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Modulation instability and short-pulse generation in media with relaxing Kerr nonlinearity and high self-steepening

I.O. Zolotovskii, D.A. Korobko, V.A. Lapin

Abstract. **The modulation instability in waveguides with high Kerr nonlinearity, characterised by a delayed nonlinear response, has been investigated with allowance for the self-steepening parameter and third-order dispersion. General expressions for the modulation gain are obtained. The influence of the waveguide parameters on the gain is analysed. It is shown that the joint effect of the delayed nonlinear response and negative nonlinearity dispersion leads to an increase in the modulation gain. The relations obtained are confirmed by numerical simulation. The results of this study can be used to design compact generators of high-frequency pulse trains.**

Keywords: *modulation instability, delayed nonlinear response, self-steepening parameter.*

1. Statement of the problem and basic equations

The technology of fabricating optical media with high Kerr nonlinearity has been intensively developed in the last years. Examples of these media are semiconductor waveguides [1–3]; nanocomposites [4, 5]; and photon-crystal structures [6], including those based on polymer and organic materials [7, 8]. Materials of this type are promising for many optoelectronic devices: pulse compressors, optical switches, logic gates, etc. [9–11]. High Kerr nonlinearity makes it possible to increase the nonlinear conversion efficiency of these devices and reduce their size (i.e., make it possible to use them in compact optoelectronic circuits).

Generators of short-pulse trains, the principle of operation of which is based on the modulation instability (MI) of a continuous wave in a nonlinear dispersive medium [12], are another type of these nonlinear converters. The modulation instability is typical of nonlinear systems that support propagation of localised waves and is determined by the balance between nonlinearity and dispersion [13]. This effect was experimentally revealed in quartz optical fibres with a constant anomalous group-velocity dispersion, where generation of a pulse train from a continuous modulated wave was observed [14, 15]. Note that standard quartz optical fibres are characterised by relatively low nonlinearity; therefore, a train of short soliton-like pulses can be generated if their length is on the order of several kilometres.

I.O. Zolotovskii, D.A. Korobko, V.A. Lapin Ulyanovsk State University, ul. L'va Tolstogo 42, 432000 Ul'yanovsk, Russia; e-mail: rafzol.14@mail.ru, korobkotam@rambler.ru, lva2013@yandex.ru

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In this study, we consider the MI in an optical waveguide with high nonlinearity. A specificity of this problem is that, when developing an MI model, one must take into account the higher order nonlinear and dispersion effects: nonlinear response delay, wavefront self-steepening, and third-order dispersion. The relations obtained can be used to design compact generators and compressors of optical pulses on the basis of highly nonlinear waveguides.

The nonlinear response of a medium is characterised by the nonlinear refractive index, which is related to the real part of the nonlinear third-order dielectric susceptibility $\chi^{(3)}$ [12, 16],

$$n_2 = \frac{3}{8n}\mathrm{Re}\chi^{(3)}. \qquad (1)$$

Here, n is the linear part of the refractive index of the medium $\bar{n}$. We restrict ourselves to the case of an isotropic medium and a linearly polarised electric field of the propagating wave. Under these conditions, the nonlinear polarisation of the medium can be represented as $P_{\mathrm{NL}} = \chi^{(3)}E^3$, where E is the electric field strength. Thus, the 'total' refractive index is related to the field intensity $|E|^2$ as

$$\bar{n} = n + n_2|E|^2. \qquad (2)$$

In the approximation of slowly varying amplitudes, the field of a wave packet propagating in a waveguide along the z axis can be written as

$$E(\boldsymbol{r},t) = U(x,y)|A(z,t)|\exp\{\mathrm{i}[(\beta(\omega)-\beta_0)z-(\omega-\omega_0)t]\},$$

where ω_0 is the packet carrier frequency, $\beta(\omega)$ is the propagation constant, and $U(x,y)$ is the field distribution in a plane oriented perpendicular to the propagation direction. The amplitudes $A(z,t)$ of a wave packet in a nonlinear waveguide satisfy the nonlinear Schrödinger equation (NLSE), which can be written as follows in the related coordinate system that moves with a group velocity $u_{\mathrm{gr}} = (\partial\beta/\partial\omega)^{-1}_{\omega=\omega_0}$ [12, 16]:

$$\mathrm{i}\frac{\partial A}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial\tau^2} + \gamma|A|^2A = 0. \qquad (3)$$

Here,

$$\tau = t - \int_0^z \frac{\mathrm{d}z}{u_{\mathrm{gr}}(z)}$$

is the time in the related coordinate system and $\beta_2 = (\partial^2\beta/\partial\omega^2)^{-1}_{\omega=\omega_0}$ is the group-velocity dispersion in the waveguide. The nonlinearity coefficient has the form

$$\gamma = \frac{n_2\omega_0}{cS_{\rm eff}}, \tag{4}$$

where

$$S_{\rm eff} = \left(\int_0^\infty |U(x,y)|^2 {\rm d}x{\rm d}y\right)^2 \left(\int_0^\infty |U(x,y)|^4 {\rm d}x{\rm d}y\right)^{-1}$$

is the effective mode area. Below we will consider the wave propagation in a single-mode waveguide with a $S_{\rm eff}$ value retained constant along its length.

Equation (3) describes adequately the propagation of wave packets with widths exceeding 0.1 ps in single-mode quartz optical fibres characterised by $n_2 \approx 3 \times 10^{-20}$ m^2 W^{-1}, which corresponds to the nonlinearity coefficient (depending on the wavelength and specific fibre realisation) $\gamma \approx 2-30$ W^{-1} km^{-1} [12]. The nonlinear refractive index n_2 of modern highly nonlinear optical materials may reach $\sim 10^{-14}-10^{-15}$ m^2 W^{-1} [1–8]; thus, the nonlinearity coefficient γ of the waveguides made of these materials may exceed the nonlinearity coefficient of quartz waveguides by 5–6 orders of magnitude. At an average modulated-wave power $P_0 \approx 1$ W, the characteristic nonlinearity length in these waveguides may be only few millimetres, which is favourable both for miniaturising optoelectronic elements and for obtaining a high degree of compression of generated soliton-like pulses. Their width can approximately be found from a rough estimate, according to which a modulated wave with a modulation period T is transformed into a train of solitons with a width τ_0 and a peak power $P = P_0T/\tau_0$. The equality of the dispersion and nonlinear lengths of soliton yields $\tau_0 \approx |\beta_2|/(\gamma P_0 T)$. Thus, if the characteristic parameters of highly nonlinear waveguides and modulated wave are used (γ = 100 W^{-1} m^{-1}, $\beta_2 = 10^{-23}$ s^2 m^{-1}, P_0 = 10 W, and $T = 10^{-12}$ s), these waveguides (with a length of only few millimetres) can generate pulses with widths on the order of several tens of femtoseconds.

However, Eqn (3) is insufficient for exact description of wave-packet propagation in highly nonlinear waveguides, because it does not include the terms responsible for the higher order nonlinear and dispersion effects, which could be neglected in the case of low nonlinearity. The most important of them is the relaxation nature of high nonlinearity, which is related to the fact that the nonlinear response of an optical medium is not instantaneous but is characterised by a finite settling time $\tau_{\rm NL}$. Despite the large variety of the mechanisms of occurrence of optical nonlinearity, it can be noted that, in almost all cases, higher nonlinearities are characterised by a longer response time [16]. The quasi-static consideration based on Eqn (3) remains valid if radiation pulse width greatly exceeds the nonlinearity settling time $\tau_{\rm NL}$. Weakly nonlinear quartz optical fibres are characterised by a time response on the order of several femtoseconds, which must be taken into account in the case of propagation of ultrashort pulses (with a width below 0.1 ps) [16]. When describing the propagation of these pulses (with widths of few field oscillation periods), one has to reject the concept of slowly varying amplitudes and consider jointly the field wave equations and the material equations describing the response of the medium. In recent years some new methods have been developed, which are based on separate description of the fast (electronic) and slow (Raman) responses of the medium [17] and on the spatial and temporal multiscale expansion of the wave equation for ultrashort pulse propagation [18].

The nonlinearity relaxation time $\tau_{\rm NL}$ in semiconductor waveguides is much longer: it ranges from several tenths to several tens of picoseconds; the nonlinear response, which significantly affects the propagation of wave packets, is also much stronger than in quartz glass [1, 7, 8, 16, 19]. Such large $\tau_{\rm NL}$ values make it possible to take into account the nonlinearity relaxation within the approximate Debye model [16, 20–22]. In addition, high nonlinearity calls for consideration of the wavefront self-steepening effect and higher order dispersion.

Let us restrict ourselves to the propagation of pulses with widths no less than 0.05 ps. In this case, we can retain the description of wave packets using slowly varying envelopes and take the nonlinear Schrödinger equation (3) as a basis. After introduction of the terms taking into account the third-order dispersion and self-steepening effect, the generalised propagation equation in a highly nonlinear waveguide can be written as the system

$$\begin{aligned} &{\rm i}\frac{\partial A}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 A}{\partial \tau^2} - {\rm i}\frac{\beta_3}{6}\frac{\partial^3 A}{\partial \tau^3} + \Delta n A - {\rm i}\frac{\mu}{\gamma}\frac{\partial}{\partial \tau}(\Delta n A) = 0, \\ &\frac{\partial \Delta n}{\partial \tau} = \frac{1}{\tau_{\rm NL}}(-\Delta n + \gamma\,|A|^2). \end{aligned} \tag{5}$$

Here, the time-dependent nonlinear response of the medium, $\Delta n(\tau)$, and the parameters of third-order dispersion $\beta_3 = (\partial^3\beta/\partial\omega^3)^{-1}_{\omega=\omega_0}$and self-steepening are introduced. The latter parameter is determined by the value and dispersion of the optical-medium nonlinearity [16, 23]:

$$\mu = \frac{2n_2}{cS_{\rm eff}} - \frac{\omega_0}{c}\frac{\partial}{\partial\omega}\Big(\frac{n_2}{S_{\rm eff}}\Big). \tag{6}$$

It is generally assumed (with a rather high accuracy) for standard quartz optical fibres that $\mu \approx 2\gamma/\omega_0$ [12], i.e., μ is a small value, which affects weakly the dynamics of a wave packet if its width exceeds 100 fs and the peak power is much smaller than 1 MW. In highly nonlinear waveguides this parameter significantly increases (not only due to the rise in the nonlinearity coefficient but also owing to the dispersion factors) and may reach $10^{-12}-10^{-11}$ W^{-1} m^{-1} s. Below we will analyse the stability of a steady-state solution for this system [$A = A_0\exp({\rm i}\gamma|A_0|^2 z)$] to small modulations and find the conditions for developing modulation instability, which are necessary for generating pulse trains from a modulated wave.

2. Modulation instability under conditions of relaxing nonlinearity

As was noted above, system (5) has a solution in the form of a continuous wave with a constant power P_0, which undergoes a nonlinear phase shift

$$A(z) = \sqrt{P_0}\exp({\rm i}\gamma P_0 z). \tag{7}$$

Let us consider a small perturbation of this wave $a(z,\tau)$, which leads to a small change in the nonlinear response δn,

$$\begin{aligned} &A(z,\tau) = (\sqrt{P_0} + a)\exp({\rm i}\gamma P_0 z), \\ &\Delta n = \gamma P_0 + \delta n. \end{aligned} \tag{8}$$

System (5), linearised in small perturbations, takes the form

$${\rm i}\frac{\partial a}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 a}{\partial\tau^2} - {\rm i}\frac{\beta_3}{6}\frac{\partial^3 a}{\partial\tau^3} + \sqrt{P_0}\,\delta n -$$

$$-\mathrm{i}\frac{\mu}{\gamma}\left(\sqrt{P_0}\,\delta n\frac{\partial \delta n}{\partial\tau}+\gamma P_0\frac{\partial a}{\partial \tau}\right)=0,$$

$$\frac{\partial\delta n}{\partial\tau}=\frac{1}{\tau_{\mathrm{NL}}}[-\delta n+\gamma\sqrt{P_0}\,(a+a^*)]. \qquad (9)$$

Proceeding from (9), one can derive a system of linear equations for the Fourier components $\tilde{a}(\Omega,k)$ and $\tilde{a}^*(-\Omega,-k)$:

$$\left[k+\frac{\beta_2}{2}\Omega^2+\frac{\beta_3}{6}\Omega^3+\frac{\gamma P_0}{1+\mathrm{i}\Omega\tau_{\mathrm{NL}}}\right.$$

$$\left.+\frac{\mu P_0}{1+\mathrm{i}\Omega\tau_{\mathrm{NL}}}(2\Omega+\mathrm{i}\Omega^2\tau_{\mathrm{NL}})\right]\tilde{a}+(\gamma P_0+\mu P_0\Omega)\frac{\tilde{a}^*}{1+\mathrm{i}\Omega\tau_{\mathrm{NL}}}=0,$$

$$(\gamma P_0-\mu P_0\Omega)\frac{\tilde{a}}{1+\mathrm{i}\Omega\tau_{\mathrm{NL}}}+\left[-k+\frac{\beta_2}{2}\Omega^2+\frac{\beta_3}{6}\Omega^3\right.$$

$$\left.+\frac{\gamma P_0}{1+\mathrm{i}\Omega\tau_{\mathrm{NL}}}-\frac{\mu P_0}{1+\mathrm{i}\Omega\tau_{\mathrm{NL}}}(2\Omega+\mathrm{i}\Omega^2\tau_{\mathrm{NL}})\right]\tilde{a}^*=0.$$

A system of equations has a nontrivial solution if its determinant is zero. Based on this condition, we find the dispersion relation between the modulation frequency Ω and the propagation constant k:

$$k=\frac{\beta_3\Omega^3}{6}+\left(1+\frac{\mathrm{i}}{2}\Omega\tau_{\mathrm{NL}}\right)\frac{2\mu P_0\Omega}{1+\mathrm{i}\Omega\tau_{\mathrm{NL}}}$$

$$\pm\frac{|\Omega|}{2}\left[\beta_2^2\Omega^2+\frac{4\beta_2\gamma P_0}{1+\mathrm{i}\Omega\tau_{\mathrm{NL}}}+\frac{4\mu^2P_0^2}{(1+\mathrm{i}\Omega\tau_{\mathrm{NL}})^2}\right]^{1/2}. \qquad (10)$$

Note that dispersion relation (10) in the limiting case $\tau_{\mathrm{NL}}=0$ passes to the relation obtained in [24] for a medium with an instantaneous nonlinear response, while in the case $\beta_3=0$ and $\mu=0$ it coincides with the result obtained in [22] for a medium characterised by relaxing nonlinearity, with higher order and nonlinearity dispersions disregarded. Thus, one can state that the results of this study generalise the results obtained in the aforementioned papers.

It follows from (10) that two dispersion branches $k(\Omega)$ can be selected; they differ not only in imaginary but also in real parts of the propagation constant. The real part is responsible for the propagation of a wave with a spatial period $z = 2\pi/\mathrm{Re}\,k$, while the imaginary part determines its amplification or attenuation; thus, one can separately consider the propagation of damping modulation and (which is most interesting) rising modulation. The frequency dependence of the gain $g(\Omega)$ can be written in the form

$$g(\Omega)=2\,\mathrm{Im}\,k=-\frac{2\mu P_0\Omega^2\tau_{\mathrm{NL}}}{1+\Omega\tau_{\mathrm{NL}}^2}\pm|\beta_2|\,|\Omega|$$

$$\times\mathrm{Re}\left(-\Omega^2-\frac{\mathrm{sgn}(\beta_2)4\gamma P_0}{|\beta_2|(1+\mathrm{i}\Omega\tau_{\mathrm{NL}})}-\frac{4\mu^2P_0^2}{\beta_2^2(1+\mathrm{i}\Omega\tau_{\mathrm{NL}})^2}\right)^{1/2}. \qquad (11)$$

In the limiting case of instantaneous response ($\tau_{\mathrm{NL}}=0$), with self-steepening disregarded ($\mu=0$), we arrive at the classical case of MI [12], which arises in a waveguide with anomalous dispersion $\beta_2<0$ in the frequency band $\Omega_{\mathrm{c}}\div\Omega_{\mathrm{c}}$ ($\Omega_{\mathrm{c}}=4\gamma P_0/|\beta_2|$). It is noteworthy that the value of the third-order dispersion β_3 does not affect the gain in the initial stage of MI. The coefficient $g(\Omega)$, which is given by expression (11), has two branches. In the most typical case, where the self-steepening parameter $\mu>0$ and

$$\frac{2\mu P_0\Omega^2\tau_{\mathrm{NL}}}{1+\Omega\tau_{\mathrm{NL}}^2}<|\beta_2|\,|\Omega|$$

$$\times\mathrm{Re}\left(-\Omega^2-\frac{\mathrm{sgn}(\beta_2)4\gamma P_0}{|\beta_2|(1+\mathrm{i}\Omega\tau_{\mathrm{NL}})}-\frac{4\mu^2P_0^2}{\beta_2^2(1+\mathrm{i}\Omega\tau_{\mathrm{NL}})^2}\right)^{1/2},$$

the branches have different signs; the branch that is larger in modulus corresponds to modulation damping, while the branch smaller in modulus corresponds to its gain. If the waveguide has a negative dispersion nonlinearity ($\mu<0$), the modulation gain becomes larger in modulus. At $\mu P_0\Omega^2\tau_{\mathrm{NL}}\to 0$ the absolute values of damping and gain become equal.

Figures 1 and 2 present the dependences of the absolute value of $g(\Omega)$ in the cases of anomalous and normal waveguide dispersion β_2 at different τ_{NL} and μ values. The γ, β_3, and P_0 values were chosen typical of highly nonlinear waveguides. The dependences are given for only positive frequencies ($\Omega>0$) because they are symmetric for negative frequencies: $g(\Omega)=g(-\Omega)$. This can be shown by repeating the above-performed procedure with system (9) for the Fourier components $\tilde{a}(-\Omega,k)$ and $\tilde{a}^*(\Omega,-k)$.

It follows from the plots in Fig. 1a that the nonlinear response rate affects strongly the value and frequency range of the modulation gain. As was noted above, in the case of instantaneous nonlinear response ($\tau_{\mathrm{NL}}=0$), the MI region in a waveguide with anomalous dispersion is limited by the lim-

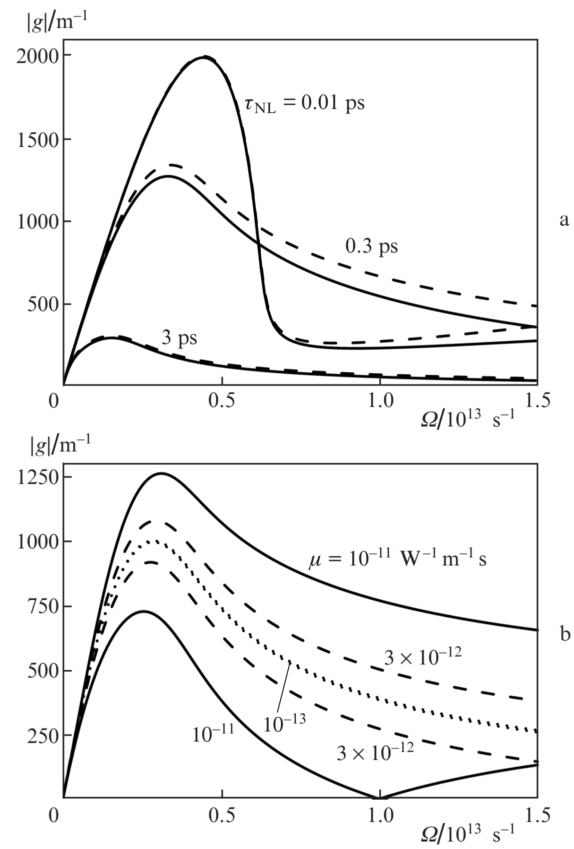

Figure 1. Absolute values of gain (damping) $g(\Omega)$ (11) at (a) waveguide anomalous dispersion $\beta_2=-10^{-22}$ s^2 m^{-1}, $\mu=10^{-12}$ W^{-1} m^{-1} s and different τ_{NL} values (solid and dashed lines correspond to gain and damping, respectively) and (b) at $\tau_{\mathrm{NL}}=0.5$ ps and different μ values. In both cases $\gamma=100$ W^{-1} m^{-1}, $\beta_3=10^{-35}$ s^3 m^{-1}, and $P_0=10$ W.

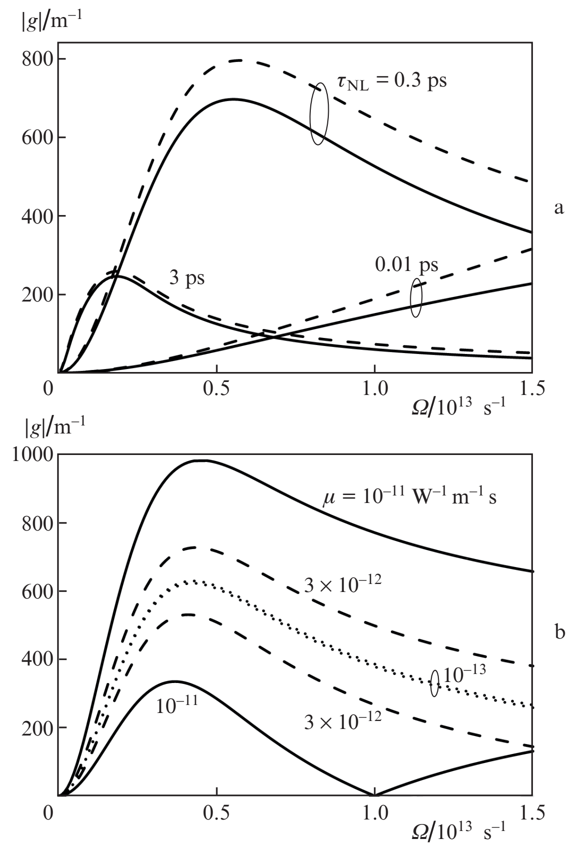

Figure 2. Absolute values of gain (damping) $g(\Omega)$ (11) at (a) waveguide anomalous dispersion $\beta_2 = -10^{-22}$ s^2 m^{-1}, $\mu = 10^{-12}$ W^{-1} m^{-1} s and different τ_{NL} values (solid and dashed lines correspond to gain and damping, respectively) and (b) at $\tau_{NL} = 0.5$ ps and different μ values. In both cases $\gamma = 100$ W^{-1} m^{-1}, $\beta_3 = 10^{-35}$ s^3 m^{-1}, and $P_0 = 10$ W.

iting frequency $\Omega_c = 4\gamma P_0\ |\beta_2|^{-1}$. With allowance for the nonlinearity inertia, the MI region becomes unbounded. At small τ_{NL} values (0.01 ps), the region of strong MI, which can be referred to as the parametric gain region, is retained on the whole. A region of weak MI, caused by nonlinearity relaxation, arises beyond the strong-MI region. With an increase in τ_{NL} (0.3 ps), the parametric MI gain decreases, and the sharp boundary between the strong- and weak-MI regions becomes blurred. At long relaxation times (3 ps), there is an unbounded region of weak MI, where the parametric gain makes a small contribution only in the low-frequency range. The maximum modulation gain, caused by delayed nonlinear response, falls in the frequency range $\Omega \approx 1/\tau_{NL}$.

The influence of the positive self-steepening parameter μ manifests itself in the 'splitting' of the dependences of gain moduli $g(\Omega)$, i.e., in a decrease in the modulation gain and in increased damping. The influence of the parameter μ on the modulation gain at a typical time τ_{NL} is shown in Fig. 1b in more detail. At small μ values (10^{-13} W^{-1} m^{-1} s) its influence is insignificant (the dotted lines almost coincide). With an increase in μ (3×10^{-12} W^{-1} m^{-1} s), the branches of the plots $|g(\Omega)|$ undergo 'splitting', i.e., the positive parameter μ suppresses MI due to both factors: parametric gain and nonlinearity relaxation. In the case of large positive μ values (10^{-11} W^{-1} m^{-1} s), the frequency range of modulation gain becomes limited; when $\Omega > 10^{13}$ s^{-1}, g changes sign and determines the modulation damping.

It is noteworthy that the situation becomes inverted for a waveguide with a negative self-steepening parameter ($\mu < 0$): at large negative μ values the modulation gain increases. This MI gain can be explained by the opposite effect of the delayed nonlinear response, which reduces the velocity of wave maxima, and the nonlinear rise in the leading edge steepness. At $\mu > 0$ the trailing edge becomes steeper [16, 25]; this factor, in combination with the delayed response, reduces the MI effect. It is important that MI gain arises only under joint effect of negative self-steepening and relaxing nonlinearity. Under conditions of an instantaneous nonlinear response, both negative and positive values of parameter μ facilitate a decrease in MI.

The peculiar effect of the delayed nonlinear response is the MI at normal ($\beta_2 > 0$) waveguide dispersion. Figure 2a shows a frequency dependence $g(\Omega)$ for a waveguide with $\beta_2 > 0$ at different τ_{NL} values. It is known that MI does not arise at an instantaneous nonlinear response in waveguides with normal dispersion, i.e., the parametric gain region is absent [the subradical expression in (11) is always negative]. In the case of fast response ($\tau_{NL} = 0.01$ ps), MI arises in the high-frequency range. With an increase in τ_{NL} to 0.3 ps and then to 3 ps, the maximum gain decreases and passes to the low-frequency range. The dependence $|g(\Omega)|$ for a waveguide with normal dispersion at different μ values and at a characteristic time τ_{NL} are shown in Fig. 2b. Note that, as in the case of anomalous dispersion, positive and negative values of the self-steepening parameter facilitate MI damping and gain, respectively.

3. Application of the obtained results to modelling pulse generators

The relations obtained in the previous section refer to the initial MI phase. Its further development leads to amplification of not only the spectral components with a modulation frequency Ω but also the next harmonics; this process is accompanied by pump depletion (the spectral component with $\Omega = 0$, which is initially presented by a continuous wave with a power P_0, can be considered as pump radiation). Under these conditions, the gain g at the modulation frequency decreases. Finally, the modulated wave decays into a sequence of soliton-like pulses (breathers) [26]. As was noted above, this process occurs in highly nonlinear waveguides at lengths on the order of few millimetres. The relations obtained can be useful to analyse the developed MI phase and design pulse-train generators based on highly nonlinear waveguides.

We investigated numerically the developed MI phase in a waveguide with high nonlinearity. The simulation was performed using the split step-by-step Fourier transform [12] of system (5) at different waveguide parameters. A harmonically modulated wave

$$A(\tau,0) = \sqrt{P_0}\Big(1 + 0.01\cos\frac{\tau}{T}\Big) \tag{12}$$

with a power $P_0 = 10$ W was considered to be the initial one.

Figure 3a shows the simulation results for the transmission of the initial modulated wave in a highly nonlinear waveguide with anomalous dispersion at different nonlinearity relaxation times τ_{NL}. The modulation period $T = 1$ ps, which corresponds to the frequency $\Omega = 2\pi \times 10^{12}$ s^{-1}. It can be seen that the time τ_{NL} has a decisive effect on the MI development. This is in agreement with the results obtained above. At short relaxation times (0.01 ps), a high-frequency train of short soliton-like pulses is formed on a small length (5.6 mm) of a highly nonlinear waveguide. With an increase in the relaxation time (0.1 ps), the modulation gain decreases, and a longer waveguide is required to form a pulse train. With a further increase in the response delay time, when it becomes comparable with the modulation period (τ_{NL} = 0.5 ps), the modulation gain is low, and the MI development is very weak. Thus, we can conclude that highly nonlinear waveguides with a long nonlinear response time are inefficient as generators of high-frequency pulse trains and other ways for reducing the relaxation time should be sought for. As an example, we will note that ionising radiation can be used to reduce τ_{NL} in semiconductor structures [27]. At the same time, a direct decrease in the relaxation time leads generally to deterioration of the nonlinear properties of the waveguide, which is, naturally, undesirable.

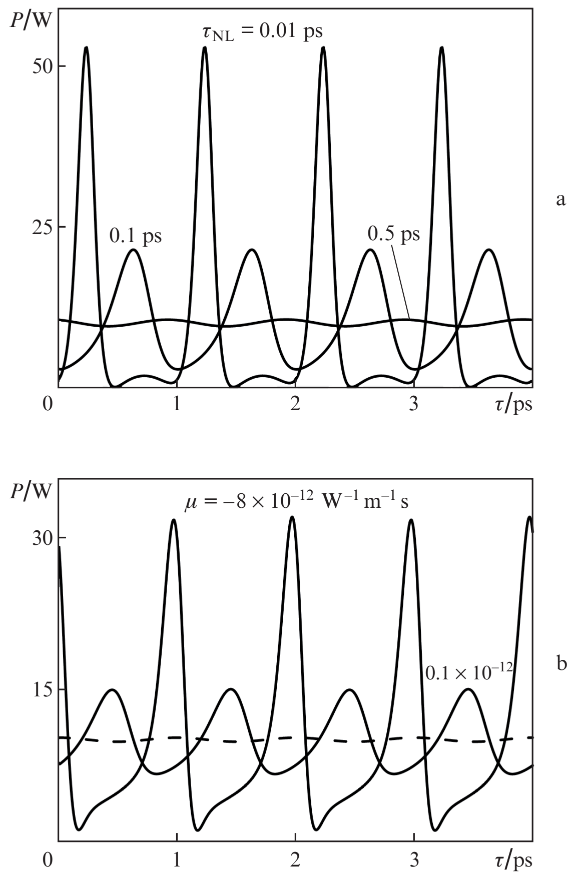

Figure 3. Modulated-signal power in the developed phase of modulation instability with P_0 = 10 W (a) after passage through a 5.6-mm-long waveguide at $\mu = 10^{-12}$ W^{-1} m^{-1} and different τ_{NL} values and (b) after passage through a 9.2-mm-long waveguide at τ_{NL} = 0.5 ps and different μ values (the dashed line shows the initial modulated wave). In both cases $\beta_2 = 5 \times 10^{-23}$ s^2 m^{-1}, $\beta_3 = -5 \times 10^{-37}$ s^3 m^{-1}, and γ = 100 W^{-1} m^{-1}.

Based on the above results, we propose the following concept of modifying a highly nonlinear pulse-train generator, which is based on the MI effect. To this end, it is necessary to design a highly nonlinear waveguide, which is characterised by negative nonlinearity dispersion and, therefore, by a self-steepening parameter $\mu < 0$. This can be done, for example, by forming a periodic structure in the waveguide [28]. The results of the previous section indicate that a highly nonlinear waveguide with an intermediate value of nonlinearity relaxation time and a negative self-steepening parameter has a higher modulation gain than a similar waveguide with positive μ and is a more efficient pulse-train generator.

Figure 3b shows the simulation results for the transmission of a modulated wave (12) with a period T = 2 ps in a highly nonlinear waveguide with a relaxation time τ_{NL} = 0.5 ps at large negative ($\mu = -8 \times 10^{-12}$ W^{-1} m^{-1} s) and small positive ($\mu \approx \gamma/\omega_0 = 0.1 \times 10^{-12}$ W^{-1} m^{-1} s) values of self-steepening parameter. It can be seen that a train of soliton-like pulses is formed in the waveguide with negative μ on a small length. The waveguide with small positive μ, which is characterised by a lower modulation gain, is less efficient as a generator of high-frequency pulse trains.

4. Conclusions

We considered the MI effect in waveguides with high Kerr nonlinearity, which is characterised by a delayed nonlinear response, taking into account the influence of the self-steepening parameter and third-order dispersion. These waveguides are promising as compact generators of short-pulse trains in optoelectronic circuits. The system of equations describing the propagation of a wave packet in a waveguide of this type is analysed for stability to small perturbations. A general expression for the gain (damping) of small harmonic modulations is derived. It is shown that in particular cases (with the self-steepening and third-order dispersion disregarded or in the absence of delayed nonlinear response) our result coincides with the known expressions for the modulation gain [22, 24]. The expressions derived by us show that the third-order dispersion does not affect the modulation gain, whereas the influence of the delayed nonlinear response and self-steepening parameter is complex and depends on the value and sign of the self-steepening parameter μ.

At small values of the waveguide self-steepening parameter $|\mu| < 1/(P_0\Omega^2\tau_{NL})$ its influence is insignificant. The peculiar effect of the delayed nonlinear response is the formation of an additional frequency range of MI with a maximum modulation gain in the vicinity of $\Omega \approx 1/\tau_{NL}$. It is important that the nonlinearity relaxation effect provides the existence of MI in a waveguide with normal dispersion, which is impossible in the case of an instantaneous response. In a waveguide with anomalous dispersion, an increase in τ_{NL} leads to blurring of the boundary between the MI region, determined by the delayed response, and the region of parametric modulation gain. The maximum gain rapidly decreases with an increase in the relaxation time τ_{NL} in waveguides with both normal and anomalous dispersions.

In the case of the instantaneous nonlinear response, the self-steepening parameter, independent of its sign, plays the role of a damping factor with respect to MI and decreases the modulation gain. Under a joint effect of nonlinear relaxation and significant self-steepening [$|\mu| > 1/(P_0\Omega^2\tau_{NL})$], the result depends on the sign of μ. A positive self-steepening parameter leads to suppression of MI, while at negative μ values, due to the 'opposite' effect of relaxing nonlinearity and increasing the leading-edge steepness, the modulation gain significantly increases. This effect can be used to design compact generators of high-frequency pulse trains based on MI in waveguides with high relaxing Kerr nonlinearity. The conclusions of our study are confirmed by the results of numerical simulation.

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