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Fourier space method for calculating the propagation of laser radiation in biaxial crystals taking into account the angle between the eigenpolarisations

S.G. Grechin, P.P. Nikolaev, A.G. Okhrimchuk

Abstract. **We have proposed a technique for calculating the propagation of laser radiation in biaxial optical crystals in arbitrary directions. The technique is based on the use of the Fourier space method and takes into account both diffraction and angle beween the eigenpolarisations of the spatial spectrum components, phase shift differences for them with account for all orders of the spatial dispersion and also the features of the boundary conditions at the input and output facets. Using internal conical refraction as an example, we have compared the results of calculations with experimental data.**

Keywords: *biaxial crystals, propagation of laser radiation, Fourier space method.*

1. Introduction

Optical crystals are widely used in a broad range of applications [1]. Initially, uniaxial crystals were generally employed. Born and Wolf [2] derived for such crystals the equations describing the propagation of radiation. In recent decades, a large number of biaxial crystals were synthesised, which considerably extend the functionality of different applications [3]. In solving many applied tasks, use is made of the crystal with the cuts in the principal planes of the crystal-optic coordinate system. In this case, their properties are described with high accuracy by the equations for uniaxial crystals. Nevertheless, in some cases it is needed to use the most arbitrary cuts. This, in particular, involves the realisation of wavelength- and temperature-noncritical interactions, i.e., birefringence and nonlinear optical frequency conversion [4–6]. In such cases, it is necessary to take into account all the processes proceeding in the crystal. For biaxial crystals, they include the angle between the eigenpolarisations with respect to the crystal coordinate system when the propagation direction of radiation changes; misorientation for different components of the spatial spectrum of the radiation beams manifests itself even upon interaction of the multiple beams with different wavelengths. This work is devoted to the account for such processes. A method is proposed for calculating the propagation of laser radiation in an arbitrary direction in biaxial crystals and the boundary conditions at the input and output facets of the crystal. The theoretical and experiemtnal results are compared for the case of conical refraction.

S.G. Grechin, P.P. Nikolaev N.E. Bauman Moscow State Technical University, 2-ya Baumanskaya ul. 5, 105005 Moscow, Russia; e-mail: paneeque@gmail.com;
A.G. Okhrimchuk Fiber Optics Research Center, Russian Academy of Sciences, ul. Vavilova 38, 119333 Moscow, Russia

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2. Optical properties of biaxial optical crystals

We present the basic expressions determining the optical properties of biaxial crystals and the features of variation in the parameters of propagating radiation. Expressions for the refractive indices of s- and f-waves have the form [2]:

$$n_{\mathrm{f}} = \sqrt{\frac{2}{B+\sqrt{B^2-4C}}},$$

$$n_{\mathrm{s}} = \sqrt{\frac{2}{B-\sqrt{B^2-4C}}},$$

$$B = s_x^2\left(\frac{1}{n_y^2}+\frac{1}{n_z^2}\right)+s_y^2\left(\frac{1}{n_x^2}+\frac{1}{n_z^2}\right)+s_z^2\left(\frac{1}{n_x^2}+\frac{1}{n_y^2}\right), \quad (1)$$

$$C = \frac{s_x^2}{n_y^2 n_z^2}+\frac{s_y^2}{n_x^2 n_z^2}+\frac{s_z^2}{n_x^2 n_y^2},$$

where n_x, n_y, n_z are the principal values of the refractive indices of the medium;

$$s_x = \sin\theta\cos\varphi, \quad s_y = \sin\theta\sin\varphi, \quad s_z = \cos\theta \quad (2)$$

are the direction cosines of the wave vector; and θ and φ are the angles that define the direction of the wave vector with respect to the principal axes of the crystal (Fig. 1).

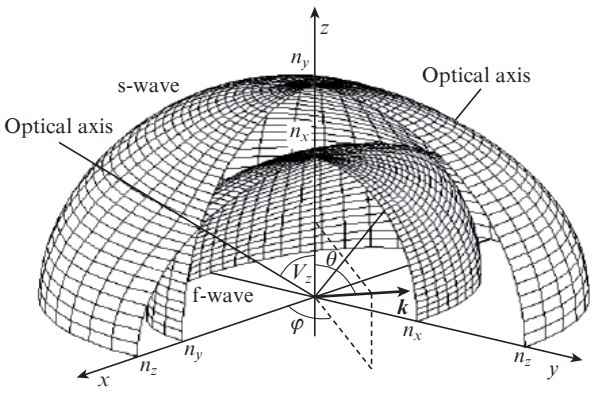

Figure 1. Normal surfaces for s- and f-waves in a biaxial crystal with a ratio of the refractive indices $n_x < n_y < n_z$.

The distributions of the characteristic surfaces for both waves are shown in Fig. 1. One of the significant differences of biaxial crystals from uniaxial ones is the angle dependence of the eigenpolarisations for both components on the direction of the wave vector $\boldsymbol{k}$. In uniaxial crystals the angles between the eigenpolarisations remain unchanged throughout the entire range of angles of the wave vector in the crystal coordinate system. For the o-components it is parallel to the xy plane, and for e-component it is perpendicular to it and lies in the cutting plane passing through the z axis and the wave vector $\boldsymbol{k}$. The angle between the eigenpolarisations for different directions of radiation propagation in the case of biaxial crystals is shown in Fig. 2. The expression for the rotation angle (angle δ in Fig. 2) has the form:

$$\tan 2\delta = \frac{\cos\theta \sin 2\varphi}{\sin^2\theta/\tan^2 V_z + \sin^2\varphi + \cos^2\theta\cos^2\varphi}, \qquad (3)$$

where V_z is the angle from the z axis to the optical axis, which is defined by the expression

$$\cos V_z = \frac{n_x}{n_y}\sqrt{\frac{n_z^2 - n_y^2}{n_z^2 - n_x^2}}. \qquad (4)$$

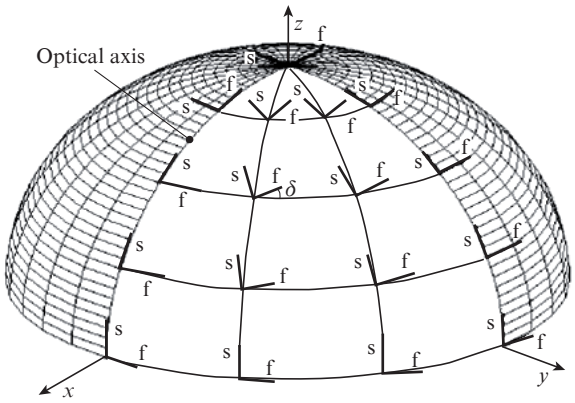

Figure 2. Orientations of eigenpolarisations for different sets of angles θ and φ in biaxial crystals.

Most strongly the angle δ changes in the vicinity of the optical axis (Fig. 3).

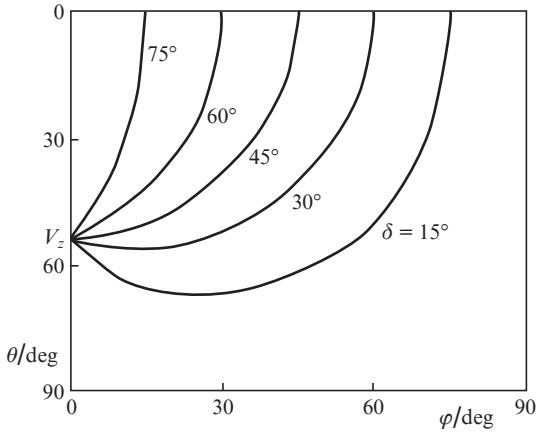

Figure 3. Dependences of the angle of rotation of eigenpolarisations on the direction of radiation propagation.

Wavelength-noncritical processes are realised in the directions located in the vicinity of the optical axis of the crystal [6]. As soon as the angle V_z has a dispersion, the angle between the eigenpolarisations for a given direction, corresponding to the process being realised, will also have a dispersion. Figure 4 shows the dispersion dependences of the angle V_z for some biaxial crystals. They imply that for different interacting waves the angle between the eigenpolarisations, for example upon parametric generation of radiation, may be several tens of degrees.

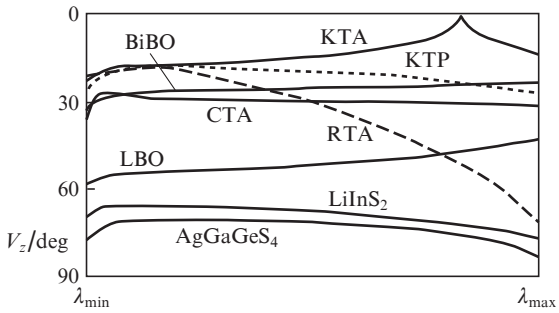

Figure 4. Dispersion of the angle to the optical axis in the biaxial crystals.

Obviously, in the general case, for radiation beams with finite divergence different spatial components (components of the spatial spectrum*) will have different orientations of eigenpolarisations. In the case of a focused laser beam, the angle of the eigenpolarisations between the edge components of the spatial spectrum can reach tens of angular minutes, and in the vicinity of the optical axis of the crystal – units of degrees.

Thus, in solving the problem of radiation propagation in a biaxial crystal in the most general case it is necessary to take into account the changes in the angle between the eigenpolarisations.

3. Methods for solving the problem of radiation propagation in a biaxial crystal

Calculation of radiation propagation in crystals requires the use of numerical methods. Already in the late 1960s conservative grid methods were developed (see, for example, [10, 11]), which can be effectively used for solving the problems of radiation propagation in uniaxial crystals, and in some special cases – in biaxial crystals. However, these methods have limitations caused by the instability, which, in particular, is manifested in solving the problems of frequency conversion when it is needed to take into account not only the group velocity mismatch, but also the dispersion spreading of the pulses of the second, third and higher orders.

The overall analysis shows that the problem of propagation and interaction should be solved in the representation, in which manifestation of limiting and related mechanisms is most simply addressed. The combined action of both diffraction and the angle between the eigenpolarisations is most simply described in the spectral representation of propagating

* Here and below by the spatial dispersion of the medium is meant the nonlocality of the response to the dielectric tensor [7–9]. In this case, it determines the dependence of the refractive index on the orientation angle of the vector $\boldsymbol{k}$.

(interacting) radiation beams – spatial spectrum components. Fourier space methods for solving the problems of nonlinear frequency conversion of radiation beams in uniaxial crystals were considered in [11–29], and of femtosecond pulses – in [30]. In both cases, use was made not of an approximate description of the dispersion of the refractive indices but of a more accurate description of the dispersion, described by the Sellmeier equation, which yielded good results (see, for example, [24, 30–32]).

One of the effective methods for solving the problem of radiation propagation is the Fourier space method, which is based on the solutions in the spectral space and allows the most accurate account of all the main mechanisms manifested in biaxial crystals. This method, the mathematical representation of which was first presented in [11–14], is based on solving a system of parabolic differential equations for slowly varying amplitudes of the spectral components. A further development of this method is the split-step method, which consists in a separate and successive solution of problems of linear propagation of radiation and nonlinear interaction with a medium. For uniaxial crystals, this method was comprehensively considered for the first time in [19, 20].

In the analysis of linear processes the most applicable is the first method, whereas in the analysis of nonlinear interaction of radiation with a medium (nonlinear optical frequency conversion, stimulated scattering, etc.) the second method is used, which is due to a higher operation speed, since the convolution of the functions is computed slower than the Fourier transform is performed by modern methods. The authors of Refs [25, 26] compared these two methods in solving the same problem. Also note that when it is necessary to take into account the boundary conditions on the sides of crystal elements (e.g., in the case of waveguide propagation of radiation), the second method is only applicable.

Here we restrict our consideration to propagation of radiation in a linear medium, since this problem can be easily generalised in the future to other practical problems.

4. Propagation of radiation in a biaxial crystal with an arbitrary cut

We will consider the process of propagation in a biaxial crystal with an arbitrary cut for a linear nonmagnetic and nongyrotropic medium. In this case, the problem can be reduced to a scalar diffraction problem for the propagation of the radiation field between two layers – the input and output facets of a biaxial crystal [33]. In combining it, for example, with the problem of nonlinear interaction, only the number of layers will change, whereas all other approaches to the solution will remain the same.

Radiation at the entrance to a biaxial medium can always be represented as a superposition of linearly polarised monochromatic waves, whose field strength is written in the complex form:

$$E(x,y) = \frac{1}{2}\{A_\mathrm{s}(x,y)\exp[\mathrm{j}\varphi_\mathrm{s}(x,y)] + \mathrm{c.c.}\}$$

$$+\frac{1}{2}\{A_\mathrm{f}(x,y)\exp[\mathrm{j}\varphi_\mathrm{f}(x,y)] + \mathrm{c.c.}\}, \qquad (5)$$

where $A_i(x,y)$ is the amplitude distribution of the radiation field and $\varphi_i(x,y)$ is the phase distribution of the radiation field.

For definiteness we assume below that a linearly polarised wave is incident on the biaxial crystal. The results obtained can be easily generalised to the case of depolarised radiation with both a uniform and a nonuniform degree of depolarisation across the radiation beam cross section.

Mathematically, propagation of radiation with the diffraction between the two layers in an arbitrary medium taken into account is reduced to the calculation of the diffraction integral (Fig. 5):

$$E_0(x_2,y_2) = -\frac{1}{\lambda z}\iint E_i(x_1,y_1)$$

$$\times \exp[\mathrm{j}k\sqrt{z^2 + (x_2 - x_1)^2 + (x_2 - x_1)^2}]\mathrm{d}x_1\mathrm{d}y_1, \qquad (6)$$

where x_i, y_i are the transverse coordinates; $E_i(x_1,y_1)$ is the initial distribution of the radiation field; $E_0(x_2,y_2)$ is the resulting distribution of the radiation field; and z is the length of the medium.

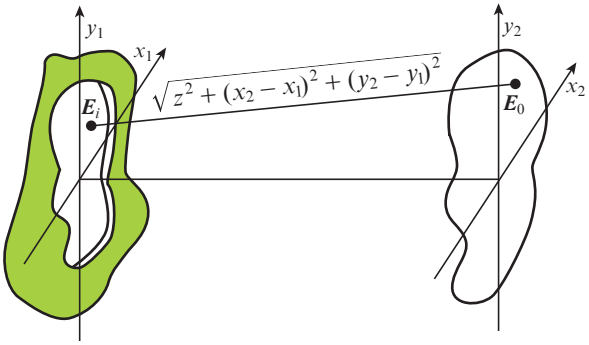

Figure 5. Geometry of the diffraction problem.

The wave number k in (6) contains all the information about the spatial dispersion of the medium and the angle between the eigenpolarisations of radiation. Since expression (6) is a convolution integral, it will be solved by the Fourier space method. The general structure of such a solution has the well-known form [34]:

$$F_i(k_x,k_y) = \iint E_i(x_1,y_2)\exp[\mathrm{j}(k_x x + k_y y)]\mathrm{d}x_1\mathrm{d}y_1, \qquad (7)$$

$$F_{0\mathrm{s,0f}}(k_x,k_y) = F_i(k_x,k_y)\exp\left[\mathrm{j}z\sqrt{k^2_\mathrm{s,f}(k_x,k_y) - k_x^2 - k_y^2}\right], \qquad (8)$$

$$E_0(x_2,y_2) = \iint F_0(k_x,k_y)\exp[-\mathrm{j}(k_x x + k_y y)]\mathrm{d}k_x\mathrm{d}k_y, \qquad (9)$$

where k_x, k_y are the components of the spatial spectrum; $F_i(k_x,k_y)$ is the spatial spectrum of initial radiation; $F_0(k_x,k_y)$ is the spatial spectrum of the resulting radiation; $k_\mathrm{s,f}(k_x,k_y)$ is the wave number $[k_\mathrm{s,f} = 2\pi n_\mathrm{s,f}(k_x,k_y)/\lambda]$, dependent on k_x and k_y; and $n_\mathrm{s,f}(k_x,k_y)$ is the refractive index for s- and f-components of radiation.

In the case under study, the initial distribution of the field $E_i(x_1,y_1)$ is expanded in a series of plane monochromatic waves of the spatial spectrum $F_i(k_x,k_y)$, taking into account the fact that each of these spatial component has its own angle of eigenpolarisations (s- and f-components). Therefore, even for linearly polarised input radiation in the general case a couple of waves (spatial components) with orthogonal

polarisations [$F_{is}(k_x,k_y)$ and $F_{if}(k_x,k_y)$] propagate, each of which has its own phase shift. In this case, all the orders of spatial dispersion are taken into account, since use is made of expression (1), rather than its approximation. When calculating $E_0(x,y)$, we should take into account all peculiarities of the expansion in the eigenpolarisations and the phase shift of each component.

Apart from the above-mentioned features of the problem in question, we should also take into account the differences in the boundary conditions on the input and output facets of the crystal, which are caused by the difference in the transmittances of the components with polarisation orientation parallel and perpendicular to the plane of incidence. Because for each component of the spatial spectrum $k_{x,y}$ the decomposition into σ- or π-polarised components is different, the transmittances for them will also differ from each other.

By using the approach presented above, the calculation of radiation propagation in an anisotropic medium consists in sequential solving the following sub-problems:

(i) Calculation of refractive indices $n_f(k_x,k_y)$ and $n_s(k_x,k_y)$ for each component of the spatial spectrum $k_{x,y}$ and for s- or f-components of eigenpolarisations of radiation.

(ii) Calculation of the rotation angles $\delta(k_x,k_y)$ of eigenpolarisations with respect to the cutting plane (the plane passing through the z axis and the vector $\boldsymbol{k}$, see Fig. 1) for each component $k_{x,y}$ of the spatial spectrum.

(iii) Calculation of transmittances of radiation on the input facet and its expansion in the eigenpolarisations of the anisotropic crystal for each component $k_{x,y}$ of the spatial spectrum.

(iv) Solution of the diffraction problem of propagation of radiation in an anisotropic medium between the input and output facets on the basis of equations (7)–(9).

(v) Calculation of transmittances for s- or f-polarisations of the radiation at the output facet and expansion in two orthogonal polarisations for each component $k_{x,y}$ of the spatial spectrum.

5. Calculation of the refractive indices and the rotation angle of eigenpolarisations

Calculation of the refractive indices for each component $k_{x,y}$ of the spatial spectrum is reduced to solving the inverse problem, i.e., finding the values of the refractive indices $n_{s,f}(k_x,k_y)$ and angles φ and θ by the specified value of k_x and k_y. It should be noted that in an anisotropic medium the spatial spectrum is equidistant in the expansion parameters k_x and k_y, but not equidistant in the angular deviations of the wave vectors of the corresponding spatial components.

To calculate the refractive indices $n_{s,f}(k_x,k_y)$, it is necessary to know the direction cosines of the wave vector of the spectral components in the crystal coordinate system. However, from the initial data for the radiation beam we only know the distribution of the spatial spectrum in the coordinate system tied to the direction of its propagation. To find the solution, we must recalculate the known distribution of the spatial spectrum from one coordinate system in the direction cosines of the wave vector in the crystal coordinate system.

Figure 6 shows the crystal coordinate system xyz and the coordinate system tied to the propagation direction of the radiation beam, $x'y'z'$ (radiation propagates along the axis z′ – vector $\boldsymbol{k}_0$). The axis x' lies in the plane zz', the axis y' – in the orthogonal plane. Then each component of the spatial spectrum of radiation with the specified values of $k_{x'}$ and $k_{y'}$ in the coordinate system $x'y'z'$ will have projections on the axis:

$$\begin{pmatrix} k_{x'} \\ k_{y'} \\ k_{z'} \end{pmatrix}, \quad k_{z'} = \sqrt{|k|^2 - k_{x'}^2 - k_{y'}^2}, \tag{10}$$

where $|k| = |k(k_{x'},k_{y'}|$ is the modulus of the wavenumber for the component of the spatial spectrum.

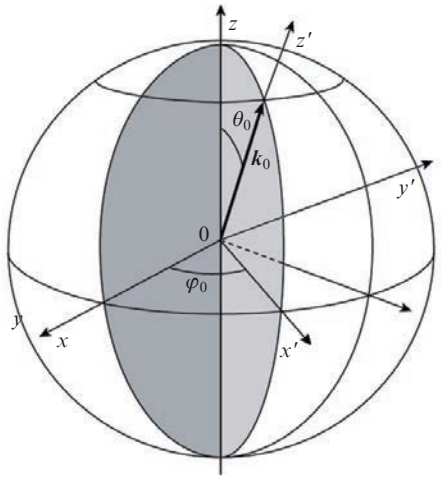

Figure 6. Transformation of the coordinate system of the radiation beam to the crystal coordinate system.

The rotation matrix used to recalculate the projections of the wavenumbers in the crystal coordinate system has the form

$$M = \begin{pmatrix} \cos\theta\cos\varphi & -\sin\varphi & \sin\theta\cos\varphi \\ \cos\theta\sin\varphi & \cos\varphi & \sin\theta\sin\varphi \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}. \tag{11}$$

Then, using (10) and (11) the direction cosines of the spectral components for (1) are calculated as follows:

$$\begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} = M \begin{pmatrix} k_{x'}/|k| \\ k_{y'}/|k| \\ k_{z'}/|k| \end{pmatrix}, \tag{12}$$

where $|k_i(k_x,k_y)| = 2\pi n_i(k_x,k_y)/\lambda$ is the modulus of the wavenumber of each component of the spatial spectrum.

The values of the direction cosines (12) are substituted into (1), and the systems of equations, obtained for each component of the spatial spectrum, are solved by iterative numerical methods (e.g., modified Newton method) in the range of refractive indices $n_x \div n_z$. The minimised function, such as the f-polarised component, has the form:

$$F = n_f - \sqrt{\frac{2}{B + \sqrt{B^2 - 4C}}} \leqslant \text{eps}, \tag{13}$$

where eps is the error in determining the refractive index. Likewise, for each spectral component the refractive indices for the s-polarised component were determined.

Figure 7 shows the example of the calculated distributions of the refractive index of s- and f-polarised components in the

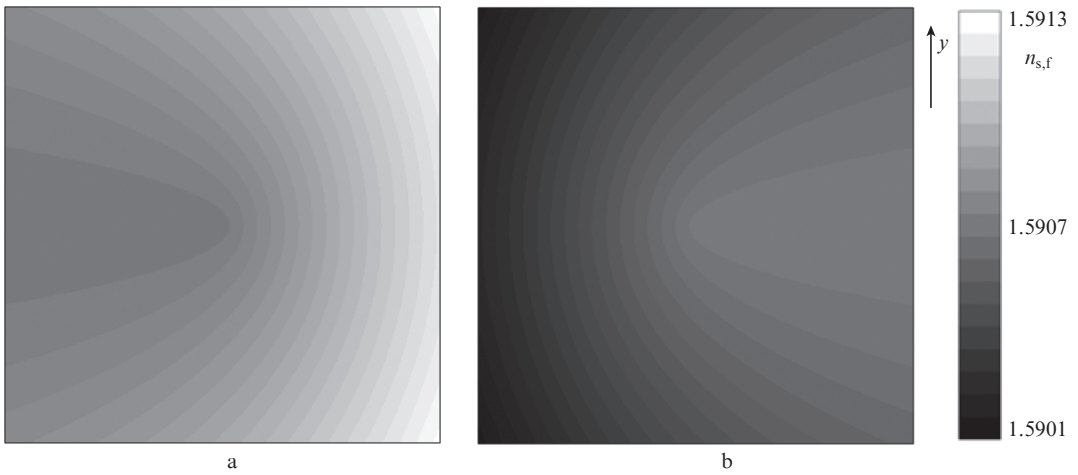

Figure 7. Refractive index distribution for (a) s- and (b) f-polarised components in the direction of the optical axis of the LBO crystal (the plane *xz* is directed horizontally).

direction of the optical axis of a biaxial LBO crystal for the spatial spectrum corresponding to the range of angular deviations ±0.3° in both coordinates.

To each component of the spatial spectrum $k_{x,y}$ there corresponds its own angle of eigenpolarisations (3). Figure 8 presents the example of the calculated distribution of the rotation angle of eigenpolarisations in the direction of the optical axis of a biaxial LBO crystal for spatial spectrum corresponding to the range of angular deviations ±0.3° in both coordinates.

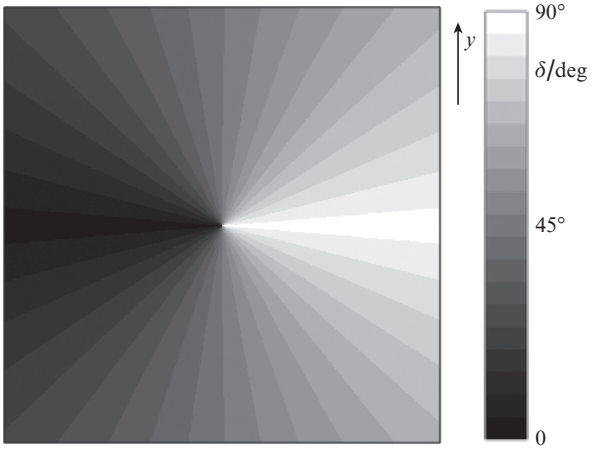

Figure 8. Distribution of the angle of rotation of eigenpolarisations in the direction of the optical axis of the LBO crystal (the plane *xz* is directed horizontally).

6. Boundary conditions on the input and output facets of a biaxial crystal

The reflection coefficient of radiation falling on the interface between the media depends on the angle of incidence and polarisation of radiation, is described by the Fresnel formulas and differs for orthogonal σ and π-polarised components of the incident wave. At the same time, in a biaxial medium there are two intrinsic s- and f-polarised components of the wave with different refractive indices. Then, to describe the wave propagation through the interface between the media, four transmittances are required for the field strength, relating the four defined polarisations. These are the coefficients [2]

$$T_i^{\sigma} = 1 - \frac{\sin(\alpha - \beta_i)}{\sin(\alpha + \beta_i)},$$
$$T_i^{\pi} = 1 - \frac{\tan(\alpha - \beta_i)}{\tan(\alpha + \beta_i)}, \tag{14}$$

where the subscript *i* corresponds to the s- or f-polarised component; α is the angle of incidence of the wave; and β_i is the angle of refraction of the wave of the s- or f-polarised component.

For each component $k_{x,y}$ of the spatial spectrum, the angle of the wave incidence on the interface between the media for the input and output facets is calculated in the same way:

$$\alpha = \arcsin \frac{\sqrt{k_x^2 + k_y^2}}{|k(k_x, k_y)|}. \tag{15}$$

Expressions for the refraction angles of the wave for s- and f-polarised components at the input and output facets are different:

$$\beta_{s,f}^{in} = \arcsin \frac{\sqrt{k_x^2 + k_y^2}}{|k(k_x, k_y)|\, n_{s,f}(k_x, k_y)}, \tag{16}$$

$$\beta_{s,f}^{out} = \arcsin\left(\frac{\sqrt{k_x^2 + k_y^2}}{|k(k_x, k_y)|}\right) n_{s,f}(k_x, k_y). \tag{17}$$

Consider how the component of the spatial spectrum of linearly polarised light is expanded in the eigenpolarisations of a biaxial crystal on its front facet (Fig. 9).

Let us consider the most general case of a linearly polarised wave E_0 incident on the interface between the media. In this case there are two projections of the wave vector $\boldsymbol{k}$ on the interface between the media – k_x and k_y, orientations and values of which determine the mutual orientation of the plane of incidence and eigenpolarisations of radiation in the crystal. This, in turn, will determine the refractive indices for each of the radiation components by the eigenpolarisations propa-

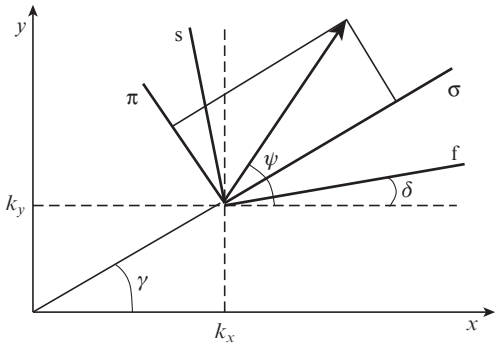

Figure 9. Expansion of the spatial component of radiation in the eigenpolarisations of a biaxial crystal at its input facet; σ and π are the orientations of the polarisation planes of radiation incident on the crystal, perpendicular and parallel planes of the interface between the media.

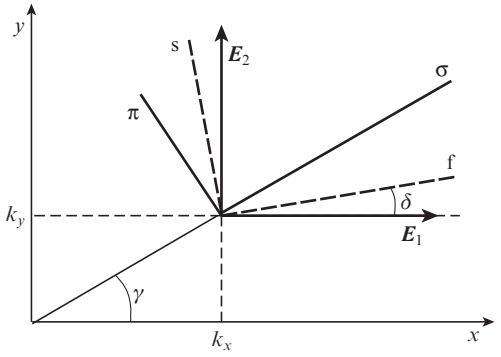

Figure 10. Propagation of radiation through the output facet of a biaxial crystal.

gating in the crystal. In the general case, the plane wave polarisation is rotated with respect to the x axis by the angle ψ. Each component of the spatial spectrum of this wave will have the same polarisation. Each mth component of the spatial spectrum in the working coordinate system is rotated by an angle $\gamma = \text{arctag}(k_y/k_x)$. The choice of the coordinate system (x and y) is due to the necessity of matching the orientations of eigenpolarisations for the zero component of the spatial spectrum with the orientation of these axes. Then, the expansion of the spatial spectrum components in σ and π-polarised components of the field strength will have the form:

$$E_\sigma = E_0\cos(\gamma - \psi), \qquad E_\pi = E_0\sin(\gamma - \psi). \tag{18}$$

Since each component of the spatial spectrum has its own rotation angle of eigenpolarisations δ, by using expressions (11) and (18) the expressions for the s- and f-polarised components of the field strength in the biaxial medium are as follows:

$$E_s = E_\sigma T_s^\sigma \sin(\gamma - \delta) - E_\pi T_s^\pi \cos(\gamma - \delta),$$
$$E_f = E_\sigma T_f^\sigma \cos(\gamma - \delta) + E_\pi T_f^\pi \sin(\gamma - \delta). \tag{19}$$

The components of the spatial radiation spectrum propagate through the output facet as follows (Fig. 10). At the output of the medium, each component of the spatial spectrum will generally have an elliptical polarisation. For definiteness, beyond the output facet we represent each such component of the spatial spectrum as a sum of two orthogonal linearly polarised components E_1 and E_2, directed along the axes x and y, respectively. S- and f-polarised components whose planes are rotated by the rotation angle of eigenpolarisations δ are incident on the output facet. Each component of the spatial spectrum is rotated by the angle $\gamma = \arctan(k_y/k_x)$. Then, using (11) the expansion of the spectral components in σ- and π-polarisations will have the form:

$$E_\sigma = E_f T_f^\sigma \cos(\gamma - \delta) + E_s T_s^\sigma \sin(\gamma - \delta),$$
$$E_\pi = E_f T_f^\pi \cos(\gamma - \delta) + E_s T_s^\pi \cos(\gamma - \delta), \tag{20}$$

where E_f and E_s are f- and s-polarised components.

In view of (20) at the output of a biaxial crystal two orthogonal linearly polarised components have the form:

$$E_1 = E_\sigma \cos\gamma + E_\pi \sin\gamma,$$
$$E_1 = E_\sigma \sin\gamma - E_\pi \cos\gamma. \tag{21}$$

7. Results of calculations

Accounting for all orders of spatial dispersion and the angle between the eigenpolarisations for different spectral components in the solution method allows one to calculate accurately the change in the profile of the radiation beam with a wide angular spectrum and its walk-off during propagation in an anisotropic medium.

The calculation was performed for the 20-mm-long nonlinear bismuth triborate (BiB_3O_6) crystal. This biaxial medium has a strong birefringence, which allows one to observe more clearly the effects of deformation of the beam profile. The principal values of the refractive indices for this crystal at a wavelength of 540 nm are: $n_x = 1.7869$, $n_y = 1.8185$, $n_z = 1.9613$. The crystal cut angle θ was chosen close to the optical crystal axis angle and was equal to 25° at $\varphi = 0°$. At the crystal input the beam has a Gaussian amplitude profile (radius of 0.2 mm at the $1/e^2$ level, divergence of 4 mrad). The plane of radiation polarisation at the crystal input lies at an angle of 45° to the orientations of its eigenpolarisations.

Figure 11 shows the profiles of the radiation beam before and after its propagation through a biaxial crystal. To emphasise the deformation of the beam profile, the cutting angle was chosen specially close to the optical axis of the crystal. It is seen that the s-and f-polarised components of the beam during its propagation through the biaxial crystal are deformed in orthogonal directions. In addition, the f-polarised component experiences a walk-off during propagation. When radiation propagates in the xz plane at the cut-off angle $\theta < V_z$, the walk-off angle of the f-polarised wave component in the linear approximation is described by the expression

$$\beta(\theta) = \frac{1}{2}\frac{(n_z^2 - n_x^2)\sin 2\theta}{n_x^2 + (n_z^2 - n_x^2)\cos^2\theta}. \tag{22}$$

For the initial data presented above, the walk-off angle is 3.86°. The results of calculations by the above-described method give an angle of 3.83°.

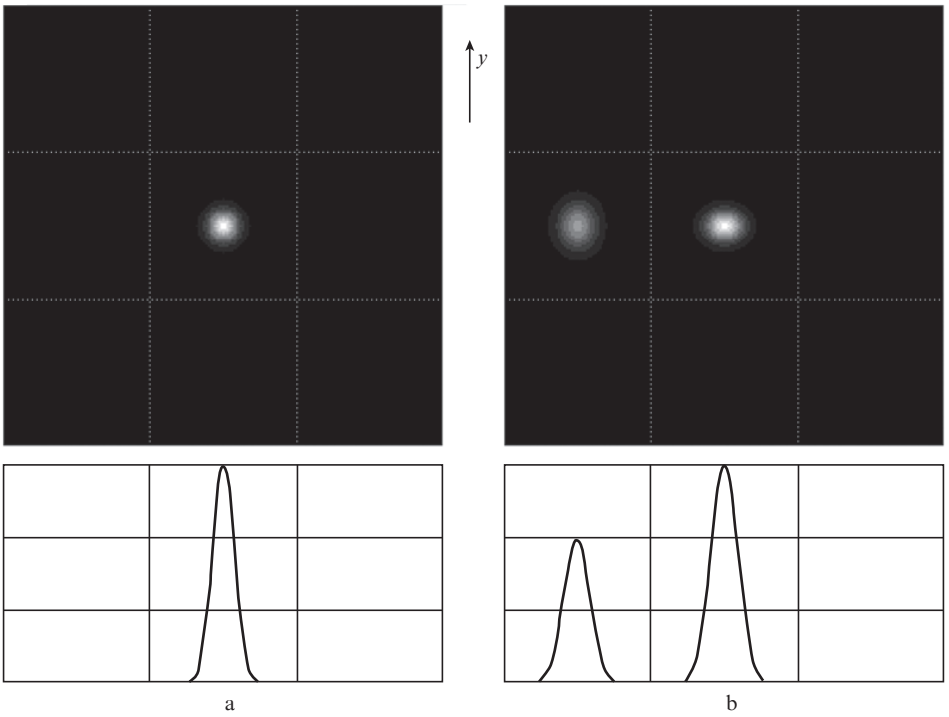

Figure 11. Profiles of the radiation beam (a) before and (b) after its passage through a biaxial crystal (the principal plane *xz* of the biaxial crystal is directed horizontally).

8. Comparison of calculation results with experimental data

The adequacy of the proposed Fourier space method for calculating the propagation of radiation in a biaxial medium can be most clearly checked by the example of calculation of the internal conical refraction – a phenomenon which is observed when a collimated unpolarised beam of monochromatic radiation propagates along the optical axis of a biaxial crystal. During propagation in a medium the transverse beam profile will diverge along the trajectory of a hollow cone, and after escaping from the medium – a hollow cylinder. Diffraction of the radiation beam leads to the fact that at the crystal output the beam profile will have the form of two thin concentric rings. Analytical description of the internal conical refraction for a nongyrotropic biaxial medium is considered in [35–37].

Schell and Bloembergen [37], using a biaxial aragonite crystal ($n_x = 1.53$, $n_y = 1.68$, $n_z = 1.685$) as an example, compared the results of experimental studies of conical refraction with theoretical calculations. The optical axis of aragonite lies at an angle $V_z = 80.4°$ to the z axis.

In the experiment, linearly polarised single-mode radiation of a 0.6328-μm He–Ne laser propagated along the optical axis of aragonite (the length of the crystal along the x axis of the crystal coordinate system was 9.5 mm). At the input facet of the crystal the beam had a Gaussian profile and the waist size $r = 34$ μm. Figures 12a, b show the experimentally measured [37] transverse profiles of the radiation beam at the output facet of the crystal after the conical refraction for two orthogonal polarisations of radiation, and Figs 12c, d – similar profiles calculated by the method described in this paper.

Figure 13 illustrates the beam cross sections at the output of aragonite along the *xz* plane for two orthogonal polarisa-

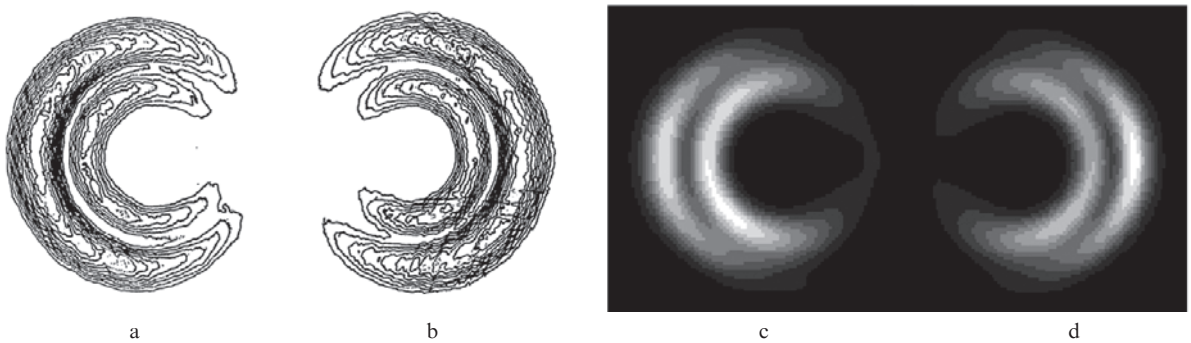

Figure 12. (a, b) Experimentally measured [37] and (c, d) calculated distributions of the radiation beam at the output facet of aragonite for linearly polarised radiation in the plane *xz* (a, b) and the orthogonal plane (b, d).

tions of linearly polarised radiation – in the plane *xz* (left peaks) and in the orthogonal plane (right peaks). It can be seen that the positions of the extrema of the experimental and calculated distributions coincide with high accuracy. Some discrepancy is due to the fact that we failed to accurately simulate the experiment of [37] due to the lack of description of the spatial structure of the beam inside the crystal.

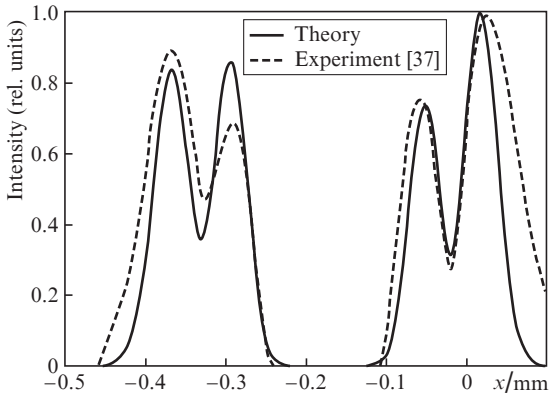

Figure 13. Comparison of the experimentally obtained [37] and calculated cross sections of the radiation beam at the output facet of aragonite along the plane *xz*.

9. Conclusions

The paper presents a method for calculating the propagation of radiation in a biaxial crystal with an arbitrary direction of the cut. The method takes into account all the orders of the spatial dispersion of the medium, which defines the action of diffraction and changes in angles of eigenpolarisations for different spatial components and features of the reflection coefficients at the input and output facets.

We have compared the experimental data and simulation results in the case of conical refraction of radiation propagating along the optical axis of the crystal. In the case of conical refraction it is required to take into account all the features of radiation propagation in the biaxial medium. Agreement of the results shows the accuracy of the data obtained using the proposed calculation method.

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