Quantum Electronics **44** (4) 341–344 (2014)

PACS numbers: 42.25.Fx; 42.25.Hz; 42.50.Ar; 42.65.Lm
DOI: 10.1070/QE2014v044n04ABEH015249

Interference of biphotons upon parametric down-conversion in the field of biharmonic pumping

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Abstract. **We report theoretical investigation of interference of biphotons emitted upon type-II collinear parametric down-conversion in the case of biharmonic pumping. Interference occurs when an optical or electronic shutter is used as an amplitude modulator in the experimental scheme. The phase of the interference is shown to depend on the time interval between the instant the shutter is opened and the instant corresponding to the maximum pump intensity. The main parameter affecting the visibility of the interference pattern is a time interval during which the shutter is open.**

Keywords: *parametric down-conversion, interference of biphotons, biharmonic pumping.*

1. Introduction

Spontaneous parametric down-conversion involves generation of coupled photon pairs or biphotons [1]. One of the important tasks of quantum optics is preparation [2] and transformation [3] of the biphoton field. The characteristics of the biphoton field are significantly affected by spatial and temporal modulation of pump radiation. The transformation of the biphoton field also occurs as a result of interference of biphotons. Interference of biphotons was mostly observed when several spatially separated areas of a nonlinear crystal were optically pumped [4, 5] or a second pump pulse delayed in time was introduced [6, 7], and also in the presence of an additional interferometer (e.g., a Michelson interferometer) in the scheme of the experiment [8, 9]. Nonclassical properties of biphoton interference in spontaneous parametric down-conversion are used to test the foundations of quantum theory [10], quantum cryptography and quantum teleportation [7, 11]. Apart from generation in spontaneous parametric down-conversion, biphotons can be generated in parametric oscillators [12] and this process can be described by the classical equations; however, it is not considered in this study.

The aim of this work is to study theoretically the effect of interference of biphotons, arising from different spectral components of the field of biharmonic pumping, on the average coincidence count rate of photons using an amplitude modulator in the detection scheme. Two principal schemes are proposed, which allow interference of biphotons to be observed under two-frequency pumping.

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Received 1 June 2013; revision received 31 January 2014
Kvantovaya Elektronika **44** (4) 341–344 (2014)
Translated by I.A. Ulitkin

2. Wave function

Consider parametric down-conversion and interference of biphotons emitted in the field of biharmonic pumping when use is made of the scheme shown in Fig. 1. Note that in this scheme some elements (filters and apertures), used in real experiments to cut off 'harmful' radiation, are omitted.

Biharmonic pumping can be obtained by using a two-mode laser or other sources of two-frequency, coherent optical radiation. The classical pump field $\boldsymbol{E}_{\rm p}$ (with amplitudes $E_{\rm p1}$, $E_{\rm p2}$; frequencies $\omega_{\rm p1}$, $\omega_{\rm p2}$; phases $\varphi_{\rm p1}$, $\varphi_{\rm p2}$; and wave vectors $\boldsymbol{k}_{\rm p1}$, $\boldsymbol{k}_{\rm p2}$ directed along the z axis) passes through a transparent crystal with quadratic nonlinearity $\chi^{(2)}$, having a length L. In the crystal, biphotons are spontaneously emitted in the same direction as the pump (along the z axis). One photon of the biphoton is emitted in the signal wave, described by the creation operator of the fieldя $\hat{E}_{\rm o}^{-}$. The signal wave has a polarisation in the plane of Fig. 1. Another photon is emitted in the idle wave, described by the creation operator of the field $\hat{E}_{\rm e}^{-}$, with a polarisation perpendicular to the signal wave. The signal and idle waves pass through an element (*2*), consisting of a set of quartz plates and producing a time delay τ between ordinary and extraordinary waves. Biphotons enter a Brown–Twiss interferometer, which consists of a beam splitter (*3*) with reflection and transmission coefficients of 50%; polarisers (*4*) and (*5*), the axes of which are located at an angle $\pi/4$ to the polarisation direction of the signal wave; photodetectors (*6*, *7*); an amplitude modulator (*8*); a controlled element (*9*); and a coincidence circuit (*10*). Part of pump radiation is delivered through the beam splitter (*1*) to controlled element (*9*), which synchronises the instant of opening of the modulator relative to the instant corresponding to the maximum modulation signal of the pump intensity. As a

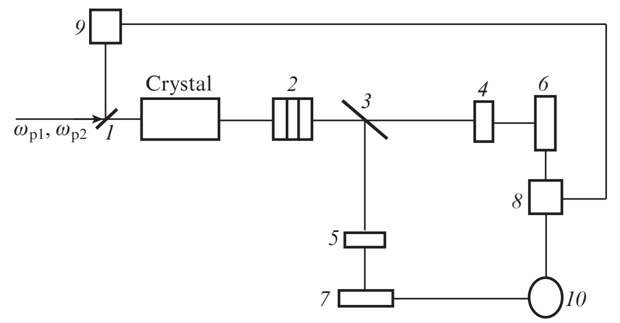

Figure 1. Schematic diagram for observing interference of biphotons, in which an electronic shutter is used as an amplitude modulator (see notations in the text).

modulator (*8*) we used an electronic shutter that opens and closes with a period of $T_0 = 2\pi/\Omega$, where $\Omega = \omega_{p2} - \omega_{p1}$ is the modulation frequency of the pump intensity. If we consider the case, when the electronic shutter (*8*) is mounted behind the photodetector, there occur errors due to self-inductance or mutual inductance of conductors or other hardware correlation. However, the element (*8*) may be fictitious (actually absent). In this case, the element (*9*) should be connected to a personal computer. The data will be processed by the computer, discarding coincidences when the shutter is closed. The personal computer is currently part of the coincidence circuit [13]. Work of the shutter is discussed in detail below.

To describe parametric down-conversion, we will use the interaction Hamiltonian between the pump field and a crystal

$$\hat{H} = \int_V d^3 r \chi^{(2)} E_p \hat{E}_o^- \hat{E}_e^- + \text{h.c.}, \quad (1)$$

where V is the volume of the crystal;

$$E_p = \sum_{j=1}^{2} E_{pj} \exp[i(k_{pj} z - \omega_{pj} t + \varphi_{pj})].$$

In the first order of perturbation theory in the diffraction-free approximation, using (1) we obtain the expression for the state vector at the crystal output:

$$|\Psi\rangle = \left[1 - \frac{i}{\hbar}\int_{-\infty}^{\infty} dt \hat{H}(t)\right]|0\rangle = |0\rangle + \sum_{j,k,k'} F_{jkk'} a_{ok}^+ a_{ek'}^+ |0\rangle, \quad (2)$$

where $|0\rangle$ is the vacuum state; a_{ok}^+ is the creation operator of an ordinary wave with a wave vector $\boldsymbol{k}$; $a_{ek'}^+$ is the creation operator of an extraordinary wave with a wave vector $\boldsymbol{k}'$; $F_{jkk'} = g_{jkk'} E_{pj} \exp(i\varphi_{pj}) \delta(\omega_{ok} + \omega_{ek'} - \omega_{pj}) h(L\Delta_{jkk'})$; ω_{ok} and $\omega_{ek'}$ are the frequencies of ordinary and extraordinary waves, respectively; $\delta(x)$ is the Dirac delta function; $\Delta_{jkk'} = k_{pj} - k - k'$; $h(x) = [1 - \exp(-ix)]/(ix)$; and $g_{jkk'}$ are the constants depending on the parameters of the crystal and interacting fields. Expressions for the constants $g_{jkk'}$ can be obtained from [3], which presents a detailed derivation of the expression for the wave function in the case of monochromatic pumping. Equation (2) is found from the expression given in [3] by adding an additional subscript j under the summation symbol.

3. Fields at the photodetectors

The interference experiments study the mutual correlation of oscillations in different spatiotemporal points. The Brown–Twiss interferometer allows one to measure the value of $\langle\Psi|\hat{E}_1^- \hat{E}_2^- \hat{E}_2^+ \hat{E}_1^+|\Psi\rangle = \langle 0|\hat{E}_2^+ \hat{E}_1^+|\Psi\rangle|^2$, and, as a consequence, the average coincidence count rate ($\hat{E}_{1,2}^+$ are the annihilation operators of the field).

The coincidence circuit (*10*) detects the average coincidence count rate

$$R_c = \lim_{T\to\infty} \frac{1}{T}\int_0^T dT_1 \int_0^T dT_2 F(T_1)|A(t_1, t_2|^2 = \lim_{N\to\infty} \frac{1}{T_0 N + \Delta T + t_0}$$
$$\times \sum_{n=0}^{N} \int_{T_0 n + t_0}^{T_0 n + \Delta T + t_0} dT_1 \int_0^{T_0 N + \Delta T + t_0} dT_2 |A(t_1, t_2)|^2, \quad (3)$$

where $F(t)$ is the function of transmission of electrons by the shutter from the photodetector to the coincidence circuit [$F(t) = 1$ stands for an open shutter and $F(t) = 0$ – for a closed shutter); t_0 is the instant when the shutter starts working ($0 \leqslant t_0 \leqslant T_0$); ΔT is the time interval during which the shutter has a 100% transmittance ($0 \leqslant \Delta T \leqslant T_0$); $A(t_1, t_2)$ is the biphoton field amplitude defined by the expression

$$A(t_1, t_2) = \langle 0|\hat{E}_2^+ \hat{E}_1^+|\Psi\rangle; \quad (4)$$

$\hat{E}_1^+$ and $\hat{E}_2^+$ correspond to the fields measured by the photodetectors (*6*) and (*7*) at the instants T_1 and T_2, respectively; $T_l - s_l/c$ ($l = 1, 2$); s_1 and s_2 are the optical path lengths from the photodetectors (*6*) and (*7*) to the crystal surface, respectively; and c is the speed of light. We assume that the detectors are arranged such that $s_1 = s_2$.

When calculating the biphoton amplitude (4) we use the Heisenberg representation. In this case, the state vector at the surface of the photodetector is determined by expression (2). Operators $\hat{E}_1^+$ and $\hat{E}_2^+$ are expressed in terms of the operators at the crystal output. Because the optical elements (*2*, *3*, *4*, *5*) are linear, then each element can be associated with a matrix. As a result of multiplication of matrices, the operators of the fields at the photodetectors $\hat{E}_{1,2}^+$ are expressed in terms of the annihilation operators of ordinary and extraordinary waves at the crystal output, a_{ok} and $a_{ek'}$:

$$\hat{E}_1^+(T_1) = \sum_{k_1} E_{k_1} \exp(i\omega_{k_1} T_s) \frac{1}{\sqrt{2}} [\exp(-i\omega_{k_1}\tau) a_{ok_1}$$
$$+ \exp(i\omega_{k_1}\tau) a_{ek_1}] \exp(-i\omega_{k_1} t_1),$$
$$\quad (5)$$
$$\hat{E}_2^+(T_2) = \sum_{k_2} E_{k_2} \exp(i\omega_{k_2} T_s) \frac{1}{\sqrt{2}} [-\exp(-i\omega_{k_2}\tau) a_{ok_2}$$
$$+ \exp(i\omega_{k_2}\tau) a_{ek_2}] \exp(-i\omega_{k_2} t_2),$$

where T_s is half the sum of the time of transmission of the signal and idle waves through the element (*2*).

4. Average coincidence count rate

Let the crystal be oriented so that the phase-matching conditions

$$\Omega_{oj} + \Omega_{ej} = \omega_{pj},$$
$$\quad (6)$$
$$\boldsymbol{K}_{oj} + \boldsymbol{K}_{ej} = \boldsymbol{k}_{pj}$$

are fulfilled. The wave vector $\boldsymbol{K}_{oj}$ corresponds to an ordinary radiation beam having the frequency Ω_{oj}j, and the wave vector $\boldsymbol{K}_{ej}$ – to an extraordinary radiation beam having the frequency Ω_{ej}.

We introduce the basic requirements to be met by the circuit elements:

1) a degenerate case $\Omega_{o1} = \Omega_{e1} = \omega_{p1}/2$ corresponding the harmonic of the pump field at a frequency ω_p is considered;

2) inequalities $\Delta T \gg \tau$, DL are met, where $D = 1/u_o - 1/u_e$, and u_o and u_e are the group velocities of ordinary and extraordinary waves;

3) the difference frequency $\omega_d = \Omega_{o2} - \Omega_{e2}$ is of same order of magnitude as Ω (or less).

The biphoton amplitude with formulas (2) and (4)–(6) taken into account is defined by the expression

$$A(t_1, t_2) = \sum_{j=1}^{2} A_j(t_1, t_2), \quad (7)$$

where

$$A_1(t_1, t_2) = \alpha_1 E_{p1} \exp[-i(\omega_{p1} t_s - \varphi_{p1})][\Pi(t_{12} + \tau) - \Pi(-t_{12} + \tau)]$$

is the biphoton amplitude corresponding to the harmonic of the pump field at a frequency ω_{p1};

$$A_2(t_1, t_2) = \alpha_2 E_{p2} \exp[-i(\omega_{p2} t_s - \varphi_{p2})]$$

$$\times\{\exp[-i\omega_d(t_{12} + \tau)]\Pi(t_{12} + \tau) - \exp[-i\omega_d(-t_{12} + \tau)]\Pi(-t_{12} + \tau)\}$$

is the biphoton amplitude corresponding to a frequency ω_{p2}; α_j are slowly varying functions, which are assumed constant throughout the region of parametric down-conversion and equal in further considerations: $\alpha_1 = \alpha_2 = \alpha$; $t_s = (t_1 + t_2)/2$; $t_{12} = t_1 - t_2$; and

$$\Pi(t) = \begin{cases} 1/(DL), 0 < t < DL, \\ 0, \text{the rest of the region } t. \end{cases}$$

Substituting (7) in (3) and integrating, for the average coincidence count rate we obtain the expression

$$R_c = R_{c1} + R_{c2} + R_{c12}, \quad (8)$$

where $R_{c1} = R_1(1 - f_{c1})$;

$$R_1 = \frac{2|\alpha|^2 E_{p1}^2 \Delta T}{DLT_0};$$

$$f_{c1} = \begin{cases} 2\tau/(DL), \tau \in]0; DL/2, \\ 2(DL - \tau)/(DL), \tau \in]DL/2; DL[, \\ 0, \text{the rest of the region } \tau; \end{cases}$$

$$R_{c2} = \frac{E_{p2}^2}{E_{p1}^2} R_{c1};$$

$$R_{c12} = R_{12}(1 - f_{c1});$$

$$R_{12} = \frac{4|\alpha|^2 E_{p1} E_{p2}}{\pi DL} \sin\frac{\Omega \Delta T}{2} \cos\phi;$$

$\phi = \Omega(t_i + \Delta T/2)$ is the phase of interference; and t_i is the time interval between the instant the shutter is opened and the instant corresponding to the maximum pump intensity, which is measured at the point of location of the photodetector (*6*). The time of the shutter opening takes into account the delay associated with the passage of an electronic signal from the photodetector to the shutter.

5. Visibility of the interference pattern

The biphoton amplitude $A(t_1, t_2)$ (7) consists of the sum of the amplitudes of biphotons, $A_j(t_1, t_2)$, corresponding to the spectral components of the pump field. Because of this, there arises interference of biphotons emitted by the crystal during the absorption of different harmonics of the pump field. Influence of interference in the measurements of the average count rate of coincidences R_c is due to the interference term R_{c12} in (8). In the case when the shutter is always open ($\Delta T = T_0$), because of the time averaging interference disappears ($R_{c12} = 0$) and the coincidence count rate is equal to the sum of the count rates of coincidences corresponding to different frequencies of the pump field. Interference manifests itself most strongly at the same pump fields ($E_{p1} = E_{p2}$) and short time intervals, during which the shutter is open ($\Delta T \ll T_0$).

Consider the effect of the interference phase ϕ on the coincidence count rate R_c by the example of two-mode pumping with the difference frequency $\Omega/2\pi$ = 150 MHz. This frequency can be obtained in the case of two-mode lasing in a laser with a 1-m-long cavity. The pump fields are identical. Let the pump propagate at an angle of 49° to the main axis of a BBO crystal. For the frequency components ω_{p1}, corresponding to the wavelength λ_{p1} = 352 nm, the phase matching is degenerate in this direction. When the crystal length is L = 0.4 mm, we obtain DL = 1 ps and $\omega_d/2\pi$ = 300 MHz. The time interval ΔT, during which the shutter is open, is chosen equal to 1 ns. At these parameters, the three requirements set forth in Section 4 have been met. Figure 2 shows the dependences of the coincidence count rate R_c on the time delay τ at different values of interference phases. The maximum value of the coincidence count rate is reached at the interference phase $\phi = 0$, which is approximately twice the coincidence count rate in the absence of interference ($\phi = \pi/2$). The minimum value of the coincidence count rate is attained at $\phi = \pi$, which is significantly less than the coincidence count rate in the absence of interference.

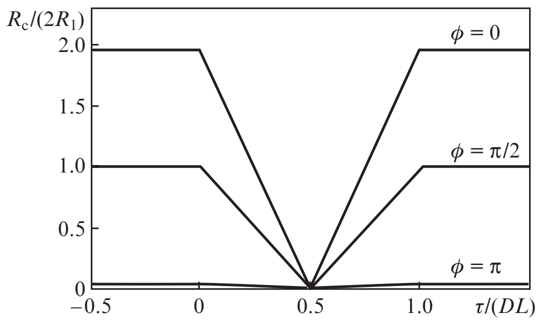

Figure 2. Dependence of the normalised average coincidence count rate $R_c/(2R_1)$ on $\tau/(DL)$ at different interference phases.

The visibility of the interference pattern is defined as the relative difference between the maximum and minimum count rates of coincidences corresponding to the different interference phases, expressed as a percentage. For the example under study, the visibility of the interference pattern is

$$V = \frac{R_c(\phi = 0) - R_c(\phi = \pi)}{R_c(\phi = 0) + R_c(\phi = \pi)} \times 100\% = 96\%.$$

Using (8), the visibility is defined by the expression

$$V = \frac{2E_{p1}E_{p2}}{E_{p1}^2 + E_{p2}^2} \frac{\sin(\Omega\Delta T/2)}{\Omega\Delta T/2} \times 100\%.$$

Figure 3 shows the dependence of the visibility of the interference pattern, V, on time ΔT at identical pump fields. In the case of $\Delta T \to 0$, the visibility tends to 100%, while at $\Delta T \to T_0$ – to zero.

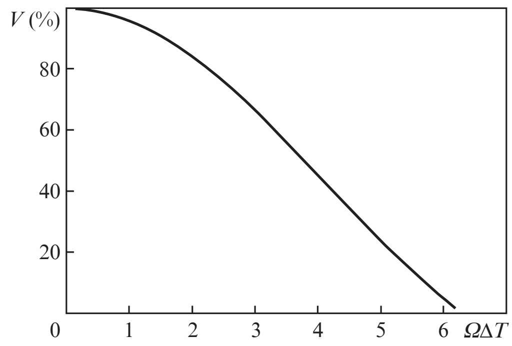

Figure 3. Dependence of the visibility of the interference pattern, V, on $\Omega\Delta T$.

6. Scheme with an optical shutter

The scheme in Fig. 4 differs from that in Fig. 1 by the presence of an optical shutter (*8*) instead of the electronic one. The optical shutter is periodically opened (100% transmittance) and closed (complete absorption of biphotons) at a frequency Ω. If the shutter is closed, two-photon states are absent at the output of the shutter in the state vector: $|\Psi\rangle = |0\rangle$. Therefore, the biphoton amplitude in the case of the open shutter is determined by expression (7), and, when the shutter is closed, it is equal to zero. The average coincidence count rate R_c is then found from the expression

$$R_c = \lim_{T\to\infty}\frac{1}{T}\int_0^T \mathrm{d}T_1\int_0^T \mathrm{d}T_2\,|A(t_1,t_2)|^2$$

and, after substituting the biphoton amplitude into it, has the form of (8). The interference phase difference for the schemes in Figs 1 and 4 is a constant. It is related to the fact that an optical shutter is located between the nonlinear crystal and the element (*2*), unlike the electronic shutter, which is located behind the photodetector.

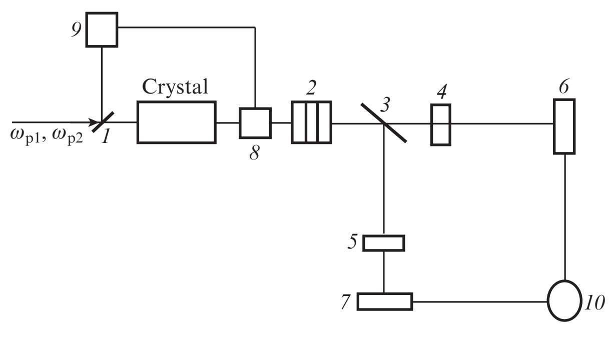

Figure 4. Schematic diagram for observing interference of biphotons, in which an optical shutter is used as an amplitude modulator (see notations in the text).

7. Conclusions

Thus, we have proposed and theoretically investigated the methods for producing interference of biphotons arising from different spectral components of the field of biharmonic pumping in the case of type-II collinear spontaneous parametric down-conversion. We consider two basic schemes, leading to equivalent results. We have demonstrated that by changing the instant of the shutter opening with respect to the instant corresponding to the maximum pump intensity modulation signal, we can control the interference phase, and by changing the time interval during which the shutter is open, it is possible to control the visibility of the interference pattern. Interference of biphotons can be used to control the average coincidence count rate of photocounts in the Brown–Twiss interferometer.

Acknowledgements. The author thanks E.G. Lariontsev for his enormous help in finalising the article for publication.

This work was supported by the Russian Foundation for Basic research (Grant No. 11-02-00080).

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