection coefficients $R_{\rm s}(\theta)$ and $R_{\rm p}(\theta)$ were measured with a Metricon-2010 prism coupler in the range of angles from 42.7° to 67.7° during scanning by a TE- and TM-polarised beam of a helium–neon laser with a wavelength $\lambda = 632.8$ nm and a diameter $d = 1.5$ mm. The divergence of the laser beam was $\Delta\theta \approx \lambda/d = 0.024°$; therefore, in calculating $R_{\rm s}(\theta)$ and $R_{\rm p}(\theta)$ one can use the approximation of a plane incident wave.

To find the reflection coefficients, we measured the angular dependences of the intensity of the laser beam reflected from the working face of the measuring prism in a situation when the sample is in optical contact with it [in this case, the dependence $R(\theta)$ exhibits an *m* line] and when the sample is removed from the prism (m lines are absent). The specular reflection coefficients were defined as the ratio of these dependences. To enhance the accuracy of $R_s(\theta)$ and $R_p(\theta)$ measurements, we performed averaging over several scans. Figure 3 presents the experimental dependences of the specular reflection coefficients $R_s(\theta)$ and $R_p(\theta)$ for the SiO_x/SiO_2 film.

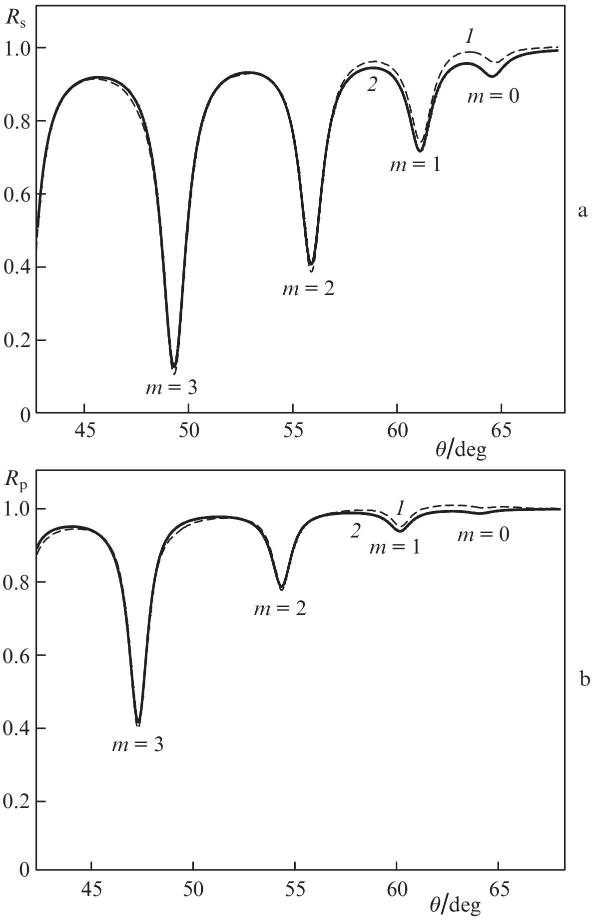

Figure 3. Angular dependences of the specular reflection coefficient for the $SiOx/SiO_2$ film – (*1*) measured and (*2*) calculated by expressions (5) and (10) at the optimum values of n_f, m_f, H_f and H_{im} in the case of (a) TE and (b) TM polarisations of the incident laser beam.

It follows from Fig. 3 that the film features four TE and four TM waveguide modes. The optical constants n_f, m_f and the film thickness H_f were determined by minimising the functional

$$\phi = \left[N^{-1}\sum_{i=1}^{N}(R_{\exp}(\theta_i) - R_{\rm thr}(\theta_i))^2\right]^{1/2},$$

determining the root-mean-square deviation between the experimental [$R_{\exp}(\theta)$] and theoretical [by expressions (5) and (10)] [$R_{\rm thr}(\theta)$] values of the reflection coefficient. We also calculated the thickness H_{im} of the air gap between the film and the prism. The optical constants of the substrate were as follows: $n_s = 1.45705$ and $m_s = 0$. The optimal values of n_f, m_f, H_f and H_{im}, which yield the best agreement between the theoretical and experimental values of $R(\theta)$ throughout the entire range of incidence angles, are listed below. It follows from Fig. 3 that the measured and calculated widths of the resonances associated with the excitation of the waveguide modes in the film are close. Note a slight discrepancy between the measured and calculated positions of the minima for the mode with $m = 0$. In our opinion, this is due to the presence of a small refractive index gradient across the film thickness. Such a gradient affects to the greatest extent the position of m lines for the lower-order modes (m = 0, 1), which are strongly localised in the film, and to a much lesser extent the positions of m lines for the higher-order modes.

The values of the SiO_x/SiO_2 film parameters calculated from the angular dependences $R_s(\theta)$ and $R_p(\theta)$ for a wavelength of $\lambda = 632.8$ nm are as follows: $n_f = 1.9677$, $m_f = 0.011$, $H_f = 1003.3$ nm and $H_{im} = 88.6$ nm [for $R_s(\theta)$] and $n_f = 1.9595$, $m_f = 0.011$, $H_f = 1012.3$ nm and $H_{im} = 75.7$ nm [for $R_p(\theta)$]. For comparison we present the values of n_f, m_f and H_{im}, calculated for the same wavelength by the transmission spectroscopy method: $n_f = 1.9664$, $m_f = 0.010$ and $H_f = 1007$ nm.

As can be seen from these data, the values of the extinction coefficient of the film, found for the case of TE and TM polarisations of the exciting laser beam, coincide and are equal to 0.011. The calculated film thicknesses differ by 9 nm, and the error in determining H_f is ±0.5%. The thickness of the air gap between the prism and the film, defined as the average of the values obtained in the case of the TE and TM polarisations is 82.2 ± 6.5 nm. Measurements performed at different thicknesses H_{im} of the gap between the measuring prism and the film showed that the measurement error in n_f is $\pm 2 \times 10^{-4}$. The difference in the values of n_f (1.9677 and 1.9595) for TE and TM polarisations we associate with the anisotropy of the film material.

To check the accuracy of the proposed method used for measuring n_f, m_f and H_f of thin films, we also determined the SiO_x/SiO_2 film parameters by transmission spectroscopy. Figure 4a presents the transmission coefficient $T(\lambda)$ measured by a Cary-50 spectrophotometer at normal incidence of the beam. The thickness and optical constants $n_f(\lambda)$ and $m_f(\lambda)$ of the film were determined by minimising the root-mean-square deviations of the experimental and theoretical curves $T(\lambda)$ with Cauchy's dispersion model $n_f(\lambda) = A_0 + A_1/\lambda^2 + A_2/\lambda^4$, $m_f(\lambda) = B_0 + B_1/\lambda^2 + B_2/\lambda^4$, where A_i and B_i ($i = 0, 1, 2$) are the constants. The calculated values of H_f, n_f and m_f are given above. Figure 4b shows the found dependences $n_f(\lambda)$ and $m_f(\lambda)$, and Fig. 4a – the corresponding theoretical curve $T(\lambda)$.

Thus, the values of n_f, m_f and H_f obtained by the method of excitation of the waveguide modes are in good agreement with the results obtained by transmission spectroscopy.

4. Conclusions

We have shown that the method of excitation of the waveguide modes allows one, at any coupling coefficient, to perform the simultaneous measurement of the refractive index n_f, extinction coefficient m_f and thickness H_f of thin films, as well as to determine the thickness H_{im} of the gap between the measuring prism and the film. We have found the optical constants and thickness of a silicon monoxide film deposited on a quartz substrate at a wavelength of 632.8 nm. The values found are in good agreement with those obtained by transmission spectroscopy. The error in determining n_f, m_f and H_f by the method described is ±0.0002, ±0.001 and ±0.5%, respectively. The method can be also used to measure optical parameters of films at other wavelengths by employing the corresponding lasers.

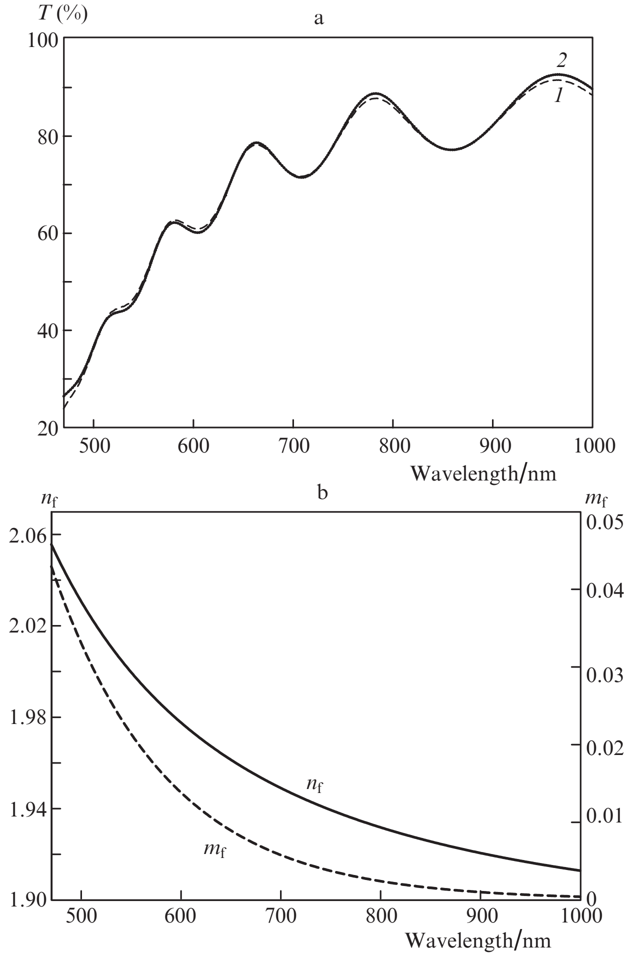

Figure 4. (a) Dependences $T(\lambda)$ for the SiO$_x$/SiO$_2$ film – (*1*) measured at normal incidence and (*2*) calculated at the optimal $n_f(\lambda)$, $m_f(\lambda)$, H_f = 1007 nm, and (b) dependences $n_f(\lambda)$ and $m_f(\lambda)$ at which the best agreement is achieved between the experimental and calculated values of $T(\lambda)$. The optical constants of the substrate are $n_s = 1.45705$ and $m_s = 0$.

When calculating the n_f, m_f and H_f values of the SiO$_x$/SiO$_2$ film, we have used the approximation of an incident plane wave and a model of a single-layer uniform film. To improve the accuracy of the method we should take into account the possible presence of transition layers, refractive index gradients $n_f(z)$ and the extinction coefficient $m_f(z)$ in a thin film, as well as the finite cross section of the exciting laser beam, as is done in ellipsometry.

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