

PACS numbers: 42.60.Da; 42.60.Fc; 42.62.Eh
DOI: 10.1070/QE2014v044n04ABEH015297

A laser gyro with a four-mirror square resonator: formulas for simulating the dynamics of the synchronisation zone parameters of the frequencies of counterpropagating waves during the device operation in the self-heating regime

E.A. Bondarenko

Abstract. **For a laser gyro with a four-mirror square resonator we have developed a mathematical model, which allows one to simulate the temporal behaviour of the synchronisation zone parameters of the frequencies of counterpropagating waves in a situation when the device operates in the self-heating regime and is switched-on at different initial temperatures.**

Keywords: *laser gyro, ring gas laser, synchronisation of counterpropagating wave frequencies.*

1. Introduction

Among the main types of laser gyros widely used in practice, we can single out the devices based on a ring gas He–Ne laser (the ratio of isotope concentrations, $[^{20}Ne]:[^{22}Ne] = 1:1$) with a flat N-mirror (N = 3, 4) resonator ensuring generation of linearly polarised radiation in the sagittal plane. The laser, usually operating at 0.6328 μm, is pumped by a dc parallel discharge obtained by a common cathode and two anodes [1–3].

According to relations (5.55)–(5.57) from [3] and to expressions (6.45)–(6.47) from [4], when the currents are balanced in the discharge arms, the resonator is fine tuned to the centre of the emission line and the losses are identical, the system of equations describing the dynamics of the dimensionless intensities I_j (j = 1, 2) and the phase difference ψ of counterpropagating waves of such a laser gyro can be written as

$$\dot{I}_1 = (\alpha - \beta I_1 - \theta I_2) I_1 - 2 r_2 \sqrt{I_1 I_2} \cos(\psi + \varepsilon_2),$$
$$\dot{I}_2 = (\alpha - \beta I_2 - \theta I_1) I_2 - 2 r_1 \sqrt{I_1 I_2} \cos(\psi - \varepsilon_1), \quad (1)$$
$$\dot{\psi} = M\Omega + r_2 \sqrt{I_2/I_1} \sin(\psi + \varepsilon_2) + r_1 \sqrt{I_1/I_2} \sin(\psi - \varepsilon_1).$$

In deriving these equations it was taken into account that the wave with j = 1 propagates in the direction of the laser gyro rotation. In system (1) α, β, and θ are the Lamb coefficients characterising the properties of the active medium; $M = (1 + K_a)M_g$ is the scale factor of the laser gyro, primarily determined by its geometrical component $M_g = 8\pi S/(\lambda L)$ and also taking into account the properties of the medium through a small parameter K_a; L is the perimeter of the axial contour; S is the covered area; Ω is the angular velocity of rotation of the device in inertial space; and r_j and ε_j are the moduli and arguments of complex integral coefficients $r_j \exp(i\varepsilon_j)$ of the linear coupling of the counterpropagating waves, characterising their interaction through backscattering, absorption and transmission of radiation on the mirrors.

In our paper [5], based on the analysis of system (1) we obtained formulas for calculating the parameters of the synchronisation zone of the frequencies of counterpropagating electromagnetic waves generated in the laser gyro. These parameters are, respectively, the coordinates $\Omega_{(-)}$ and $\Omega_{(+)}$ of the left and right boundaries of the synchronisation zone on axis of the angular velocity Ω, the coordinate of its centre $\Omega_{(0)} = (\Omega_{(+)} + \Omega_{(-)})/2$ and the half-width of this zone $\Omega_s = (\Omega_{(+)} - \Omega_{(-)})/2$. The relations obtained in [5] supplement the results of earlier theoretical studies [3, 6–13] and have the form

$$\Omega_{(\pm)} = \pm \frac{\sqrt{r_p^2 + \mu^2 r_m^2 \pm 2\mu(r_2^2 - r_1^2)}}{\sqrt{1-\mu^2}\,M},$$

$$\Omega_{(0)} = \frac{\sqrt{r_p^2 + \mu^2 r_m^2 + 2\mu(r_2^2 - r_1^2)} - \sqrt{r_p^2 + \mu^2 r_m^2 - 2\mu(r_2^2 - r_1^2)}}{2\sqrt{1-\mu^2}\,M}, \quad (2)$$

$$\Omega_s = \frac{\sqrt{r_p^2 + \mu^2 r_m^2 + 2\mu(r_2^2 - r_1^2)} + \sqrt{r_p^2 + \mu^2 r_m^2 - 2\mu(r_2^2 - r_1^2)}}{2\sqrt{1-\mu^2}\,M}.$$

In view of the condition $|r_2 - r_1| \ll (r_1 + r_2)/2$ (see, for example, [3]) implemented in practice, expressions (2) can be approximately written in a more compact form:

$$\Omega_{(\pm)} = \Omega_{(0)} \pm \Omega_s,$$

$$\Omega_{(0)} = \frac{\mu(r_2^2 - r_1^2)}{\sqrt{(1-\mu^2)(r_p^2 + \mu^2 r_m^2)}\,M}, \quad (3)$$

$$\Omega_s = \frac{\sqrt{r_p^2 + \mu^2 r_m^2}}{\sqrt{1-\mu^2}\,M},$$

where

$$r_p = \sqrt{r_1^2 + r_2^2 + 2 r_1 r_2 \cos\varepsilon_{12}}; \quad r_m = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos\varepsilon_{12}};$$

$$\varepsilon_{12} = \varepsilon_1 + \varepsilon_2; \quad \mu = \frac{2 r_1 r_2 \sin\varepsilon_{12}}{\alpha_m r_p} \quad (|\mu| < 1); \quad (4)$$

$$\alpha_m = \alpha_p \frac{1-h}{1+h}; \quad \alpha_p = \alpha = \frac{c}{L}(g - \Gamma); \quad h = \frac{\theta}{\beta}.$$

E.A. Bondarenko National Technical University of Ukraine ‘Kiev Polytechnic Institute’, prosp. Pobedy 37, Building 28, 03056 Kiev, Ukraine; e-mail: ea_bndrk@ukr.net

Received 16 August 2013; revision received 14 November 2013
Kvantovaya Elektronika **44** (4) 364–370 (2014)
Translated by I.A. Ulitkin

Here r_p and r_m are the combinations of parameters of the linear coupling of counterpropagating waves; α_p and α_m are the inverse relaxation times of the sum and difference of the intensities of counterpropagating waves, respectively; g is the linear unsaturated gain of the active medium; Γ is the resonator losses per trip; h is the parameter depending on the total pressure of the He–Ne mixture [14]; and μ is a quantity characterising the effect of the active-medium gain on the parameters of the synchronisation zone. (Equations (2) and (3) are valid under the condition of weak coupling of counterpropagating waves, which suggests that in the entire range of working discharge currents used in laser gyros, the ratios r_p/α_p and r_m/α_m are much less than unity. In modern devices operating at sufficiently large excesses of the pump power over the threshold [3], the condition is usually satisfied.)

At the laser gyro design stage, in developing, for example, the methods of its tests in a heat chamber, of paramount importance is the problem of construction of a mathematical model that would allow one to simulate the temporal behaviour of parameters $\Omega_{(-)}$, $\Omega_{(+)}$, $\Omega_{(0)}$, Ω_s of the synchronisation zone of the counterpropagating wave frequencies of the device in a situation when it is switched on at different values of the initial temperature, and thereafter operates in the self-heating regime.

In the literature known to the author, this problem was not considered in detail; therefore, the solution to this problem is the aim of this paper. The data presented below can complement the results of earlier works in this area [11, 15–22].

2. Description of a laser gyro

Following [3], as an example we will choose a laser gyro with a four-mirror square resonator having a nominal arm length l = 50 mm and a perimeter $L = 4l$ = 200 mm. According to [3], such a device is characterised by the half-width of the synchronisation zone, $\Omega_s \approx 0.05$ deg s^{-1}. The angular resolution q_θ of the laser gyro is 2.61″, and the geometric scaling factor is M_g = 496459. The gyroscope operates at a total pressure of the He–Ne mixture of 6.5 Torr and a five-fold excess of the gain g over the losses Γ (i.e., the parameter of relative excitation is $N_{rel} = g/\Gamma = 5$). In order not to present (together with the comments) cumbersome formulas for calculating the small parameter K_a, as well as expressions for the estimates of β and θ, we assume $M = M_g = 496459$ and, in addition, set h = 0.652.

The laser gyro described was considered in [23] in studying the dependence of the parameters $\Omega_{(-)}$, $\Omega_{(+)}$, $\Omega_{(0)}$, and Ω_s of the synchronisation zone of the counterpropagating wave frequencies on the active-medium gain g. From this work we borrow all the data necessary for description.

Thus, following [23], we assume that the resonator of the laser gyro in question is formed by two flat signal mirrors (M_1, M_2) and two spherical mirrors (M_3, M_4) with radii of curvature R = 1000 mm mounted on the piezocorrectors (mirrors are numbered clockwise). For flat mirrors M_1 and M_2 we have specified the following energy parameters: the integral coefficient of light scattering, K^{f}_{scat}, into the full solid angle 4π sr; absorption losses, Γ^{f}_{absorp}; and useful transmission losses, Γ^{f}_{transm}. For spherical mirrors M_3 and M_4 we have specified the integral coefficient of light scattering, K^{s}_{scat}, and absorption losses Γ^{s}_{absorp}. Moreover, we have specified the diffraction losses due to the presence of an aperture in the laser gyro resonator. Then, the total losses of the gyroscope, Γ, can be calculated as

$$\Gamma = \Gamma_{mirr} + \Gamma_{diffr},$$
$$\Gamma_{mirr} = 2(K^{f}_{scat} + \Gamma^{f}_{absorp} + \Gamma^{f}_{transm} + K^{s}_{scat} + \Gamma^{s}_{absorp}), \quad (5)$$

where Γ_{mirr} are the mirror losses and Γ_{diffr} are the diffraction losses. As in [23], we will use $K^{f}_{scat} = 5\times10^{-6}$, $\Gamma^{f}_{absorp} = 55\times10^{-6}$, $\Gamma^{f}_{transm} = 60\times10^{-6}$, $K^{s}_{scat} = 10\times10^{-6}$, and $\Gamma^{s}_{absorp} = 50\times10^{-6}$. Then, $\Gamma_{mirr} = 360\times10^{-6}$. Furthermore, we set $\Gamma_{diffr} = 40\times10^{-6}$. As a result, we obtain $\Gamma = 400\times10^{-6}$.

Then, according to formula (7) from [23], as applied to this laser gyro resonator, we calculate complex integral coefficients $r_j\exp(i\varepsilon_j)$ of the linear coupling between counterpropagating waves by using the expression

$$\frac{L}{c} r_j \exp(i\varepsilon_j) = a_f\left\{\exp\left[i\left(\frac{\pi}{2} - \chi_f \pm \varphi_1\right)\right] + \exp\left[i\left(\frac{\pi}{2} - \chi_f \pm \varphi_2\right)\right]\right\}$$
$$+ a_s\left\{\exp\left[i\left(\frac{\pi}{2} - \chi_s \pm \varphi_3\right)\right] + \exp\left[i\left(\frac{\pi}{2} - \chi_s \pm \varphi_4\right)\right]\right\}$$
$$+ b_f[\exp(\pm i\varphi_1) + \exp(\pm i\varphi_2)] + b_s[\exp(\pm i\varphi_3) + \exp(\pm i\varphi_4)], \quad (6)$$

which describes the result of summation of complex local coupling coefficients of these waves with respect to all four mirrors. (Here and below the upper arithmetic signs in the formulas correspond to $j = 1$, and the lower ones – to $j = 2$.)

From expression (6) we obtain the resulting relations for the estimates of r_j and ε_j ($j = 1, 2$):

$$r_j = \frac{c}{L}\sqrt{A_j^2 + B_j^2}, \ \varepsilon_j = \frac{\pi}{2} - \arctan\frac{A_j}{B_j}, \quad (7)$$

where

$$A_j = a_f[\sin(\chi_f \mp \varphi_1) + \sin(\chi_f \mp \varphi_2)]$$
$$+ a_s[\sin(\chi_s \mp \varphi_3) + \sin(\chi_s \mp \varphi_4)]$$
$$+ b_f(\cos\varphi_1 + \cos\varphi_2) + b_s(\cos\varphi_3 + \cos\varphi_4);$$
$$B_j = a_f[\cos(\chi_f \mp \varphi_1) + \cos(\chi_f \mp \varphi_2)] \quad (8)$$
$$+ a_s[\cos(\chi_s \mp \varphi_3) + \cos(\chi_s \mp \varphi_4)]$$
$$\pm b_f(\sin\varphi_1 + \sin\varphi_2) \pm b_s(\sin\varphi_3 + \sin\varphi_4).$$

Expressions (8) feature two groups of parameters. The parameters of the first group – a_f, χ_f, b_f and a_s, χ_s, b_s – respectively characterise individual properties of flat and spherical mirrors. In particular, a_f and a_s are the moduli of local complex dimensionless coefficients of counterpropagating wave coupling through backscattered radiation on flat and spherical mirrors; χ_f and χ_s are the 'loss angles' due to scattering by these mirrors; b_f are the moduli of local complex dimensionless coefficients of counterpropagating wave coupling through absorption and transmission of radiation by flat mirrors; and b_s are the moduli of local complex dimensionless coefficients of counterpropagating wave coupling through absorption by spherical mirrors. The above parameters will be considered as the known constants. On the basis of formulas (8) from [23] (which for the sake of brevity we do not present here) for the above characteristics of the mirrors, we have for these quantities the following numerical estimates: $a_f = 1.15\times10^{-6}$, $a_s = 1.72\times10^{-6}$, $\chi_f = 461''$, $\chi_s = 652''$, $b_f = 5.91\times10^{-8}$, $b_s = 2.72\times10^{-8}$.

The second group parameters in expressions (8) are the phase angles φ_n (n = 1,..., 4), which describe the influence of a change in the axial contour geometry [24, 25] of the laser gyro resonator when the device is switched on and subsequently operates in the self-heating regime. These values are not yet known. Looking ahead, we note only that under these conditions they will change over time [which, in turn, will lead to a deviation of the synchronisation zone parameters $\Omega_{(-)}$, $\Omega_{(+)}$, $\Omega_{(0)}$, $\Omega_{\rm s}$]. Therefore, to solve this problem we need to derive the corresponding formulas for calculating φ_n.

3. Additional information required for the formulation of the problem

In this section we will introduce into consideration the physical quantities necessary for the formulation of the problem, define them and present some numerical estimates. We will also dwell on the circumstances of laser gyro switching, in particular, on its automatic extremal perimeter control system (PCS).

We assume that at a fixed reference temperature $T_{q\lambda}$ of the laser gyro monoblock (for example, when $T_{q\lambda}$ = 25 °C), an integer (even or odd) number q of wavelengths λ is fitted on the perimeter L of the resonator axial contour, i.e.,

$$L = L_{q\lambda} = q\lambda = 4l. \tag{9}$$

We assume that when the laser gyro monoblock is heated by ΔT_λ degrees relative to the reference value of $T_{q\lambda}$, the perimeter of resonator axial contour increases (in the case of a switched off PCS) by the value of ΔL_λ, equal to one wavelength λ, i.e.,

$$\Delta L_\lambda = \lambda = L K_{\rm TE} \Delta T_\lambda. \tag{10}$$

Here $K_{\rm TE}$ is the coefficient of a relative linear thermal expansion of the material of which the laser gyro monoblock is made. We assume that the material is a Zerodur glass ceramic [3, p. 3-7], for which according to [2, p. 95] $K_{\rm TE} \approx 5.27\times10^{-8}$ 1/°C. Without introducing a large error, in the calculations below $K_{\rm TE}$ = 6.328× 10^{-8} 1/°C, which will make it possible to use a more convenient rounded value ΔT_λ = 50 °C that (for a given L = 200 mm) can be found from (10). The parameter ΔT_λ in its physical sense is conveniently defined as the intermode temperature interval of the laser gyro.

Let the current temperature T of the laser gyro monoblock vary with time t according to the law

$$T = T_{\rm ini} + \Delta T_{\rm SH}(t),\ \ T_{\rm ini} = T_{q\lambda} + \Delta T_{\rm ini}, \\ \Delta T_{\rm SH}(t) = \Delta T_{\rm SH}^{\max}[1 - \exp(-t/\tau_{\rm SH})], \tag{11}$$

where $T_{\rm ini}$ is the initial temperature of the monoblock at time t = 0 of switching-on of the laser gyro; $\Delta T_{\rm ini}$ is the initial increment in the monoblock temperature relative to the basic value of $T_{q\lambda}$; $\Delta T_{\rm SH}(t)$ is the increment in the monoblock temperature increasing in time (relative to $T_{\rm ini}$) during the laser gyro operation in the self-heating regime; $\Delta T_{\rm SH}^{\max}$ is the maximum increment in the monoblock temperature after the termination of the thermal transition process; $\tau_{\rm SH}$ is the time constant of the monoblock self-heating. According to experimental data [3] (see Fig. 4.2 on p. 3-17), for the given device $\Delta T_{\rm SH}^{\max}$ = 7 °C, and roughly, $\tau_{\rm SH}$ = 2400 s. We will use these values in the calculations below.

The values introduced allow us to consider now the circumstances of the laser gyro switching on. Thus, let the device be switched on at time t = 0. Then, its PCS, according to the algorithm of its work, with two controlled spherical mirrors M_3 and M_4 mounted on piezocorrectors will first adjust the axial contour perimeter L so that an integer number of wavelengths λ is fitted on it, i.e., will provide the resonance condition

$$L = L_{(q+k)\lambda} = (q+k)\lambda \quad (k = 0, \pm 1, \pm 2, ...), \tag{12}$$

and then for the entire time of operation of the gyro will stabilise the value of L, continuously compensating for the thermal expansion of the monoblock. (We assume that the PCS works without errors and almost instantly; we neglect its searching motion and take into account only the working motion aimed at maximising the generation power.)

In formula (12) the sum $q + k$ is the resulting index of the longitudinal working mode, at which, after an initial PCS adjustment generation will occur, and k is the 'adjustment' index that is automatically 'selected' by the system so as to ensure the fulfilment of the two-sided inequality

$$T_{q\lambda} + (k - 1/2)\Delta T_\lambda < T_{\rm ini} \leqslant T_{q\lambda} + (k + 1/2)\Delta T_\lambda. \tag{13}$$

Thus, for example, at the above parameters $T_{q\lambda}$ = 25 °C, ΔT_λ = 50 °C and the initial laser gyro monoblock temperature $T_{\rm ini}$ ranging from 1 to 50 °C, the index k will be automatically 'selected' equal to zero by the system. When $T_{\rm ini}$ varies from 51 to 100 °C, the index is equal to unity. In other words, depending on the initial temperature, generation in the first case occurs on the qth longitudinal mode at $L = L_{q\lambda} = q\lambda$, and in the second – on the (q + 1)th mode at $L = L_{(q+1)\lambda} = (q+1)\lambda$.

Strictly speaking, the value $M_{\rm g}$ of geometric scale factor of the laser gyro in question during its operation on the qth longitudinal mode will be different from the corresponding value on the (q + 1)th mode. However, this difference is very small and, therefore, will not be considered below.

To solve the problem, we assume that the material from which the laser gyro monoblock is made is homogeneous and isotropic, and the monoblock is thermally deformed only in the axial plane of the resonator.

4. Formulation and solution of the problem

For the laser gyro in question we need to obtain, taking (9)–(13) into account, such expressions for φ_n (n = 1, ..., 4) appearing in formulas (7) and (8) for r_j and ε_j (j = 1, 2), which together with the initial relations (3) and (4) would allow one to simulate the temporal behaviour of the synchronisation zone parameters $\Omega_{(-)}$, $\Omega_{(+)}$, $\Omega_{(0)}$, and $\Omega_{\rm s}$ of the counterpropagating wave frequencies in a situation when the device operates in the self-heating regime, its switching on occurring at different initial temperatures.

To calculate the values of φ_n we use relation (9) [23]:

$$\varphi_n = \frac{4\pi}{\lambda} S_n. \tag{14}$$

Here S_n is the distance measured along the axial contour (clockwise) between the reference plane (located at the origin of the coordinates) and the centre of the mirror M_n. The origin of the coordinates is chosen on the surface of the mirror M_1 at a point where the centre of the light spot of a Gaussian beam is located (at this point the axial contour touches the surface of the mirror M_1 and is 'reflected' from it).

To estimate the values of S_n, in expression (14) we will use formula (10) [23]:

$$S_n = -t_n \sin\theta_n + \sum_{m=1}^{n} L_{m-1}^{(m)}, \quad (15)$$

where $L_0^{(1)} \equiv 0$ and $L_{m-1}^{(m)}$ ($m = 2, 3, 4$) is the length of the laser gyro resonator arm between the mirrors M_{m-1} and M_m (which is the distance measured along the axial contour between the centres of light spots of the Gaussian beam on the surfaces of these mirrors); t_n is the displacement of the centre of the light spot of the Gaussian beam at the surface of the mirror M_n relative to its centre (which is measured in the axial plane to the right); and θ_n is half the angle between the arms of the laser gyro resonator at the mirror M_n (in our case, $\theta_n = \pi/4$). It follows from (15) that

$$S_1 = -\frac{\sqrt{2}}{2}t_1,\ S_2 = -\frac{\sqrt{2}}{2}t_2 + L_1^{(2)},\ S_3 = -\frac{\sqrt{2}}{2}t_3 + L_1^{(2)} + L_2^{(3)},$$
$$S_4 = -\frac{\sqrt{2}}{2}t_4 + L_1^{(2)} + L_2^{(3)} + L_3^{(4)}. \quad (16)$$

The methods for calculating the quantities t_n and $L_{n-1}^{(n)}$ for plane N-mirror misaligned (i.e., with displaced mirrors) laser gyro resonators of arbitrary (flat) form, containing, in the general case, plane-parallel plates in the arms, have been proposed in [26] and [27], respectively. Based on these techniques, with the laser gyro resonator under study, when, in the case of the switched off PCS, all four mirrors due to thermal expansion of the monoblock move together with its mounting surfaces in the axial plane linearly, normally and outwards to a distance w_n, for the mentioned values we can obtain the expressions

$$t_1 = \frac{1}{8-3\xi}[(2-\xi)(w_1 - w_3) + (6-2\xi)(w_2 - w_4)],$$
$$t_2 = \frac{1}{8-3\xi}[(6-2\xi)(-w_1 + w_3) + (2-\xi)(-w_2 + w_4)], \quad (17)$$
$$t_3 = t_4 = \frac{2}{8-3\xi}(w_1 - w_2 - w_3 + w_4)$$

and

$$L_1^{(2)} = L_3^{(4)} = l + (\sqrt{2}/2)(w_3 + w_4),$$

$$L_2^{(3)} = l + \frac{\sqrt{2}/2}{8-3\xi}[(8-2\xi)w_1 + (8-4\xi)w_2 + \xi(-w_3 + w_4)], \quad (18)$$

$$L_4^{(1)} = l + \frac{\sqrt{2}/2}{8-3\xi}[(8-4\xi)w_1 + (8-2\xi)w_2 + \xi(w_3 - w_4)],$$

where $\xi = pl$ is a small dimensionless parameter introduced for brevity and $p = 2(\sqrt{2}/R)$ is the optical power of each of the spherical mirror in the axial plane. Given $l = 50$ mm, $R = 1000$ mm, we have the following numerical estimates: $p = 0.0028$ mm^{-1}, $\xi = 0.14$.

By substituting expressions (17) and (18) into (16) we obtain

$$S_1 = \frac{\sqrt{2}/2}{8-3\xi}[(2-\xi)(-w_1 + w_3) + (6-2\xi)(-w_2 + w_4)],$$

$$S_2 = l + \frac{\sqrt{2}/2}{8-3\xi}[(6-2\xi)(w_1 + w_4) + (2-\xi)(w_2 + w_3)],$$

$$S_3 = 2l + \frac{\sqrt{2}/2}{8-3\xi}[(6-2\xi)(w_1 + w_4) + (10-4\xi)(w_2 + w_3)], \quad (19)$$

$$S_4 = 3l + \frac{\sqrt{2}/2}{8-3\xi}[(6-2\xi)w_1 + (10-4\xi)w_2$$
$$+ (18-7\xi)w_3 + (14-5\xi)w_4].$$

In order to make use of formulas (19) below, we need an expression for the axial contour perimeter L of the laser gyro resonator. The perimeter of the resonator is by definition equal to the sum of all the lengths of its arms, i.e., $L = L_1^{(2)} + L_2^{(3)} + L_3^{(4)} + L_4^{(1)}$. Thus, taking into account (9) and (18)

$$L = q\lambda + \Delta L,\ \ \Delta L = \sqrt{2}(w_1 + w_2 + w_3 + w_4), \quad (20)$$

where ΔL is the perimeter increment caused by linear, normal movements w_n of all the four mirrors of the laser gyro resonator due to thermal expansion of the monoblock when its temperature T rises by ΔT ($\Delta T = T - T_{q\lambda}$) degrees relative to the basic value $T_{q\lambda}$.

Given that in this laser gyro resonator the mirrors M_1 and M_2 are signal and the mirrors M_3 and M_4 are mounted on piezocorrectors and controlled by the PCS, we represent w_n in (20) in the expanded form:

$$w_1 = w_2 = h_{\Delta T},\ \ w_3 = h_{\Delta T} - w + h_{\rm PCS},$$
$$w_4 = h_{\Delta T} + w + h_{\rm PCS}. \quad (21)$$

Here $h_{\Delta T}$ is movement of each mirror M_n together with the mounting surface of the monoblock due to a temperature increment ΔT; w are the oppositely directed displacements of the mirrors M_3 and M_4 (the mirror M_4 moves from the resonator at a distance w, and the mirror M_3, on the contrary, moves into the resonator at exactly the same distance) specified [23] for the purpose of initial adjustment of the parameters r_j and ε_j of the linear coupling of counterpropagating waves; and $h_{\rm PCS}$ are the PCS-controlled like-directed displacements of the mirrors M_3 and M_4.

To explicitly define the law governing the variation of the $h_{\rm PCS}$ values in a situation when the PCS is switched on and operates normally, we substitute (21) into (20):

$$L = q\lambda + \sqrt{2}(4h_{\Delta T} + 2h_{\rm PCS}). \quad (22)$$

Comparing (22) with expression (12), which represents in a mathematical form the problem whose solution is provided by the PCS, we obtain

$$(q + k)\lambda = q\lambda + \sqrt{2}(4h_{\Delta T} + 2h_{\rm PCS}), \quad (23)$$

whence

$$h_{\rm PCS} = \frac{\sqrt{2}}{4}k\lambda - 2h_{\Delta T}. \quad (24)$$

Then, by substituting (24) into (21) we find refined relations for w_n, which already take into account the fact of the normal PCS operation:

$$w_1 = w_2 = h_{\Delta T},\ \ w_3 = \frac{\sqrt{2}}{4}k\lambda - h_{\Delta T} - w,$$
$$w_4 = \frac{\sqrt{2}}{4}k\lambda - h_{\Delta T} + w. \quad (25)$$

Taking (25) into account expressions (19) take the form

$$S_1 = \frac{k\lambda}{4} - \sqrt{2}\,h_{\Delta T} + \frac{\sqrt{2}}{2}\frac{4-\xi}{8-3\xi}w,$$

$$S_2 = l + \frac{k\lambda}{4} + \frac{\sqrt{2}}{2}\frac{4-\xi}{8-3\xi}w,$$

$$S_3 = 2l + \frac{k\lambda}{2} - \frac{\sqrt{2}}{2}\frac{4-2\xi}{8-3\xi}w, \quad (26)$$

$$S_4 = 3l + k\lambda - \sqrt{2}\,h_{\Delta T} - \frac{\sqrt{2}}{2}\frac{4-2\xi}{8-3\xi}w.$$

Relations (26) include the nominal arm length l of the laser gyro resonator. On the basis of (9), we have $l = q\lambda/4$. Substituting this expression into (26) and then (26) into (14), we find

$$\varphi_1 = \pi k - 4\pi\frac{\sqrt{2}\,h_{\Delta T}}{\lambda} + 2\pi\frac{4-\xi}{8-3\xi}\frac{\sqrt{2}\,w}{\lambda},$$

$$\varphi_2 = \pi(q+k) + 2\pi\frac{4-\xi}{8-3\xi}\frac{\sqrt{2}\,w}{\lambda},$$

$$\varphi_3 = 2\pi(q+k) - 2\pi\frac{4-2\xi}{8-3\xi}\frac{\sqrt{2}\,w}{\lambda}, \quad (27)$$

$$\varphi_4 = 3\pi q + 4\pi k - 4\pi\frac{\sqrt{2}\,h_{\Delta T}}{\lambda} - 2\pi\frac{4-2\xi}{8-3\xi}\frac{\sqrt{2}\,w}{\lambda}.$$

Formulas (27) can be simplified by omitting the term $2\pi(q + k)$ in the expression for φ_3, and by omitting the term $4\pi k$ and substituting πq instead of $3\pi q$ into the expression for φ_4. As a result, we obtain

$$\varphi_1 = \pi k - 4\pi\frac{\sqrt{2}\,h_{\Delta T}}{\lambda} + \varphi_{\rm f},$$

$$\varphi_2 = \pi(q+k) + \varphi_{\rm f}, \quad \varphi_3 = \varphi_{\rm s}, \quad (28)$$

$$\varphi_4 = \pi q - 4\pi\frac{\sqrt{2}\,h_{\Delta T}}{\lambda} + \varphi_{\rm s},$$

where

$$\varphi_{\rm f} = 2\pi\frac{4-\xi}{8-3\xi}\frac{\sqrt{2}\,w}{\lambda}, \quad \varphi_{\rm s} = -2\pi\frac{4-2\xi}{8-3\xi}\frac{\sqrt{2}\,w}{\lambda} \quad (29)$$

are the phase angles [23] depending on the oppositely directed displacements w of the spherical mirrors M_3 and M_4.

Now the resulting expressions (28) must be supplemented with the calculation formula for $h_{\Delta T}$. On the one hand, in the case of $h_{\rm PCS} = 0$ (PCS is switched off) it follows from (22) that the thermal expansion of the perimeter caused by the temperature increment ΔT of the monoblock is $4\sqrt{2}\,h_{\Delta T}$. On the other hand, the same value is equal to $q\lambda K_{\rm TE}\Delta T$. Thus, $4\sqrt{2}\,h_{\Delta T} = q\lambda K_{\rm TE}\Delta T$, which, with $\Delta T = T - T_{q\lambda}$, yields

$$h_{\Delta T} = (\sqrt{2}/8)q\lambda K_{\rm TE}(T - T_{q\lambda}). \quad (30)$$

As a result, substituting (30) into (28), we finally obtain

$$\varphi_1 = \pi k - \pi q K_{\rm TE}(T - T_{q\lambda}) + \varphi_{\rm f}, \quad \varphi_2 = \pi(q+k) + \varphi_{\rm f},$$

$$\varphi_3 = \varphi_{\rm s}, \quad \varphi_4 = \pi q - \pi q K_{\rm TE}(T - T_{q\lambda}) + \varphi_{\rm s}. \quad (31)$$

Formulas (31) are the desired relations. The current temperature T of the laser gyro monoblock is determined by (11).

To use relations (31) for modelling the dynamics of the synchronisation zone parameters of the counterpropagating wave frequencies of the laser gyro in question, we must first specify (or, more precisely, calculate) two characteristic values of the parameter q. This can be done by using the formulas

$$q = q_{\rm floor} = {\rm floor}(L/\lambda), \quad q = q_{\rm ceil} = {\rm ceil}(L/\lambda), \quad (32)$$

where $q_{\rm floor}$ and $q_{\rm ceil}$ are two integers of the parameter q differing by unity. One of them during the calculations will be even/odd, and the other, on the contrary, odd/even. [The MATLAB function floor(x) of a real argument x returns the value rounded to the nearest integer $x_1 \leqslant x$, while the function ceil(x) acquires the value rounded to the nearest integer $x_2 \geqslant x$.] The obtained values of the parameter q should be then substituted into formulas (31). Note that the simulation results in the first and second cases will be qualitatively different.

Finally, with respect to the parameter k in (31): Its value ($k = 0, \pm 1, \pm 2, \ldots$) should be chosen so that to ensure the fulfilment of inequality (13) at the initial temperature $T_{\rm ini}$ of the laser gyro monoblock.

Thus, formulas (31) obtained in this section for calculating values of φ_n together with expressions (3), (4), (7), (8), (11), (13) and (32) form a mathematical model that allows the modelling of the dynamics of the synchronisation zone parameters $\Omega_{(-)}$, $\Omega_{(+)}$, $\Omega_{(0)}$, $\Omega_{\rm s}$ of the counterpropagating wave frequencies of the laser gyro during its operation in the self-heating regime.

5. Examples of modelling the dynamics of the half-width of the synchronisation zone of counterpropagating wave frequencies in the laser gyro under study

Unfortunately, limitations on the volume of the article do not allow us to present in full the results of modelling of the dynamics of all four synchronisation zone parameters [$\Omega_{(-)}$, $\Omega_{(+)}$, $\Omega_{(0)}$, and $\Omega_{\rm s}$] of the counterpropagating wave frequencies of the laser gyro in question, i.e., for all possible quantities appearing in (31). Therefore, we consider only the simplest case when oppositely directed controlled displacements w of the mirrors M_3 and M_4 are absent ($w = 0$). To this end we assume $\varphi_{\rm f} = \varphi_{\rm s} = 0$ in (31). In this situation, the linear coupling of counterpropagating waves during the laser gyro operation in the self-heating regime will always remain symmetric ($r_1 = r_2$) and the displacement $\Omega_{(0)}$ of the centre of the synchronisation zone of the frequencies of these waves along the axis of the angular velocity Ω, as follows from (3), will be zero. Thus, the problem reduces to the modelling of the dynamics of only one synchronisation zone parameter– its half-width $\Omega_{\rm s}$.

In addition, with the aim of further simplification of the problem, we confine ourselves to a single case (of two possible) when, for example, an even number of wavelengths λ is fitted on the axial contour perimeter of the resonator at a basic temperature $T_{q\lambda}$ of the laser gyro monoblock. By using (32) with L = 200 mm we have $q_{\rm floor}$ = 316055 and $q_{\rm ceil}$ = 316056. Therefore, we should use $q = q_{\rm ceil}$ = 316056 in (31).

Now, to start modelling the dynamics of $\Omega_{\rm s}$, we must specify (and for the convenience of the reader gather in one place) numerical values of all variables used in the calculations:

1. Formulas (3) involve the parameter M. For the laser gyro in question it equals 496459.

2. Expressions (4) contain the parameter α_m. As found above, the resonator losses Γ of the gyro are 400×10^{-6}. The device operates at five-fold excess of the gain g over the losses Γ, i.e., $g = 5\Gamma = 2000\times10^{-6}$. Therefore, by using (4) with α_p = 2400000 s^{-1} and $h = 0.652$, we have α_m = 505810 s^{-1}.

3. In relations (8) $a_f = 1.15\times10^{-6}$, $a_s = 1.72\times10^{-6}$, χ_f = 461″, χ_s = 652″, $b_f = 5.91\times10^{-8}$, $b_s = 2.72\times10^{-8}$.

4. Finally, in formulas (11), (13) and (31) $T_{q\lambda}$ = 25 °C, ΔT_λ = 50 °C, $K_{TE} = 6.328\times10^{-8}$ 1/°C, ΔT_{SH}^{max} = 7 °C, τ_{SH} = 2400 s.

Figure 1 shows the results of modelling the dynamics of the synchronisation zone half-width Ω_s of the counterpropagating frequencies of the laser gyro in question when it worked five times during four hours at the initial monoblock temperatures T_{ini} = 5, 15, 25, 35 and 45 °C from the interval 1 – 50 °C. To ensure that inequality (13) is met at the mentioned T_{ini}, the parameter k in formulas (31) was chosen to be zero. This means that in all five tests the device, every time after its switching on, worked on the same 'even' longitudinal mode (because its resulting index $q + k$ = 316056 is an even number).

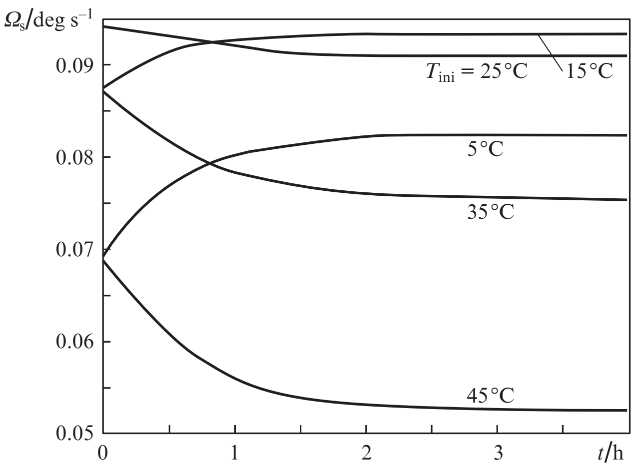

Figure 1. Results of modelling of the dynamics of the synchronisation zone half-width Ω_s of counterpropagating wave frequencies of the laser gyro during its five four-hour starts at the initial temperatures T_{ini} of the monoblock in the interval 1 – 50 °C for the case k = 0, q = 316056.

The results of modelling the Ω_s dynamics, when the laser gyro worked five times during four hours at the initial monoblock temperatures T_{ini} = 55, 65, 75, 85 and 95 °C from the interval 51 – 100 °C are presented in Fig. 2. With such values of T_{ini} the parameter k in (31) was taken equal to unity. Therefore, in these tests the gyro, every time after its switching on, worked on the same 'odd' longitudinal mode (because its resulting index $q + k$ = 316057 is an odd number).

From these graphs it is clear that the synchronisation zone half-width Ω_s of the counterpropagating wave frequencies of the laser gyro in question during its operation in the self-heating regime changes over time. The character of the Ω_s dynamics is determined by parameter K_{TE}, the law $T = T(t)$ of the increase in the laser gyro monoblock temperature and, most significantly, by the initial value T_{ini} at time t = 0 when the device is switched on. These circumstances should, apparently, be taken into account when developing the method for experimental assessment of Ω_s at the stage of certification of the laser gyro parameters.

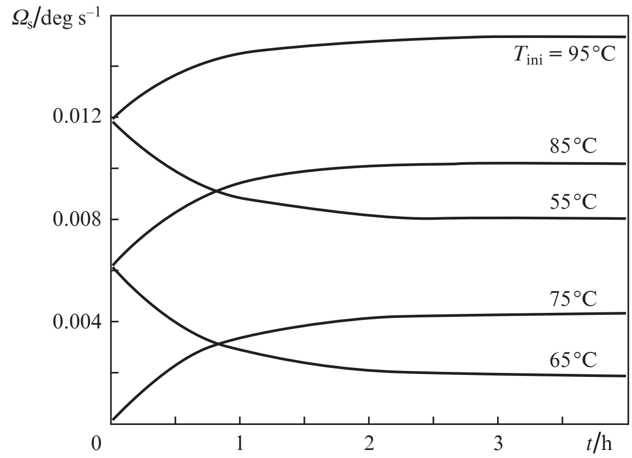

Figure 2. Results of modelling of the dynamics of the synchronisation zone half-width Ω_s of counterpropagating wave frequencies of the laser gyro during its five four-hour starts at the initial temperatures T_{ini} of the monoblock in the interval 51 – 100 °C for the case k = 1, q = 316056.

The results of modelling suggest that the measurement of the synchronisation zone half-width Ω_s of the counterpropagating wave frequencies of the laser gyro in question should be carried out in a wide range of temperatures (on a controlled uniaxial rotating bench placed in a heat chamber). The most reliable estimate of Ω_s should be its maximum measured value. The Ω_s for any given stabilised value of the air temperature in the chamber should be measured immediately after switching on the laser gyro, rather than after termination of thermal transients occurring in it.

6. Conclusions

In this paper, for a laser gyro with a four-mirror square resonator we have obtained formulas (31), which together with (3), (4), (7), (8), (11), (13) and (32) form a mathematical model that allows one to simulate the temporal behaviour of the synchronisation zone parameters of the frequencies of counterpropagating waves in a situation when the device operates in the self-heating regime, and is switched on at different initial temperatures.

References

1. Chow W.W., Gea-Banacloche J., Pedrotti L.M., Sanders V.E., Schleich W., Scully M.O. *Rev. Mod. Phys.*, **57**, 61 (1985).
2. Wilkinson J.R. *Prog. Quantum Electron.*, **11**, 1 (1987).
3. Aronowitz F., in *Optical Gyros and their Application* (Neuilly-sur-Seine, France, RTO AGARDograph 339, 1999) p. 3-1.
4. Menegozzi L.N., Lamb W.E. Jr. *Phys. Rev.*, **8**, A2103 (1973).
5. Bondarenko E.A. *Kvantovaya Elektron.*, **41**, 824 (2011) [*Quantum Electron.*, **41**, 824 (2011)].
6. Landa P.S., Lariontsev E.G. *Radiotekhn. Elektron.*, **15**, 1214 (1970).
7. Andronova I.A., Bershtein I.L. *Izv. Vyssh. Uchebn. Zaved., Ser. Radiofiz.*, **14**, 698 (1971).
8. Landa P.S. *Opt. Spektrosk.*, **32**, 383 (1972).
9. Zeiger S.G., Klimontovich Yu.L., Landa P.S., Lariontsev E.G., Fradkin E.E. *Volnovye i fluktuatsionnye protsessy v lazerakh* (Wave and Fluctuation Processes in Lasers) (Moscow: Nauka, 1974).

10. Landa P.S. *Avtokolebaniya v sistemakh s konechnym chislom stepenei svobody* (Self-oscillations in Systems with a Finite Number of Degrees of Freedom) (Moscow: Nauka, 1980).
11. Rodloff R. *IEEE J. Quantum Electron.*, **QE-23**, 438 (1987).
12. Khromykh A.M. *Elektron. Tekhn., Ser. Lazer Tekhn. Optoelektron.*, **2** (54), 44 (1990).
13. Azarova V.V., Golyaev Yu.D., Dmitriev V.G. *Kvantovaya Elektron.*, **30**, 96 (2000) [*Quantum Electron.*, **30**, 96 (2000)].
14. Birman A.Ya., Petrukhin E.A., Savushkin A.F. *Kvantovaya Elektron.*, **6**, 2626 (1979) [*Sov. J. Quantum Electron.*, **9**, 1557 (1979)].
15. Astakhov K.V., Golyaev Yu.D., Makhin P.V., Mel'nikov A.V., Tikhmenev N.V. *Giroskopiya i navigatisya*, No. 2 (9), 25 (1995).
16. Astakhov K.V., Batovrin V.K., Golyaev Yu.D., Drozdov M.S., Mel'nikov A.V. *Giroskopiya i navigatisya*, No. 4 (11), 24 (1995).
17. Astakhov K.V., Batovrin V.K., Golyaev Yu.D., Drozdov M.S., Mel'nikov A.V., Tikhmenev N.V., Yasnov S.A. *Trudy IV Sankt-Peterburgskoi Mezhd. Konf. po Integrirovannym Navigatsionnym Sistemam* (Proc. IV St. Petersburg Int. Conf. on Integrated Navigation Systems (St. Petersburg, 1997) pp 146–152.
18. Astakhov K.V., Batovrin V.K., Mel'nikov A.V., Golyaev Yu.D., Tikhmenev N.V. *Pribory i sistemy upravleniya*, No. 3, 15 (1997).
19. Astakhov K.V., Batovrin V.K., Golyaev Yu.D., Drozdov M.S., Mel'nikov A.V., Tikhmenev N.V. *Trudy V Sankt-Peterburgskoi Mezhd. Konf. po Integrirovannym Navigatsionnym Sistemam* (Proc. V St. Petersburg Int. Conf. on Integrated Navigation Systems (St. Petersburg, 1998) pp 194–200.
20. Molchanov A.V., Morozov D.A., Stepanov A.Yu., Chirkin M.V. *Trudy XIV Sankt-Peterburgskoi Mezhd. Konf. po Integrirovannym Navigatsionnym Sistemam* (Proc. XIV St. Petersburg Int. Conf. on Integrated Navigation Systems (St. Petersburg, 2007) pp. 38–40.
21. Vasin I.A., Molchanov A.V., Morozov D.A., Chirkin M.V. *Trudy XV Sankt-Peterburgskoi Mezhd. Konf. po Integrirovannym Navigatsionnym Sistemam* (Proc. XV St. Petersburg Int. Conf. on Integrated Navigation Systems (St. Petersburg, 2008) pp. 68–70.
22. Molchanov A.V., Stepanov A.Yu., Chirkin M.V. *Aviakosm. Priborostroen.*, No. 3, 9 (2008).
23. Bondarenko E.A. *Kvantovaya Elektron.*, **42**, 465 (2012) [*Quantum Electron.*, **42**, 465 (2012)].
24. Ishchenko E.F. *Zh. Prikl. Spektrosk.*, **11**, 456 (1969).
25. Ishchenko E.F. *Otkrytye opticheskie rezonatory* (Open Optical Resonators) (Moscow: Sov. Radio, 1980).
26. Bondarenko E.A., in *Mechanics of Gyroscopic Systems* (Kiev, KPI, 2010, Issue 22) pp 22–32
27. Bondarenko E.A. *Kvantovaya Elektron.*, **19**, 171 (1992) [*Quantum Electron.*, **22**, 154 (1992)].