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Criteria for admissible values of smooth aberrations for nondiffractive laser beams*

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Abstract. **We have derived analytical expressions, verified by the methods of numerical simulation, to evaluate the angular divergence of nondiffractive laser beams containing smooth aberrations, i.e., spherical defocusing, astigmatism and toroid. Using these expressions we have formulated the criteria for admissible values of smooth aberrations.**

Keywords: *laser radiation, wavefront, smooth wavefront aberration.*

1. Introduction

In optics, including laser optics, known are rigorous analytical expressions for calculating the characteristics of diffraction beams. Thus, for a Gaussian beam with an initial radius w_0 of its cross section (at the 1/e level of the field amplitude) at a distance z, a new value of the transverse radius $w(z)$ is determined from the expression [1]

$$w^2(z) = w_0^2\left[1 + \left(\frac{\lambda z}{\pi w_0^2}\right)\right], \tag{1}$$

where λ is the wavelength. For such a beam an analytical expression is also known for the angular divergence θ (at the 0.5 intensity level) in the presence of a spherical component of the wavefront with a radius of curvature R [2]:

$$\theta = \ln 2\left[\left(\frac{w_0}{R}\right)^2 + \left(\frac{\lambda}{\pi w_0}\right)^2\right]^{1/2}. \tag{2}$$

The Maréchal rule [3] allows a reduction in the intensity I to be calculated on the axis of a diffraction-limited beam in the case of perturbation of the wavefront with a standard deviation (SD) $\sigma_{\rm wf} \leqslant \lambda/14$:

$$I = I_0\left[1 - \left(\frac{2\pi\sigma_{\rm wf}}{\lambda}\right)^2\right]. \tag{3}$$

Also known is the analytical expression for the joint account of the angular radiation divergence φ_1 of a nondiffractive beam ($\Omega = \pi\varphi_1^2/4$ is the solid angle, which contains half the beam power) with a Gaussian angular distribution of the local slopes of the wavefront and the error of the guidance system $\sigma_{\rm gs}$ that also obeys the Gaussian distribution. With the same contribution of both factors, a new value of the time-averaged angular divergence φ_2 is determined from the expression [4]

$$\varphi_2^2 = \varphi_1^2 + 8\ln 2\sigma_{\rm gs}^2. \tag{4}$$

Relations (1)–(4) almost exhaust the set of analytical expressions to account for the influence of aberrations on the characteristics of the laser radiation.

In the above formulas, joint consideration of the initial characteristics of radiation and of the introduced uncorrelated wavefront perturbations leads to the procedure of a root-mean-square addition or subtraction to calculate the resulting characteristics of radiation.

For lasers with a nondiffractive beam quality similar analytical expressions are not available, although these lasers are of great practical importance, since laser radiation with a high average power, as a rule, has a nondiffractive quality [5–7] and the angular divergence of radiation at the output of these lasers is more than an order of magnitude higher than the diffraction divergence. There are realistic projects of remote energy supply to spacecrafts [6] and laser engines [7], which will employ sufficiently high-power lasers with nondiffractive angular beam divergence. To correctly construct such laser systems, it is needed to derive analytical expressions, which take into account the contribution of nondiffractive smooth wavefront aberrations into the angular divergence of radiation.

In this paper we have found the criteria for admissible values of smooth wavefront aberrations for laser radiation of nondiffractive quality. From our point of view, these criteria allow us to solve three groups of problems. The first group arises from the need to develop optical laser channels with small angular divergence of radiation.

Let us enumerate the problems we can solve with the help of these criteria. We assume that the angular divergence of laser radiation is equal to φ and the beam aperture – to a. One of the questions, which arises during the adjustment of an optical system, is about the value of the error, such as spherical defocusing, related to the amplitude (wavefront deflection) $h_{\rm s}$, so that the angular divergence of radiation does not change significantly.

This group of problems also involves the admissible value of astigmatism, which arises at oblique incidence of the beam on the rotating mirror at an angle α. Any flat mirror has a finite radius of curvature, which is due to a manufacture error or thermal distortion of mirrors, including the case when they are irradiated by high-power laser pulses. Regardless of

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the cause of curvature, the wavefront after reflection in the meridional plane obtains a jump-like additive

$$h_{\mathrm{m}} = \frac{a^2}{4R\cos\alpha}, \tag{5}$$

and in the sagittal plane – an additive

$$h_{\mathrm{s}} = \frac{a^2\cos\alpha}{4R}. \tag{6}$$

The astigmatism amplitude h_{a} is given by

$$h_{\mathrm{a}} = h_{\mathrm{m}} - h_{\mathrm{s}} = \frac{a^2}{4R}\frac{\sin^2\alpha}{\cos\alpha}. \tag{7}$$

We have formulated the criteria for the admissible value h_{a}, which insignificantly increases the angular divergence of a beam with an aperture a.

Finally, this group of problems also includes the one about the admissible amplitude of circular toroid-type aberrations arising under the circular loading of laser mirrors. The shape of aberrations resembles an axisymmetric Zernike polynomial

$$Z = \sqrt{7}\,(20\rho^6 - 30\rho^4 + 12\rho^2 - 1),$$

where $\rho = \sqrt{x^2 + y^2}$. Thus, the first group of problems is related to the influence of smooth aberrations on the angular divergence φ of the beam.

The second group of problems arises when evaluating the performance of an adaptive system [8]: At what values of the above-listed smooth aberrations does an adaptive system not require their compensation (since such compensation does not make sense)?

The third group of problems is related to the possibility of high-precision measurement of aberrations, such as the fact whether there are physical or technical limitations on implementation of sufficient accuracy of measurements.

In this paper we restrict our consideration to the first group of problems, whose solution is of key importance for the second and third groups.

2. Methodical approach

The criteria for the admissible values of smooth aberrations at a given angular divergence φ and beam aperture a can be designed by using analytical expressions, which jointly take into account both the intrinsic angular divergence of the beam and its wavefront aberrations. Obviously, in this case the additional terms in the analytical expression for the total angular divergence of the beam must be significantly less than the initial divergence φ.

First, let us agree to what is meant by a model of a beam with an angular divergence that is higher than the diffraction divergence. It is known [2, 3, 9] that in the case of spherical deflection of a diffraction-limited beam wavefront with a deflection $h_{\mathrm{s}} \geqslant \lambda$, it is needed to apply geometrical optics approximation. Further analysis will be given in this range of the wavefront perturbation amplitudes. Figure 1a shows the wavefront characterised by the SD σ_1 of its angular local slopes in the case of their Gaussian distribution. The power density distribution at the aperture of diameter a is assumed uniform. We call such a beam a Gaussian-like beam. Then, the angular intensity dependence has the form

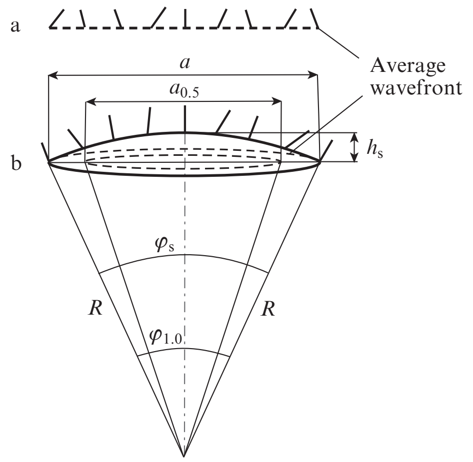

Figure 1. Model of a Gaussian-like beam (a) without a spherical component and (b) with a spherical component having a radius of curvature R.

$$I(\phi) = \frac{P_0}{2\pi\sigma_1^2}\exp\left(-\frac{\phi^2}{2\sigma_1^2}\right), \tag{8}$$

where ϕ is the polar angle. The Gaussian angular intensity distribution over the coordinate ϕ in (8) is wider than the diffraction distribution. For the beam in question we have the following relation between the angular divergence (i.e., the solid angle, which contains half the power of the beam) with the SD of the wavefront tilt [4]:

$$\varphi = 2\sqrt{2\ln 2}\,\sigma_1 \approx 2.36\sigma_1. \tag{9}$$

The Gaussian-like angular intensity distribution is typical of high-power laser systems with a large number of mirrors and numerous uncorrelated perturbations factors and is confirmed experimentally.

3. Derivation of analytical relations and results of numerical simulations

3.1. Criteria for the admissible value of spherical defocusing aberrations

Figure 1a shows the wavefront of a nondiffractive beam with an aperture a and SD σ_1. On the average, the wavefront is a plane.

Figure 1b presents the same wavefront with an added spherical component having a radius of curvature $R \gg a$. The wavefront is a sphere with an average radius of curvature R, which has local slopes with the SD σ_2. With the constraint $h_{\mathrm{s}} \ll a \ll R$, the following relation

$$h_{\mathrm{s}} = \frac{a^2}{8R} \tag{10}$$

is valid. The solid angle $\Omega_{1.0}$ of the cone, where a set of all rays of the beam with a spherical wavefront uniformly distributed over the aperture propagates, is defined as

$$\Omega_{1.0} = \frac{\pi}{4}\varphi_{1.0}^2 = \frac{\pi}{4}\left(\frac{a}{R}\right)^2. \tag{11}$$

In the solid angle

$$\Omega_{0.5} = \frac{\pi}{4}\left(\frac{\varphi_{1.0}}{\sqrt{2}}\right)^2 \qquad (12)$$

half of all rays propagates. A flat angle φ_s we are interested in, which corresponds to the solid angle $\Omega_{0.5}$, is given by

$$\varphi_s = \frac{4\sqrt{2}h_s}{a} \approx \frac{5.64h_s}{a}. \qquad (13)$$

Thus, in the geometrical optics approximation ($h_s \geqslant \lambda$) formula (13) is valid when φ_s is about six and more diffraction limits λ/a.

We define the SD σ_s of local slopes of a spherical wavefront (Fig. 1). For a section of a sphere with a chord of length $a \ll R$ and the wavefront deflection $h_s \ll a$ (Fig. 1b) the local slopes are described by the expression

$$\theta(x) = x/R,$$

where x is the current coordinate on an aperture of diameter a. The SD of local slopes of the sphere section σ_s on the chord of length a is determined from the ratio

$$\sigma_s^2 = \frac{R}{a}\int_{-a/2}^{+a/2}\left(\frac{x}{R}\right)^2 \mathrm{d}\left(\frac{x}{R}\right) = \frac{16}{3}\left(\frac{h_s}{a}\right)^2. \qquad (14)$$

The angular divergence of the laser beam in the case of smooth aberrations is taken into account by adding the quadratic quantities from the analytical expressions because the quantities being added are uncorrelated. Thus, the new values of the SD, taking into account the two factors, are found from the expression

$$\sigma_2^2 = \sigma_1^2 + \sigma_s^2 = \sigma_1^2\left[1 + \frac{16}{3}\left(\frac{h_s}{\sigma_1 a}\right)^2\right], \qquad (15)$$

and, consequently, the angular divergence φ_2 is related to the initial one by the expression

$$\varphi_2^2 = \varphi_1^2\left[1 + \frac{16}{3}\left(\frac{h_s}{\sigma_1 a}\right)^2\right], \qquad (16)$$

which can also be written in a simplified form [4]:

$$\varphi_2 = 2.36\sigma^2. \qquad (17)$$

The criterion for the admissible defocusing is the ratio

$$\frac{16}{3}\left(\frac{h_s}{\sigma_1 a}\right)^2 \ll 1, \text{ whence } h_s \ll \frac{\sqrt{3}}{4}\sigma_1 a. \qquad (18)$$

In the case of an axially symmetric initial beam and axially symmetric wavefront perturbation, formulas (15)–(17) can be derived differently, provided that $h_s \ll a\sigma_1$. We assume that the discrete values of the local wavefront slopes φ_i are obtained by the Hartmann method in N equidistant points on the aperture of diameter a.

The SD σ_1 of local slopes is determined from the expression (at $N \gg 1$)

$$\sigma_1^2 = \frac{1}{N}\sum_i^N \varphi_i^2. \qquad (19)$$

When we add a spherical component of radius R on an aperture of diameter a, there appears a regular wavefront perturbation with an amplitude in the centre h_s; in this case, $h_s \ll a \ll R$. Then, the first local slope (from the centre) on the right semi-aperture will have an angular increment $a/(RN)$ and become equal to

$$\varphi_{+1} + \frac{a}{RN}, \qquad (20)$$

and the last will be equal to

$$\varphi_{N/2} + \frac{N}{2}\frac{a}{RN}. \qquad (21)$$

The first local slope (from the centre) on the left semi-aperture will be equal to

$$\varphi_{-1} - \frac{N}{2}\frac{a}{RN}, \qquad (22)$$

and the last will be equal to

$$\varphi_{-N/2} - \frac{N}{2}\frac{a}{RN}. \qquad (23)$$

The new value of the SD σ_2 is determined from the expression

$$\sigma_2^2 = \frac{1}{N}\left[\left(\varphi_{+1} + \frac{a}{NR}\right)^2 + \ldots + \left(\varphi_{N/2} + \frac{N}{2}\frac{a}{NR}\right)^2 + \left(\varphi_{-1} - \frac{a}{NR}\right)^2 + \ldots + \left(\varphi_{-N/2} - \frac{N}{2}\frac{a}{NR}\right)^2\right]$$

$$= \sigma_1^2 + \frac{1}{N}\left(\frac{a}{NR}\right)^2\left[1^2 + 2^2 + \ldots + \left(\frac{N}{2}\right)^2\right] + \frac{1}{N}\frac{a}{NR}\left[1^2 + 2^2 + \ldots + \left(\frac{N}{2}\right)^2\right]$$

$$= \sigma_1^2 + \frac{2}{N^3}\frac{8h}{a}\left[1^2 + 2^2 + \ldots + \left(\frac{N}{2}\right)^2\right]. \qquad (24)$$

Without loss of generality of the approach, we assume that the large number N is even. Using the relation [10]

$$\sum_{k=1}^N k^2 = \frac{N(N+1)(2N+1)}{6} \approx \frac{2N^3}{6} = \frac{N^3}{3} \quad \text{at } N \gg 1 \qquad (25)$$

and expression (24), we finally obtain

$$\sigma_2^2 = \sigma_1^2 + \frac{16}{3}\left(\frac{h_s}{a}\right)^2 = \sigma_1^2\left[1 + \frac{16}{3}\left(\frac{h_s}{a}\right)^2\right], \qquad (26)$$

which coincides with (15). Then, the angle φ_s, corresponding to half the radiation power, in view of (14) is

$$\varphi_s = 2\sqrt{2\ln 2}\,\sigma_s = 2.36\sqrt{\frac{16}{3}\left(\frac{h_s}{a}\right)} \approx 5.45\frac{h_s}{a}.$$

It is important that this expression virtually coincides with (13), we have obtained in the geometrical optics approximation.

Figure 2 shows the results of numerical simulation of the angular power distribution of a diffraction-limited beam in a circular aperture with spherical defocusing, and Fig. 3 shows the fraction of the total laser radiation power in a cone with a flat angle $\varphi_s = 5.6h_s/a$. One can see from Fig. 3 that formula (13) for spherical defocusing is fairly accurate at $h_s \geqslant 3\lambda$ and can also be used at $h_s \geqslant \lambda$.

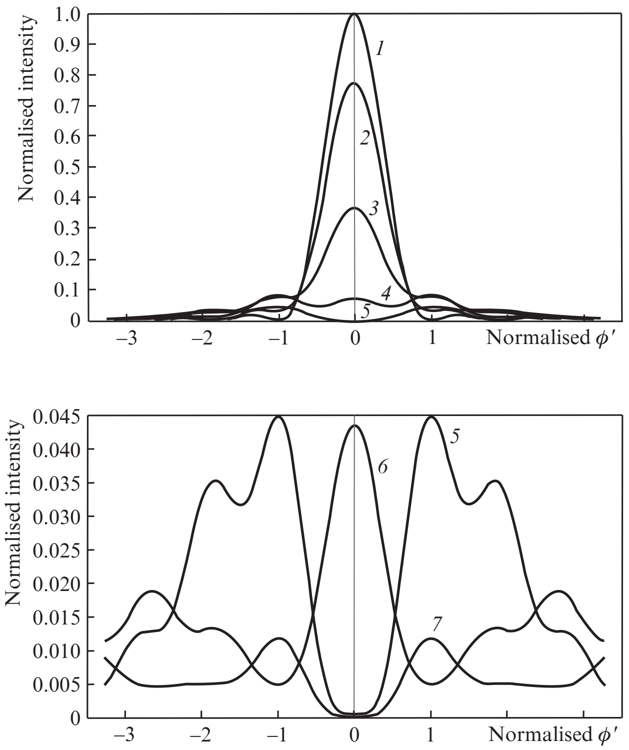

Figure 2. Far-field intensity distribution in the case of the spherical wavefront deflection h_s = (*1*) 0, (*2*) $\lambda/4$, (*3*) $\lambda/2$, (*4*) $3\lambda/4$, (*5*) λ, (*6*) $3\lambda/2$ and (*7*) 2λ. Hereinafter, ϕ' is the angular coordinate ϕ normalised to the angular divergence of a diffraction-limited beam $1.22\lambda/a$.

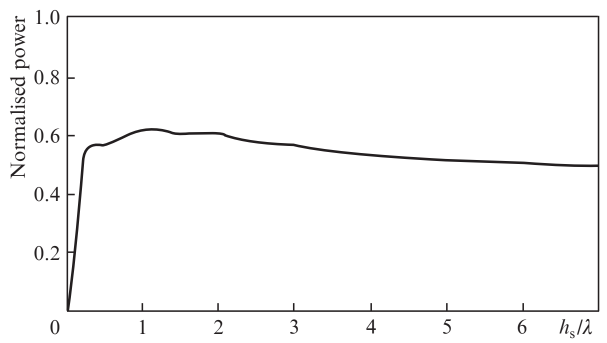

Figure 3. Dependence of the fraction of the total power P in a solid angle with a corresponding plane angle $\varphi_s = 5.6h_s/a$ on the spherical wavefront deflection h_s.

3.2. Criterion for the admissible value of toroid aberrations

Analytical apparatus we have developed for the case of spherical defocusing describes more accurately the axially symmetric aberration, which we have conventionally called a toroid (Fig. 4).

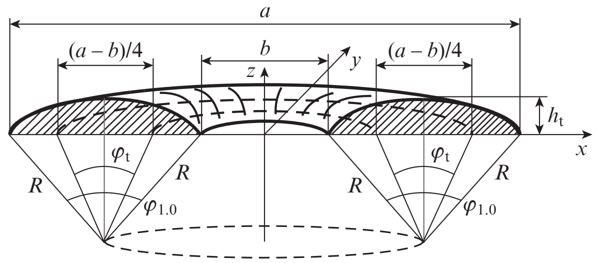

Figure 4. Axially symmetric aberration of toroid type and the angular divergence introduced.

For simplicity, we assume that the profile of the toroid-type aberrations in the cross section is close to a circular segment. Then, the current deflection z can be described with good accuracy by the expression (at $b = a\varepsilon \leqslant \rho \leqslant a$)

$$z = h_t\left[1 - \left(\frac{4\rho - a - b}{a - b}\right)^2\right] = h_t\left\{1 - \left[\frac{4\rho}{a(1-\varepsilon)} - \frac{1+\varepsilon}{1-\varepsilon}\right]^2\right\},$$

where $\varepsilon = b/a$ is the ratio of the inner and outer diameters of the circular loading zone.

At $R = [(a-b)/2]^2/(8h_t)$ the plane angle corresponding to the solid angle containing the half the power is given by the relation

$$\varphi_t = \frac{a-b}{4R} = \frac{8h_t}{a(1-\varepsilon)}. \tag{27}$$

The results of numerical simulation of the angular power distribution at $\varepsilon = 0$ are presented in Fig. 5, and Fig. 6 shows the fraction of the total laser radiation power in a cone with a plane angle φ_t (27) at $\varepsilon = 0$. Figures 7 and 8 demonstrate the results in the case of $\varepsilon = 0.5$. It follows from Figs 6 and 8 that formula (27) is valid at $h_t > \lambda$.

The dispersion of the angular distribution of the slopes with the account for the aberration of toroid type is found from the expression

$$\sigma_2^2 = \sigma_1^2 + \left(\frac{\varphi_t}{2.36}\right)^2 = \sigma_1^2 + \left[\frac{1}{2.36}\frac{8h_t}{a(1-\varepsilon)}\right]^2$$

$$= \sigma_1^2 + \frac{35}{3}\left[\frac{h_t}{a(1-\varepsilon)}\right]^2 = \sigma_1^2 + \sigma_t^2,$$

where the dispersion σ_t^2, caused by the toroid aberration, is added to the initial dispersion σ_1^2.

The angular divergence of the radiation, taking into account the toroid aberration and the initial divergence φ_1, is determined from the expression

$$\varphi_2^2 = \varphi_1^2\left\{1 + \frac{35}{3}\left[\frac{h_t}{\sigma_1 a(1-\varepsilon)}\right]^2\right\}. \tag{28}$$

The criterion for the admissible value of the toroid-type aberration is the ratio

$$\frac{35}{3}\left[\frac{h_t}{\sigma_1 a(1-\varepsilon)}\right]^2 \ll 1, \text{ or } h_t \ll \frac{\sqrt{3}\,\sigma_1 a(1-\varepsilon)}{6}. \tag{29}$$

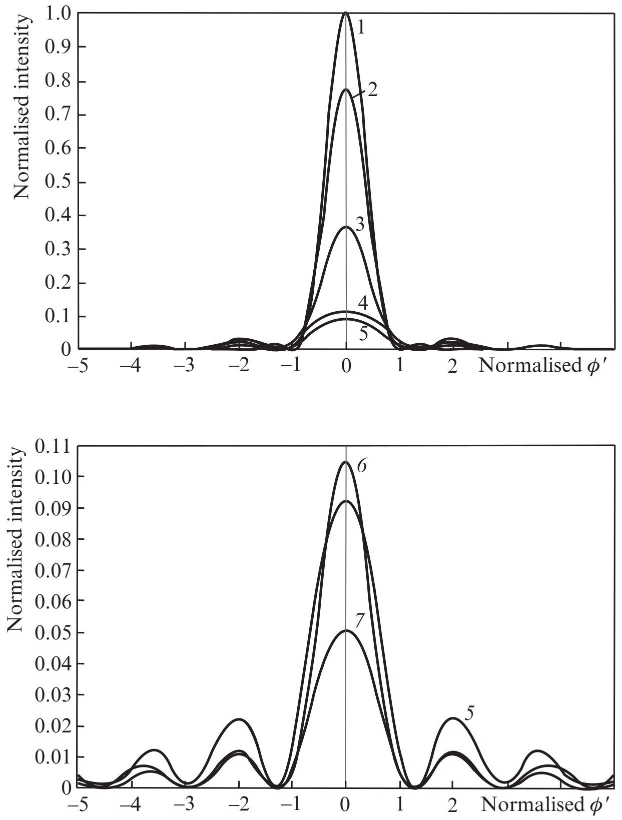

Figure 5. Far-field intensity distribution in the case of the toroidal wavefront deflection $h_{\rm t}$ = (*1*) 0, (*2*) $\lambda/4$, (*3*) $\lambda/2$, (*4*) $3\lambda/4$, (*5*) λ, (*6*) $3\lambda/2$ and (*7*) 2λ at $\varepsilon = 0$.

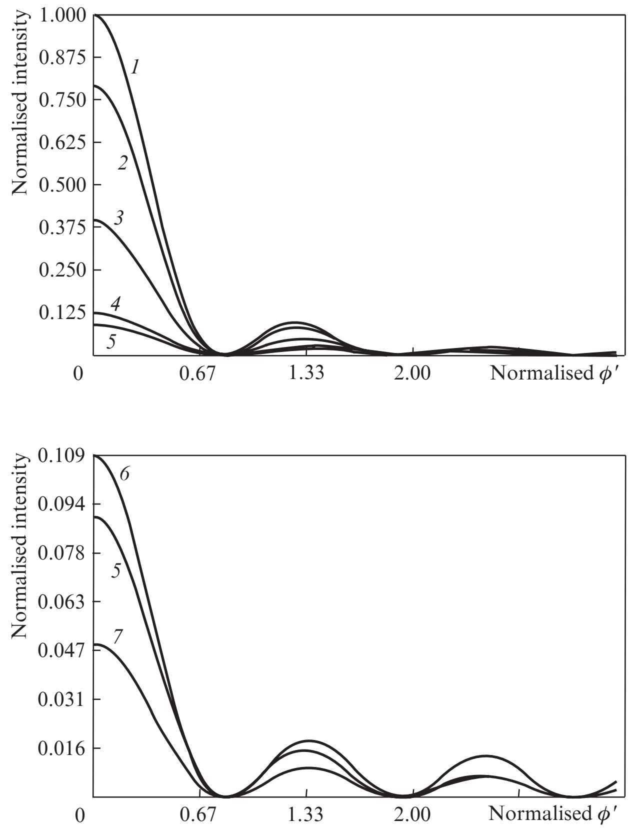

Figure 7. Far-field intensity distribution in the case of the toroidal wavefront deflection $h_{\rm t}$ = (*1*) 0, (*2*) $\lambda/4$, (*3*) $\lambda/2$, (*4*) $3\lambda/4$, (*5*) λ, (*6*) $3\lambda/2$ and (*7*) 2λ at $\varepsilon = 0.5$.

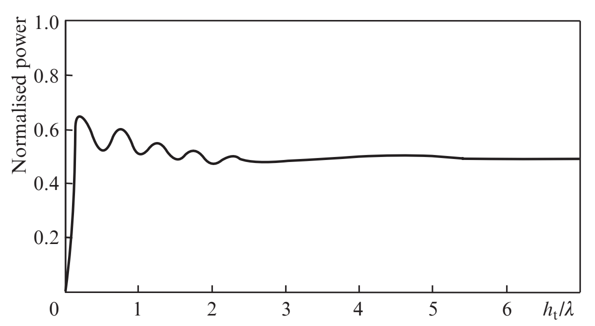

Figure 6. Dependence of the fraction of the total power P in a solid angle with a corresponding plane angle $\varphi_{\rm t} = 8h_{\rm t}/[a(1-\varepsilon)]$ at $\varepsilon = 0$ on the toroidal wavefront deflection $h_{\rm t}$.

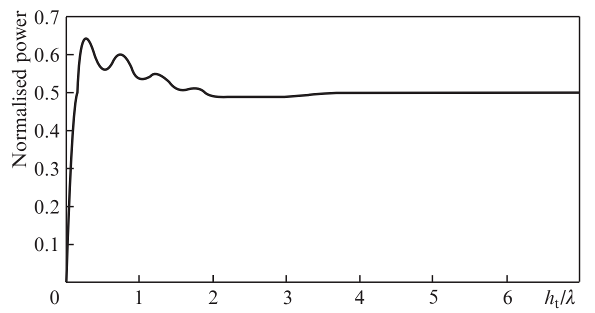

Figure 8. Dependence of the fraction of the total power P in a solid angle $\Omega_{0.5}$ with a corresponding plane angle $\varphi_{\rm t} = 8h_{\rm t}/[a(1-\varepsilon)]$ at $\varepsilon = 0.5$ on the toroidal wavefront deflection $h_{\rm t}$.

3.3. Criteria for the admissible value of astigmatism

Consider the effect of astigmatism on the angular divergence of the beam, bearing in mind that the cylindrical wavefront perturbation affects the diffraction pattern in both planes. For calculations we represent astigmatism as a cylindrical wavefront $z = -x^2/(2R)$ (Fig. 9). Figure 9 also shows the expressions for calculating the divergence of radiation due to the cylindrical wavefront.

The SD of the angular slopes σ_x and σ_y with respect to x and y axes, resulting from the initial SD and astigmatism, can be written at $h_{\rm a} \ll \sigma_1 a$ in the form

$$\sigma_x = \sqrt{\sigma_1^2 + \sigma_{\rm a}^2} = \sqrt{\sigma_1^2 + \left(\frac{3.2h_{\rm a}}{2.36a}\right)^2}$$

$$= \sqrt{\sigma_1^2 + \frac{11}{6}\left(\frac{h_{\rm a}}{a}\right)^2} = \sigma_1\left[1 + \frac{11}{12}\left(\frac{h_{\rm a}}{\sigma_1 a}\right)^2\right], \ \ \sigma_y = \sigma_1.$$

Thus, the geometric mean value of the variance due to astigmatism has the form

$$\sigma_{\rm a}^2 = \sigma_x\sigma_y = \sigma_1^2\left[1 + \frac{11}{12}\left(\frac{h_{\rm a}}{\sigma_1 a}\right)^2\right], \tag{30}$$

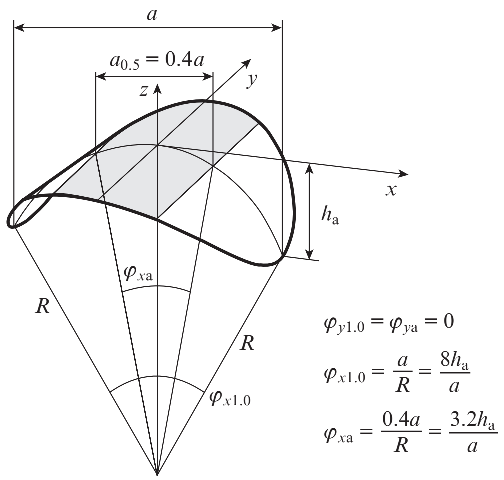

Figure 9. Wavefront aberration of astigmatism type in the form of a cylindrical surface $z = -x^2/(2R)$, bounded by a circular aperture of diameter a.

where $(11/12)(h_a/a)^2$ is the additive introduced by astigmatism. Taking into account the initial divergence φ_1 the angular divergence caused by astigmatism is determined from the expression

$$\varphi_2^2 = \varphi_1^2\left[1 + \frac{11}{12}\left(\frac{h_a}{\sigma_1 a}\right)^2\right]. \qquad (31)$$

The criterion for the admissible value of astigmatism is the ratio

$$\frac{11}{12}\left(\frac{h_a}{\sigma_1 a}\right)^2 \ll 1, \text{ or } h_a \ll \sigma_1 a. \qquad (32)$$

To estimate the divergence of the cylindrical wavefront at the half power level, use was made of the angular size φ_a of an equivalent circle corresponding an ellipse with angular sizes of the axes $\theta_x = 3.2h_a/a$ and $\theta_y = 1.22\lambda/a$. Then

$$\varphi_a = \sqrt{\theta_x\theta_y} = \sqrt{3.2\frac{h_a}{a}\cdot 1.22\frac{\lambda}{a}} \approx \frac{2}{a}\sqrt{\lambda h_a}. \qquad (33)$$

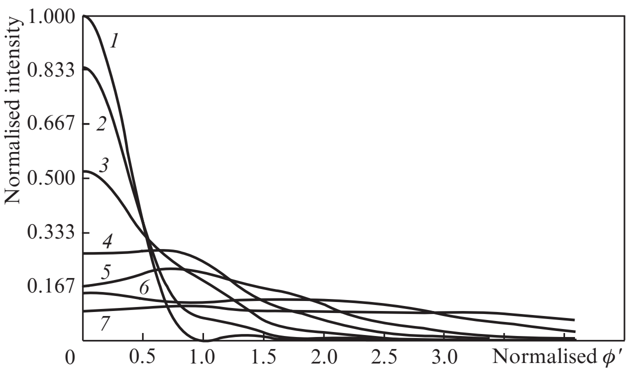

Figure 10. Far-field intensity distribution in the case of deflection h_a of a cylindrical wavefront along the axis x, equal to (*1*) 0, (*2*) $\lambda/4$, (*3*) $\lambda/2$, (*4*) $3\lambda/4$, (*5*) λ, (*6*) $3\lambda/2$ and (*7*) 2λ.

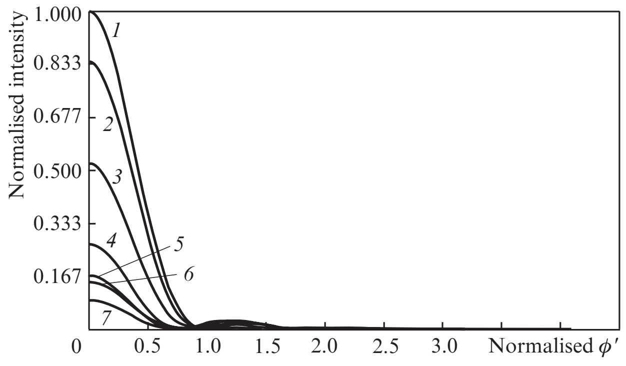

Figure 11. Far-field intensity distribution in the case of deflection h_a of a cylindrical wavefront along the axis y, equal to (*1*) 0, (*2*) $\lambda/4$, (*3*) $\lambda/2$, (*4*) $3\lambda/4$, (*5*) λ, (*6*) $3\lambda/2$ and (*7*) 2λ.

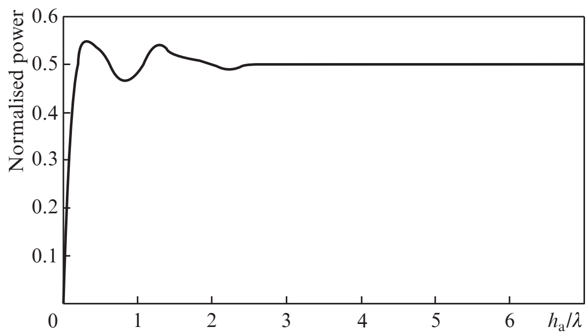

Figure 12. Dependence of the fraction of the total power P in a solid angle $\Omega_{0.5}$ with a plane angle $\varphi_a = (2/a)\sqrt{\lambda h_a}$ on the deflection h_a of a cylindrical wavefront.

The results of numerical simulation of the angular intensity distribution for a circular aperture are presented in Figs 10 and 11, and Fig. 12 shows the fraction of the total laser radiation power in a cone with a plane angle $\varphi_a = (2/a)\sqrt{\lambda h_a}$.

4. Examples of application of the criteria

Consider the examples of application of the criteria we have derived for the three above-mentioned types of aberrations, but at a constant angular divergence and aperture. We assume that at the output of the aperture with $a = 1$ m the laser beam has an angular divergence $\varphi_1 = 5\times10^{-5}$ rad ($\sigma_1 = (5/2.36)\times10^{-5}$ rad $= 2.12\times10^{-5}$ rad). These calculations corresponding to the three formulated criteria are summarised in Table 1. One can see that the most significant aberration results from a toroid, and the least significant – from astigmatism.

Table 1. Admissible values of the amplitudes of smooth aberrations for a beam with an angular divergence of 5×10^{-5} rad and a 1-m aperture.

Type of wavefront aberrations	Criteria	Admissible values of aberrations
Spherical defocusing	$h_s \ll \sqrt{3}\sigma_1 a/4$	$h_s \leqslant 1$ μm
Toroid	$h_t \ll 0.3\sigma_1 a(1-\varepsilon)$	$h_t \leqslant 0.3$ μm ($\varepsilon = 0.5$)
Astigmatism	$h_a \ll \sigma_1 a$	$h_a \leqslant 2$ μm

5. Conclusions

Thus, we have derived simple analytical expressions for the angular divergence of nondiffractive beams when smooth aberrations, such as spherical defocusing, astigmatism and toroid, are introduced.

The results of mathematical simulation of the angular power distribution of the diffraction-limited beams with the smooth aberrations introduced have shown the correctness of the derived analytical expressions for the angular divergence.

The proposed criteria are an important tool of analysis in designing laser systems. They can be used to evaluate the effectiveness of adaptive optical systems in suppressing smooth wavefront aberrations.

There is a practically important question: Are there physical and technical capabilities to measure such a small value of smooth wavefront distortion (~1 μm) on an aperture 1 m in diameter at least in the laboratory? We have technically realised such a capability by means of linear adaptive optics using the wavefront doubled-frequency spherical probing method [11]. Therefore, the criteria proposed will be of practical use.

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