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Determination of nanovibration amplitudes using frequency-modulated semiconductor laser autodyne

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Abstract. **The method for measuring nanovibration amplitudes using the autodyne signal of a semiconductor laser at several laser radiation wavelengths is described. The theoretical description of the frequency-modulated autodyne signal under harmonic vibrations of the reflector is presented and the relations for its spectral components are derived using the expansions into the Fourier and Bessel series. The results of numerical modelling based on the proposed method for measuring the reflector nanovibration amplitudes are presented that make use of the low-frequency spectrum of the autodyne signal from the frequency-modulated laser autodyne and the solution of the appropriate inverse problem. The experimental setup is described; the results of the measurements are presented for the nanovibration amplitudes and the autodyne signal spectra under the reflector nanovibrations.**

Keywords: *laser autodyne, external optical feedback, frequency modulation of laser radiation, autodyne signal spectrum, nanovibration amplitude measurements.*

1. Introduction

The possibility to measure the nanovibration amplitudes using laser interferometry methods was discussed in Refs [1–3]. However, as shown in Ref. [2], the implementation of such measurements requires calibration of the recording system before each measurement. Besides, the results of the measurements are largely dependent on the precision of the interference signal phase measurements.

To measure the nanovibration amplitude we propose to use the current modulation of the laser radiation that allows implementation of simultaneous measurements at several wavelengths of laser radiation. This offers a possibility to perform the nanovibration amplitude measurements without auxiliary excitation of mechanical oscillations with a micron-scale amplitude in the studied object, which is necessary for the calibration procedure.

A number of papers are known that demonstrate successful application of current modulation of a semiconductor laser with the aim of measuring distances, motion parameters, and vibrations of objects [4, 5–11]. It is of interest to produce measuring systems employing the autodyne detection effect in a semiconductor laser. In comparison with interference systems, the autodyne measuring systems, based on such effect, possess smaller dimensions, weight, and energy consumption [4, 12–18].

In particular, the authors of Ref. [6] demonstrate the possibility of determining the distance to a reflector under the linear variation in the laser injection current using the change in the interference signal modulation frequency with light source decoupling. In Ref. [5] for this aim the method is used that exploits the amplitude ratio of harmonics in the low-frequency spectrum of the frequency-modulated autodyne signal.

An important parameter for performing the measurements of the object motion characteristics by means of a laser autodyne system is the magnitude of the external optical feedback. In an autodyne system the regime, in which the autodyne signal is analogous to the interference one with decoupling from the radiation source, is possible with low feedback magnitudes. When the feedback is enhanced, the autodyne signal becomes distorted, in contrast to the case when the interference occurs in a system, separated from the signal source by a decoupling element. As shown in Refs [4, 19–21], the magnitude of the external optical feedback exerts essential influence on the autodyne signal shape of a semiconductor laser radiation source and, as a result, affects the precision of determining the parameters of reflector motion in autodyne systems.

The authors of Ref. [22] used the current modulation in a semiconductor laser autodyne to account for the influence of the external optical feedback magnitude on the measured value of the nanovibration amplitude. In this case the preliminary calibration of the autodyne signal was carried out by exciting auxiliary mechanical oscillations of the reflector.

In laser interferometry the frequency modulation has found application in creating high-precision instruments for measuring distances, displacements, velocities, and vibrations in the micron range of displacements [23–29]. Using the ratio of even and odd harmonic amplitudes in the low-frequency spectrum of the autodyne signal [29] allows determination of the distance between the laser autodyne and the reference reflector within the range up to 8 cm with micron resolution. The use of current modulation of a semiconductor laser for measuring vibrations is generally hampered by the complexity of the mathematical description of the inverse problem.

The goal of the present work is to substantiate theoretically and experimentally the possibility of measuring nanovibration amplitudes using the low-frequency spectrum of the autodyne signal from a frequency-modulated semiconductor laser autodyne.

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2. Autodyne signal formation in the case of optical multifrequency modulation of semiconductor laser radiation

A semiconductor laser with external optical feedback can be described within the frameworks of the Lang–Kobayashi model of a compound cavity [30]. In this model the dynamics of single-mode laser radiation can be described by the equations for the complex electric field with delayed argument and for the concentration of charge carriers [23, 24]. In Ref. [20] it is shown that the autodyne signal can be expressed in the regime of steady-state semiconductor laser oscillation, which makes it possible to pass from a set of differential equations to a nonlinear equation for the radiation power of the semiconductor laser autodyne basing on the use of low-signal analysis.

When the frequency-modulated radiation of a semiconductor laser is reflected from a reflector vibrating with a nanometre-scale amplitude, its power can be determined from the low-signal analysis of the differential equations, describing the complex electric field with delayed argument and the charge carrier concentration, and takes the form [20]

$$P(j(t)) = P_1(j(t)) + P_2(j(t))\cos(\omega(j(t))\tau(t)), \quad (1)$$

where $P_1(j(t))$ is the amplitude component of the power, independent of the distance from the external reflector; $P_2(j(t))$ is the amplitude component of the power, depending on the phase incursion $\omega\tau$ of the wave in the system with an external reflector; $\tau(t)$ is the external cavity roundtrip time for the laser radiation; and $\omega(j(t))$ is the semiconductor laser radiation frequency, depending on the pumping current density $j(t)$ and the feedback magnitude.

The frequency and the amplitude components of the radiation generated by a frequency-modulated semiconductor laser are determined by the expressions:

$$\omega(j(t)) = \omega_0 + \omega_a\sin(\Omega_1 t + \varepsilon_1), \quad (2)$$

$$P_1(j(t)) = I_1\sin(\Omega_1 t + \varepsilon_1),\ P_2(j(t)) = I_2\sin(\Omega_1 t + \varepsilon_1), \quad (3)$$

where ω_0 is the eigenfrequency of the semiconductor laser diode radiation; ω_a is the modulation amplitude of the semiconductor laser diode radiation frequency; Ω_1 is the modulation frequency of the laser diode supply current; ε_1 is the initial phase; and I_1, and I_2 are the current modulation amplitudes of the components P_1 and P_2.

When the reflector vibrates harmonically, the external cavity roundtrip time for the laser radiation is determined by the relation

$$\tau(t) = \tau_0 + \tau_a\sin(\Omega_2 t + \varepsilon_2), \quad (4)$$

where τ_0 is the external cavity roundtrip time for the laser radiation with the immobile reflector; $\tau_a = 2\xi/c$; ξ is the amplitude of the reflector oscillation; ε_2 is the initial phase; and Ω_2 is the reflector oscillation frequency.

3. The autodyne laser system spectrum under current modulation of laser diode and reflector vibration

In order to determine the reflector nanovibration amplitude it is necessary to derive an expression for the autodyne signal spectral components. Generally, for arbitrary relationship between the laser radiation modulation frequency and the reflector vibration frequency the analysis of expression (1) is difficult because of the complexity of mathematical processing of the signal, formed in such system. Practically it is possible to implement the case of synchronisation of the frequency and the initial phase of reflector vibrations with the initial phase and the frequency of laser radiation modulation. In this case the solution of the inverse problem of extracting the nanovibration characteristics from the autodyne signal is essentially simplified.

To derive an analytical expression, let us consider the case, when the modulation frequency of laser radiation is equal to the reflector vibration frequency ($\Omega_1 = \Omega_2 = \Omega$) and the initial phases are also equal ($\varepsilon_1 = \varepsilon_2 = \varepsilon$). As will be shown below, these conditions are easily implemented in the experimental setup. Then, with relations (2)–(4) taken into account, the power of the semiconductor laser radiation can be presented in the form

$$P(j(t)) = I_1\sin(\Omega t + \varepsilon) + I_2\sin(\Omega t + \varepsilon) \times\cos(\omega_0\tau_0 + (\omega_a\tau_0 + \omega_0\tau_a)\sin(\Omega t + \varepsilon)). \quad (5)$$

Having introduced the notations $\theta = \omega_0\tau_0$ and $\sigma = \omega_a\tau_0 + \omega_0\tau_a$, let us present the expression for $P(t)$ in the form of a series expansion over Bessel functions [5]

$$\begin{aligned} P(t) = {} & [I_1\sin\varepsilon + I_2\cos\theta J_0(\sigma)\sin\varepsilon - I_2\cos\theta J_2(\sigma)\sin\varepsilon]\cos(\Omega t) \\ & + [I_1\cos\varepsilon + I_2\cos\theta J_0(\sigma)\cos\varepsilon \\ & - I_2\cos\theta J_2(\sigma)\cos\varepsilon]\sin(\Omega t) + \sin\theta I_2\cos(2n\Omega t) \\ & + \sin\theta I_2\sum_{n=1}^{\infty}(J_{2n+1} - J_{2n-1})\sin(2n\varepsilon)\sin(2n\Omega t) \\ & + \cos\theta I_2\sum_{n=1}^{\infty}(J_{2n} - J_{2n+2})\sin((2n+1)\varepsilon)\cos((2n+1)\Omega t) \\ & + \cos\theta I_2\sum_{n=1}^{\infty}(J_{2n} - J_{2n+2})\cos((2n+1)\varepsilon)\sin((2n+1)\Omega t), \end{aligned} \quad (6)$$

where J_n is the nth order Bessel function of the first kind.

Using the Fourier expansion of $P(t)$

$$P(t) = 1/2a_0 + \sum_{n=1}^{\infty}[a_{2n}\cos(2n\Omega t) - b_{2n}\sin(2n\Omega t)] + \sum_{n=1}^{\infty}[a_{2n-1}\cos((2n-1)\Omega t) - b_{2n-1}\sin((2n-1)\Omega t)], \quad (7)$$

one can obtain the following expressions for the Fourier coefficients:

$$a_1 = I_1\sin\varepsilon + I_2\cos\theta J_0(\sigma)\sin\varepsilon - I_2\cos\theta J_2(\sigma)\sin\varepsilon, \quad (8)$$

$$b_1 = I_1\cos\varepsilon + I_2\cos\theta J_0(\sigma)\cos\varepsilon - I_2\cos\theta J_2(\sigma)\cos\varepsilon, \quad (9)$$

$$a_{2n} = \sin\theta I_2\sum_{n=1}^{\infty}(J_{2n-1} - J_{2n+1})\cos(2n\varepsilon), \quad (10)$$

$$b_{2n} = \sin\theta I_2\sum_{n=1}^{\infty}(J_{2n+1} - J_{2n-1})\sin(2n\varepsilon), \quad (11)$$

$$a_{2n+1} = \cos\theta I_2 \sum_{n=1}^{\infty} (J_{2n} - J_{2n+2}) \sin((2n+1)\varepsilon), \quad (12)$$

$$b_{2n+1} = \cos\theta I_2 \sum_{n=1}^{\infty} (J_{2n} - J_{2n+2}) \cos((2n+1)\varepsilon). \quad (13)$$

Introducing the coefficients C_1, C_2, C_3, C_4, equal in absolute value to the even and odd spectral components of the signal and defined as

$$C_1 = \sqrt{a_1^2 + b_1^2}, \quad (14)$$

$$C_{2n} = \sqrt{a_{2n}^2 + b_{2n}^2}, \quad (15)$$

$$C_{2n+1} = \sqrt{a_{2n+1}^2 + b_{2n+1}^2}, \quad (16)$$

it is possible to get their values in the form

$$C_1 = \cos\theta I_2 [J_0(\sigma) - J_2(\sigma)] + I_1, \quad (17)$$

$$C_2 = \sin\theta I_2 [J_1(\sigma) - J_3(\sigma)], \quad (18)$$

$$C_3 = \cos\theta I_2 [J_2(\sigma) - J_4(\sigma)], \quad (19)$$

$$C_4 = \sin\theta I_2 [J_3(\sigma) - J_5(\sigma)], \quad (20)$$

i.e., for $n = 1, 2, \ldots$

$$C_{2n} = \sin\theta I_2 [J_{2n-1}(\sigma) - J_{2n+1}(\sigma)], \quad (21)$$

$$C_{2n+1} = \cos\theta I_2 [J_{2n}(\sigma) - J_{2n+2}(\sigma)]. \quad (22)$$

Equations (17)–(22) determine the relation between the spectral components of the frequency-modulated autodyne signal and the Bessel functions of the first kind.

In order to find the nanovibration amplitude ξ that enters the parameter σ, we use the ratio of amplitudes of the second and the fourth harmonic (or the $2n$th and the $2n + 2$th) harmonics:

$$C_2/C_4 = [J_1(\sigma) - J_3(\sigma)]/[J_3(\sigma) - J_5(\sigma)]. \quad (23)$$

$$C_{2n}/C_{2n+2} = [J_{2n-1}(\sigma) - J_{2n+1}(\sigma)]/[J_{2n+1}(\sigma) - J_{2n+3}(\sigma)]. \quad (24)$$

To solve Eqns (23), (24) with respect to the unknown parameter σ, one should know the current modulation parameters of the laser autodyne, in particular, the frequency modulation amplitude of the semiconductor laser diode ω_a.

4. Numerical modelling and solving the inverse problem to determine the reflector nanovibration amplitude

The numerical modelling using the proposed method in application to the frequency-modulated semiconductor laser autodyne was carried out with the following parameters: the laser radiation wavelength $\lambda = 654$ nm, $\omega_a = 30\times10^8$ rad s^{-1}, $\Omega = 500$ Hz, $\xi = 55$ nm. Taking the parameters of the laser autodyne used in the experimental setup into account, let us choose $I_1/I_2 = 6$.

As an example, Fig. 1a illustrates the time dependence of the radiation power of the frequency-modulated laser diode under the reflector vibrations with the amplitude $\xi = 55$ nm, while Fig. 1b represents the corresponding spectrum. It is seen that in the spectrum of the autodyne signal four harmonics are observed, their amplitudes being $C_1 = 6.44$, $C_2 = 0.14$, $C_3 = 0.31$, $C_4 = 0.11$.

To determine the current modulation parameters of the laser autodyne from the autodyne signal spectrum, the value of the calibrating quantity $\sigma_m = \omega_a \tau_0$ is calculated using Eqns (23), (24).

Figure 1c presents the calibrating frequency-modulated autodyne signal for the motionless reflector, and Fig. 1d shows its spectrum, in which one can also observe four harmonics (their amplitudes are $C_1 = 5.74$, $C_2 = 0.27$, $C_3 = 0.18$, $C_4 = 0.03$). The solution of Eqn (23) or Eqn (24) allows the determination of the corresponding value of the calibrating quantity σ_m.

To determine the nanovibration amplitude it is necessary to calculate the argument σ of the first-kind Bessel function from Eqns (23), (24) using the amplitude ratio of the second

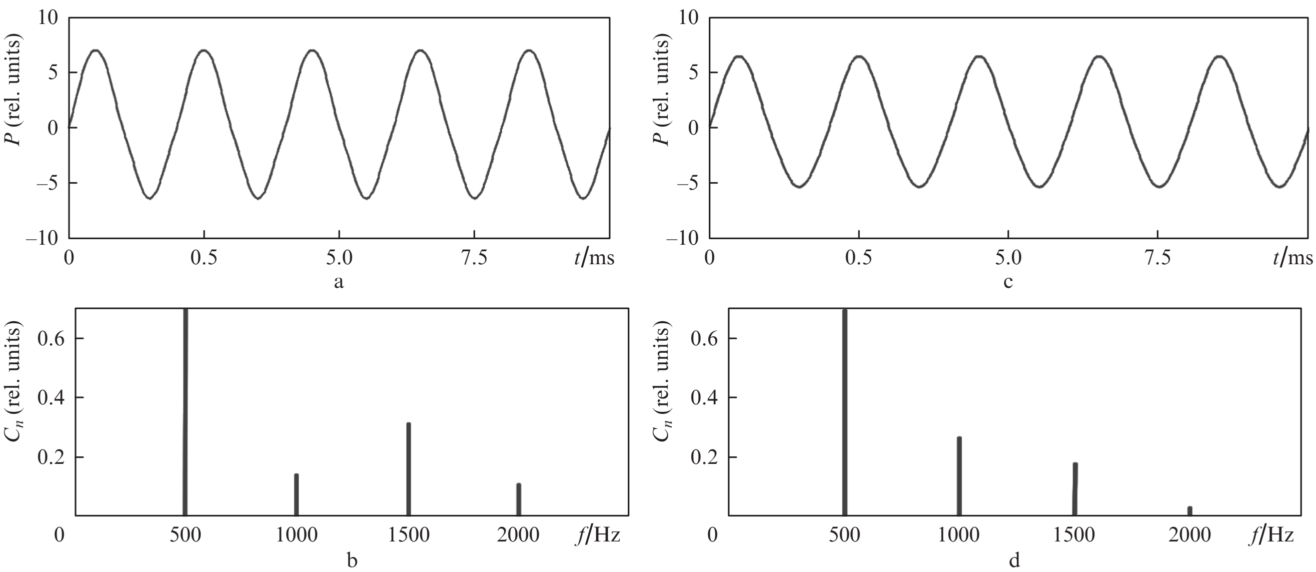

Figure 1. (a) Frequency-modulated autodyne signal under the reflector vibrations with $\xi = 55$ nm and (b) its spectrum, as well as (c) the calibrating frequency-modulated autodyne signal with immobile reflector and (d) its spectrum.

and the fourth spectral harmonics of the frequency-modulated autodyne signal under the reflector vibrations.

Keeping in mind that $\tau_a = 2\xi/c$ and $\sigma_m = \omega_a\tau_0$, we arrive at the relation for the nanovibration amplitude:

$$\xi = \frac{c}{2}\frac{\sigma - \sigma_m}{\omega_0}. \qquad (25)$$

To determine the error of mathematical modelling, a 10% random error was introduced into the initial signal using the rnd function of the Mathcad 14 mathematical package. The nanovibration amplitude, calculated using Eqn (25), appeared to agree with the initial value used when setting the problem, the discrepancy being 2.9%. With 5% artificial random error introduced, the discrepancy was reduced to 1.6%.

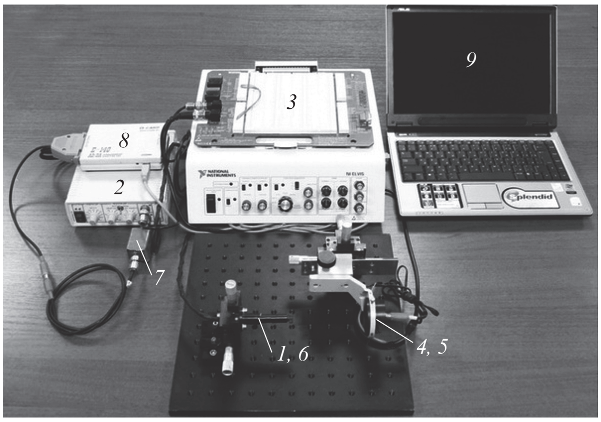

Figure 2. Appearance of the experimental setup: (*1*) semiconductor laser; (*2*) supply current control unit; (*3*) signal generator based on the NI ELVIS platform; (*4*) piezoceramic plate; (*5*) object; (*6*) photodetector; (*7*) ac filter; (*8*) ADC; (*9*) computer.

5. Measuring the amplitude of the object nanovibration by means of the frequency-modulated laser autodyne

The measurements were performed using the setup whose photograph is presented in Fig. 2.

The setup incorporated a frequency-modulated semiconductor laser autodyne using the laser diode RLD-650(5), based on quantum-well structures and generating a diffraction-limited single spatial radiation mode with the wavelength 654 nm (*1*). The radiation from the laser diode was incident onto the object (*5*), mounted on the piezoceramic plate (*4*). The laser radiation spot diameter on the object surface was 1 mm. The radiation wavelength modulation was executed at the frequency $\Omega = 500$ Hz by modulating the laser supply current using the signal generator (*3*), incorporated in NI ELVIS, the Educational Laboratory Virtual Instrumentation Suite (National Instruments, USA). The variation of the laser diode supply current was implemented by varying the supply voltage, applied to the semiconductor structure from the supply current control unit (*2*). The vibrations of the piezoceramic plate were excited by applying variable voltage with the amplitude 20 or 10 mV from the generator (*3*), which corresponds to the plate vibrations with the amplitude about 55 and 30 nm, respectively. The frequency and the initial phase of the piezoceramic plate vibrations were synchronised with the frequency and the initial phase of the laser radiation wavelength by means of the software of NI ELVIS (*3*), in which the parameters of the generated signal were specified.

We found experimentally that the efficient modulation of the laser diode supply current occurs for the amplitude of the modulating signal 20 mV, providing the presence of the 2nd and the 4th spectral components in the spectrum of the autodyne signal. Note, that the laser diode was fed from the current source. The reflected radiation was directed into the laser cavity; the change in the laser output power was recorded using the photodetector (*6*). The detected and amplified signal from the photodetector passed the ac filter (*7*) and arrived at the input of the analogue-to-digital (ADC) converter (*8*) (sampling frequency 100 kHz), connected to the computer (*9*).

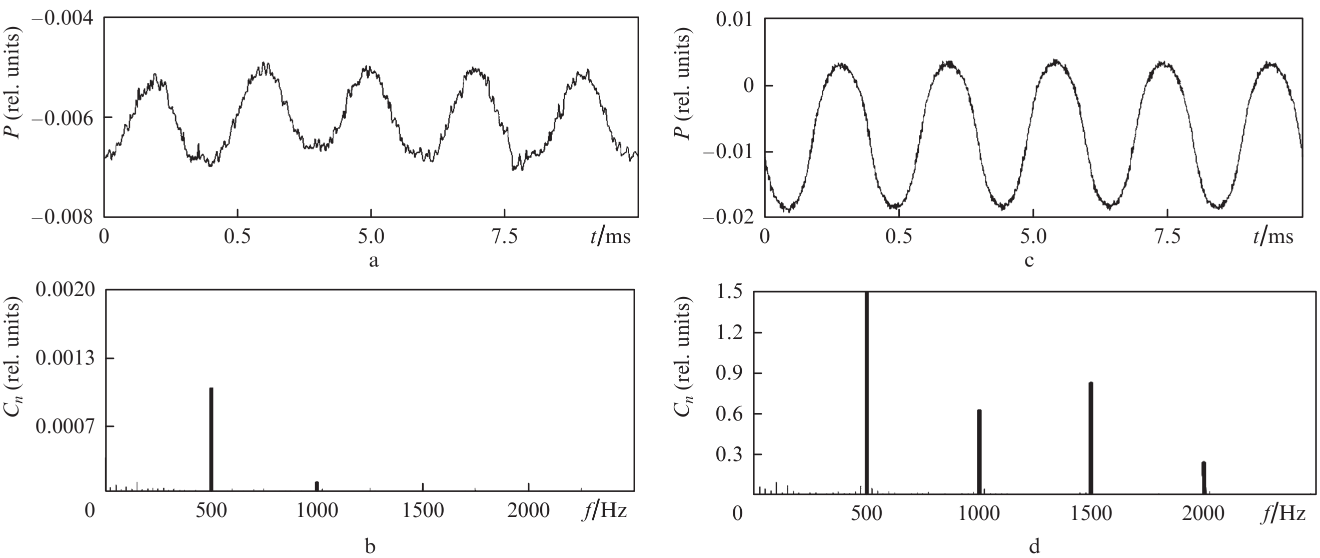

Figure 3. (a) Experimental autodyne signal under the reflector vibrations in the absence of current modulation and (b) its spectrum, as well as (c) the experimental frequency-modulated autodyne signal under the reflector vibrations and (d) its spectrum.

In the course of measurements the vibrations with the frequency $\Omega = 500$ Hz and the amplitudes $\xi = 55$ nm, measured using the independent technique, were excited in the piezoceramic plate by the generator.

Figures 3a, b represent the autodyne signal in the absence of current modulation and its spectrum that allows a precise measurement of the harmonic amplitudes only, which is insufficient for measuring the vibration amplitude without the autodyne signal calibration.

To calibrate the autodyne signal, the modulating voltage 20 mV with the frequency 500 Hz was applied to the semiconductor laser diode. The recorded frequency-modulated autodyne signal and its spectrum is presented in Figs 3c, d. The value of σ_m, calculated from several measurements with the immovable reflector, amounted to 2.05.

Using the determined values of σ and σ_m, the reflector vibration amplitude was found from the relation (25). Its experimentally measured value amounted to 55×10^{-9} m with the mean square deviation $\Delta = \pm4.5\times10^{-9}$ m. When the voltage applied to the piezoceramic was reduced by 2 times, the vibration amplitudes was measured to be 30×10^{-9} m ($\Delta = \pm5.2\times10^{-9}$ m).

Thus, in the present paper it is shown that for the frequency-modulated laser radiation it is possible to solve the inverse problem of reconstructing the nanovibration amplitude from the set of the signal spectral components, obtained by decomposing the autodyne signal into the Bessel and Fourier series. Using the computer modelling and the practically implemented measuring scheme, it was shown that the piezoceramic plate vibration amplitude can be determined with the error smaller than 20%.

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