INTEGRATED OPTICAL WAVEGUIDES

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Method of adiabatic modes in research of smoothly irregular integrated optical waveguides: zero approximation

A.A. Egorov, L.A. Sevast'yanov, A.L. Sevast'yanov

Abstract. **We consider the application of the method of adiabatic waveguide modes for calculating the propagation of electromagnetic radiation in three-dimensional (3D) irregular integrated optical waveguides. The method of adiabatic modes takes into account a three-dimensional distribution of quasi-waveguide modes and explicit ('inclined') tangential boundary conditions. The possibilities of the method are demonstrated on the example of numerical research of two major elements of integrated optics: a waveguide of 'horn' type and a thin-film generalised waveguide Luneburg lens by the methods of adiabatic modes and comparative waveguides.**

Keywords: *integrated optical waveguides, smoothly irregular waveguides, method of adiabatic waveguide modes, method of comparative waveguides, dispersion relation, numerical modelling.*

1. Introduction

In our previous works [1 – 11], with the asymptotic method and method of coupled waves we obtained analytical expressions for fields of deforming waves of a four-layer smoothly irregular integrated optical three-dimensional waveguide in the zero- and first-order approximations of the perturbation theory. Our consideration is based on a solution presented in the form of finite asymptotical series known as the adiabatic approximation. The quasi-wave equations obtained from theoretical consideration are solved in the zero- and first-order approximations of the asymptotic method [1 – 3]. According to the theoretical analysis, the modes of a smoothly irregular integrated optical waveguide are weakly hybridised quasi-TE and quasi-TM modes. Retaining the summands proportional to a dielectric function gradient in the boundary conditions and in the solution of quasi-waveguide equations makes allowance for a vector character of propagation of a monochromatic electro-magnetic field along smoothly irregular domains of a multilayer multimode integrated optical waveguide. It was established that in such consideration, shifts of complex propagation constants arise in weakly coupled quasi-TM and quasi-TE modes. It was found that at real (and positive) values of the dielectric function and magnetic susceptibility of smoothly irregular waveguide medium these shifts are purely imaginary and distinct for different quasi-waveguide modes [1 – 3].

Our consideration is based on the method of short-wavelength asymptotics [12, 13] with a solution U presented in the form of the asymptotic series

$$U \sim \sum_m u_m / k_0^m,$$

with terms proportional to $k_0^{-m} = (2\pi/\lambda)^{-m}$, where λ is the wavelength of monochromatic light in vacuum and k_0 is the modulus of the wave vector $\boldsymbol{k}_0$. In the visible wavelength range we have $\lambda \to 0$ ($k_0 \to \infty$), which suggests employment of the solution in the form of finite asymptotic series known as the adiabatic approximation [1 – 14].

The method of short-wavelength asymptotics was first formulated by Debye for solving the scalar wave equation in the form

$$u(x,\lambda) = \exp[\mathrm{i}\lambda S(x)] \sum_m (\mathrm{i}\lambda)^{-m} \varphi_m(x),$$

where $S(x)$ and $\varphi_j(x)$ are certain functions. The idea of Debye was then developed for solving Maxwell's equations [13]. The approximation in which the zero- and first-order terms are retained in this series is called 'adiabatic approximation' in the literature on theoretical and mathematical physics and 'approximation of geometrical optics' (the geometrical diffraction theory) in works on electromagnetic (optical) diffraction (see, for example, [15]).

Maslov substantially improved the asymptotic method for solving problems in mathematical physics (especially the Schrödinger equation) in a number of papers; the corresponding results are presented, for example, in monograph [16]. This approach describes the classes of solutions corresponding to spatial waves propagating in a nonuniform space. Nevertheless, there are also other important classes of waves which do not coincide with the spatial waves. In particular, integrated optics and waveguide optoelectronics study the so-called surface waves. Such waves cannot be directly described by the methods of Debye or Maslov. However, these methods were modified by the authors of the present work. The modification suggests a partial separation of variables (see, for example, [3, 10]) based on averaging the adiabatic solution over fast variables [3] with a following 'asymptotic' continuation in those variables (by analogy with the method of averaging [17]). Note that the Fourier method used for separating variables in regular waveguides is not applicable in our case.

A.A. Egorov A.M. Prokhorov General Physics Institute, Russian Academy of Sciences, ul. Vavilova 38, 119991 Moscow, Russia; e-mail: yegorov@kapella.gpi.ru;
L.A. Sevast'yanov Peoples' Friendship University of Russia, ul. Miklukho-Maklaya 6, 117198 Moscow, Russia; Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia; e-mail: sevast@sci.pfu.edu.ru;
A.L. Sevast'yanov Peoples' Friendship University of Russia, ul. Miklukho-Maklaya 6, 117198 Moscow, Russia; e-mail: alsevastyanov@gmail.com

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In addition, the known method of spectral decomposition of field over a complete system of guided (eigen) modes and radiation modes of a regular waveguide [18–24] (or the mode method in [22]) is not applicable for finding a solution because the propagation constant in this case is a complex value which brings a problem of orthogonality for corresponding modes. Note that in contrast to approximate methods of geometrical optics and geometrical diffraction we search for (and find) solutions for Maxwell's equations that are explicit equations of electromagnetic theory for wave propagation and diffraction. The two-dimensional ('tracing') rays involved in our constructed 'adiabatic waveguide modes' constitute an averaged (energy) frame of 'asymptotic decomposition' satisfying electrodynamic equations rather than geometrical optical equations (for details see [1, 3, 10]).

In the present work, possibilities of our method of adiabatic modes are demonstrated by the example of the numerical study of two most important elements of integrated optics, namely, a waveguide of 'horn' type and a thin-film generalised waveguide Luneburg lens (TGWLL). Recall that this is the lens in which the focal sphere does not coincide with its surface and the image plane is separated from lens centre by a distance greater than its radius. A TGWLL is a planar analogue [1–11, 18, 19] of the spatial generalised Luneburg lens [20]. An integrated optical waveguide of 'horn' type is the waveguide with widening. Such a waveguide can be obtained, for example, by gradually thickening the waveguide layer along the appropriate axis.

2. General statement of the problem

Propagation of guided and quasi-guided modes in smoothly irregular 'horn' type and TGWLL waveguides (Fig. 1) are described by Maxwell's equations, which for a nonabsorbing nonuniform and isotropic medium with lacking sources can be written in the SI system in the following form [21, 22]:

$$\mathrm{rot}\boldsymbol{H} = \varepsilon\frac{\partial \boldsymbol{E}}{\partial t}, \quad \mathrm{rot}\boldsymbol{E} = -\mu\frac{\partial \boldsymbol{H}}{\partial t}. \tag{1}$$

In a multilayer integrated optical waveguide the electromagnetic field that is a solution to system (1) should satisfy the tangential boundary conditions at the interfaces between layers $\boldsymbol{E}^{\tau}|_1 = \boldsymbol{E}^{\tau}|_2$, $\boldsymbol{H}^{\tau}|_1 = \boldsymbol{H}^{\tau}|_2$.

Solutions to Maxwell's equations (1) will be searched for in the form [1–5]:

$$\begin{Bmatrix}\boldsymbol{E}(x,y,z,t)\\ \boldsymbol{H}(x,y,z,t)\end{Bmatrix} = \begin{Bmatrix}\boldsymbol{E}(x;y,z)\\ \boldsymbol{H}(x;y,z)\end{Bmatrix}\frac{\exp[\mathrm{i}\omega t - \mathrm{i}\varphi(y,z)]}{\beta^{1/2}(y,z)}, \tag{2}$$

where

$$\beta_y(y,z) = k_0^{-1}\frac{\partial\varphi}{\partial y}; \quad \beta_z(y,z) = k_0^{-1}\frac{\partial\varphi}{\partial z};$$

$$\beta(y,z) = k_0^{-1}\left[\left(\frac{\partial\varphi}{\partial y}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2\right]^{1/2}; \quad k_0 = \frac{\omega}{c};$$

ω is the angular frequency of monochromatic radiation; and c is the velocity of light. The eikonal (phase)

$$\varphi(y,z) = k_0\int^{y,z}\beta(y',z')\mathrm{d}s(y',z')$$

in (2) is found by integrating along rays after the dispersion relationship has been solved and the rays and wave fronts in the horizontal plane have been separately calculated; here $\mathrm{d}s = \sqrt{\mathrm{d}y^2 + \mathrm{d}z^2}$ is the ray length element.

The dispersion relationships for the TGWLL were obtained by Southwell [18] in the approximation where 'inclined' tangential boundary conditions were substituted for their projections onto the horizontal plane. The allowance made for the conditions $|\partial h/\partial y| \neq 0$, $|\partial h/\partial z| \neq 0$ introduces into the Southwell relationships a small correction with respect to the parameter δ, where $\delta = \max|\nabla_{y,z}\beta|(k_0\beta^2)^{-1}$.

While solving a system of ordinary differential equations for adiabatic waveguide modes with the help of the parameter δ [1–5] which is a two-dimensional analogue of the value $|\nabla\varepsilon/\varepsilon|$, coupled quasi-waveguide modes arise in a 3D smoothly irregular waveguide [1–11] which are similar to hybrid modes and have six field components in contrast to three components of the TE- and TM-modes [21, 22]. In the general case, the conditions $\partial\boldsymbol{E}/\partial y \equiv 0$, $\partial\boldsymbol{H}/\partial y \equiv 0$ are not satisfied for these modes, i.e., there exist field variations in the y axis direction.

The Cartesian coordinate system is oriented as follows. The x axis crosses waveguide layers and is perpendicular to the layers in regular domains, whereas the y and z axes are parallel to the layers at these domains. Guided and quasi-guided modes of a waveguide propagate along the z axis. A

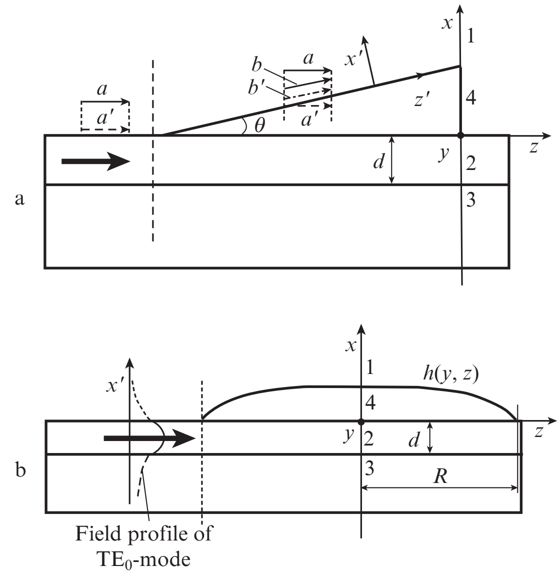

Figure 1. Transverse cross sections of investigated waveguide structures of (a) 'horn' and (b) TGWLL types. A three-layer regular integrated optical waveguide is formed by the surrounding medium or covering layer (air) with the refractive index n_c (1), the first waveguide layer (a regular part of integrated optical structure) with the refractive index n_f (2), the substrate with the refractive index n_s (3), and the second waveguide layer (irregular part of the integrated optical structure) with the refractive index n_{layer} (4). The propagation direction of the TE- or TM-mode is shown by a bold arrow. A component of the wave vector field (solid line a with arrow) and its projection onto the horizontal plane (dashed line a') are shown in the left side of Fig. 1a. In the right side of the figure are shown: the same component of the wave vector field (solid line a with the arrow), its projection onto the horizontal plane (dashed line a' with the arrow), the real component of the wave vector field on the inclined plane (solid line b with the arrow) and its projection onto the inclined plane (dot-and-dash line b' with the arrow). Here, the laboratory (auxiliary) coordinate system x', y', z' is used; $h(z)$ is a thickness of the layer forming the wedge. In Fig. 1b: $h(y,z)$ is the thickness of the layer forming the Luneburg lens and R is the radius of lens aperture.

substitution of (2) into (1) gives after certain transformations the following results.

For the longitudinal components $E_z(x;y,z)$, $H_z(x;y,z)$ we obtain the second-order equations

$$\frac{d^2E_z}{dx^2}+\chi^2E_z=\frac{\partial\ln\chi_z^2}{\partial y}\left(p_yE_z+\frac{p_z}{ik_0\varepsilon}\frac{dH_z}{dx}\right), \tag{3}$$

$$\frac{d^2H_z}{dx^2}+\chi^2H_z=\frac{\partial\ln\chi_z^2}{\partial y}\left(p_yH_z-\frac{p_z}{ik_0\mu}\frac{dE_z}{dx}\right). \tag{4}$$

By using the longitudinal components $E_z(x;y,z)$, $H_z(x;y,z)$ and their derivatives we obtain the field components $E_x(x;y,z)$, $E_y(x;y,z)$, $H_x(x;y,z)$, and $H_y(x;y,z)$:

$$\chi_z^2H_y=\left(p_yp_z+\frac{\partial p_z}{\partial y}\right)H_z-ik_0\varepsilon\frac{dE_z}{dx},\qquad \chi_z^2H_x=p_z\frac{dH_z}{dx}+ik_0\varepsilon p_yE_z, \tag{5}$$

$$\chi_z^2E_y=\left(p_yp_z+\frac{\partial p_z}{\partial y}\right)E_z+ik_0\mu\frac{dH_z}{dx},\qquad \chi_z^2E_x=p_z\frac{dE_z}{dx}-ik_0\mu p_yH_z. \tag{6}$$

In relationships (3)–(6) we used the notations

$$\chi_z^2=k_0^2\varepsilon\mu+p_zp_z+\frac{\partial p_z}{\partial z},\quad \chi^2=\chi_z^2+p_yp_y+\frac{\partial p_y}{\partial y},$$

$$p_y=-ik_0\beta_y-(2\beta)^{-1}\frac{\partial\beta}{\partial y},\quad p_z=-ik_0\beta_z-(2\beta)^{-1}\frac{\partial\beta}{\partial z},$$

in order to represent the derivatives of the field strengths E_m (here $m=x,y,z$) as follows:

$$\frac{\partial E_m}{\partial y}=p_yE_m,\quad \frac{\partial^2E_m}{\partial y^2}=\left(p_yp_y+\frac{\partial p_y}{\partial y}\right)E_m,$$

$$\frac{\partial E_m}{\partial z}=p_zE_m,\quad \frac{\partial^2E_m}{\partial z^2}=\left(p_zp_z+\frac{\partial p_z}{\partial z}\right)E_m.$$

Derivatives of the field strengths H_m are written in a similar way.

In the case of smooth irregularities under consideration (that is, at $\delta<1$) in the zero-order approximation with respect to the small parameter δ the following equations are valid for the longitudinal components

$$\frac{d^2E_z^0}{dx^2}+k_0^2(\varepsilon\mu-\beta^2)E_z^0=0, \tag{7}$$

$$\frac{d^2H_z^0}{dx^2}+k_0^2(\varepsilon\mu-\beta^2)H_z^0=0, \tag{8}$$

and for the transverse and vertical components the following differential expressions can be written:

$$H_y^0=\frac{1}{k_0^2(\varepsilon\mu-\beta_z^2)}\left(-k_0^2\beta_y\beta_zH_z^0-ik_0\varepsilon\frac{dE_z^0}{dx}\right), \tag{9}$$

$$E_x^0=\frac{1}{k_0^2(\varepsilon\mu-\beta_z^2)}\left(-ik_0\beta_z\frac{dE_z^0}{dx}-k_0^2\mu\beta_yH_z^0\right), \tag{10}$$

$$E_y^0=\frac{1}{k_0^2(\varepsilon\mu-\beta_z^2)}\left(ik_0\mu\frac{dH_z^0}{dx}+k_0^2\beta_y\beta_zE_z^0\right), \tag{11}$$

$$H_x^0=\frac{1}{k_0^2(\varepsilon\mu-\beta_z^2)}\left(-ik_0\beta_z\frac{dH_z^0}{dx}+k_0^2\varepsilon\beta_yE_z^0\right). \tag{12}$$

In the present work we consider the integrated optical waveguide structures shown in Fig. 1. A substrate [resides in the range $I_s=\{(x,y,z):x\in(-\infty,-d];\,y,z\in(-\infty,+\infty)\}$] made of the material with the refractive index n_s is covered by the main waveguide layer [in the range $I_f=\{(x,y,z):x\in[-d,0];\,y,z\in(-\infty,+\infty)\}$] of thickness d made of the material with the refractive index $n_f\geqslant n_s$. The main waveguide layer is additionally covered by the layer with the refractive index $n_{layer}\geqslant n_s$ of variable thickness $x=h(z)$ or $x=h(y,z)$ (the domain of non-zero thickness is limited by the yz plane). The additional waveguide layer resides in the domain $I_{layer}=\{(x,y,z):x\in[0,h(z)];\,y,z\in(-\infty,+\infty)\}$. A coating cover made of the material with the refractive index $n_c\leqslant n_s$, n_f, n_{layer} is above all those layers. This cover (air in our case) resides in the range $I_c=\{(x,y,z):x\in[h(z),+\infty);\,y,z\in(-\infty,+\infty)\}$.

At each of the interfaces between two media (boundaries between ranges I_j) the following tangential boundary conditions hold true

$$\boldsymbol{E}^\tau|_{-d-0}=\boldsymbol{E}^\tau|_{-d+0},\quad \boldsymbol{H}^\tau|_{-d-0}=\boldsymbol{H}^\tau|_{-d+0},\qquad \boldsymbol{E}^\tau|_{-0}=\boldsymbol{E}^\tau|_{+0},\quad \boldsymbol{H}^\tau|_{-0}=\boldsymbol{H}^\tau|_{+0}, \tag{13}$$

$$\boldsymbol{E}^\tau|_{h(z)-0}=\boldsymbol{E}^\tau|_{h(z)+0},\quad \boldsymbol{H}^\tau|_{h(z)-0}=\boldsymbol{H}^\tau|_{h(z)+0}. \tag{14}$$

In addition, the following boundary conditions hold true at infinity

$$|\boldsymbol{E}^\tau{}_{|x\to\pm\infty}|<+\infty,\quad |\boldsymbol{H}^\tau{}_{|x\to\pm\infty}|<+\infty.$$

For conditions (13) the tangent planes to the interfaces are horizontal; hence, system (13) can be reduced to independent subsystems for the TE- and TM-eigenmodes. For conditions (14) these planes in the general case are not horizontal and $\partial h/\partial y\neq0$; hence, the tangential field components in the general case are linear combinations comprising all three Cartesian components of fields with nontrivial coefficients. This circumstance prevents the boundary condition system (14) from separating to two independent subsystems for the TE- and TM-modes, which results in 'coupled' polarisations of quasi-waveguide modes propagating in an irregular integrated optical waveguide.

3. Explicit tangential boundary conditions in the method of adiabatic waveguide modes

Boundary conditions (13) for the horizontal planes of interfaces between layers in the component notations take the form:

$$H_z(x)|_{x=-d-0}=H_z(x)|_{x=-d+0},\ E_y(x)|_{x=-d-0}=E_y(x)|_{x=-d+0},$$

$$E_z(x)|_{x=-d-0}=E_z(x)|_{x=-d+0},\ H_y(x)|_{x=-d-0}=H_y(x)|_{x=-d+0},$$

$$H_z(x)|_{x=-0} = H_z(x)|_{x=+0},\; E_y(x)|_{x=-0} = E_y(x)|_{x=+0},$$

$$E_z(x)|_{x=-0} = E_z(x)|_{x=-d+0},\; H_y(x)|_{x=-0} = H_y(x)|_{x=+0}.$$

Among the three components of the tangential field $\boldsymbol{E}^{\tau(0)}$ in boundary conditions (14) only the components $E_y^{\tau(0)}$ and $E_z^{\tau(0)}$ are linearly independent. Similarly, among the three components of the magnetic field only the components $H_y^{\tau(0)}$ and $H_z^{\tau(0)}$ are linearly independent. Hence, it suffices to write out boundary conditions (14) only for these components.

Consider conditions (14) in more detail. At the point $(h(y,z),y,z)^T$ of the interface $x = h(y,z)$ the tangent plane is given by the expression $\mathrm{d}x - (\partial h/\partial y)\mathrm{d}y - (\partial h/\partial z)\mathrm{d}z = 0$.

The components of electric and magnetic field strengths $[\boldsymbol{E}^{\tau(0)})$ and $(\boldsymbol{H}^{\tau(0)}]$, tangential to this plane, in the component notation are as follows:

$$E_y^{\tau(0)} = \frac{\dfrac{\partial h}{\partial y}E_x^{(0)} + \left[1+\left(\dfrac{\partial h}{\partial z}\right)^2\right]E_y^{(0)} - \dfrac{\partial h}{\partial y}\dfrac{\partial h}{\partial z}E_z^{(0)}}{1+\left(\dfrac{\partial h}{\partial y}\right)^2+\left(\dfrac{\partial h}{\partial z}\right)^2},$$

$$E_z^{\tau(0)} = \frac{\dfrac{\partial h}{\partial z}E_x^{(0)} - \dfrac{\partial h}{\partial y}\dfrac{\partial h}{\partial z}E_y^{(0)} + \left[1+\left(\dfrac{\partial h}{\partial y}\right)^2\right]E_z^{(0)}}{1+\left(\dfrac{\partial h}{\partial y}\right)^2+\left(\dfrac{\partial h}{\partial z}\right)^2}, \tag{15}$$

$$H_y^{\tau(0)} = \frac{\dfrac{\partial h}{\partial y}H_x^{(0)} + \left[1+\left(\dfrac{\partial h}{\partial z}\right)^2\right]H_y^{(0)} - \dfrac{\partial h}{\partial y}\dfrac{\partial h}{\partial z}H_z^{(0)}}{1+\left(\dfrac{\partial h}{\partial y}\right)^2+\left(\dfrac{\partial h}{\partial z}\right)^2},$$

$$H_z^{\tau(0)} = \frac{\dfrac{\partial h}{\partial z}H_x^{(0)} - \dfrac{\partial h}{\partial y}\dfrac{\partial h}{\partial z}H_y^{(0)} + \left[1+\left(\dfrac{\partial h}{\partial y}\right)^2\right]H_z^{(0)}}{1+\left(\dfrac{\partial h}{\partial y}\right)^2+\left(\dfrac{\partial h}{\partial z}\right)^2}. \tag{16}$$

Thus, each of the tangential components of the electric field is contributed by all the three electric field components, that is, by both polarisations. Similarly behave the components of the magnetic field; thus, strict boundary conditions in the method of adiabatic modes, in contrast to the method of cross-sections [21, 23, 24], take into account 'coupling' of two polarisations actually arising in the inclined domains of the waveguide. In the smoothly irregular domain of a waveguiding structure with inclined incidence of waves (onto the interfaces between media comprising the waveguide) the polarisations are coupled and a linearly polarised mode (modes) is transformed to the mode with coupled state, the complicated polarisation structure of which can be found by solving a system of equations and relationships combining both polarisations. Note that this nonclassical state differs from the state of a classical hybrid mode which also has six field components because the classical hybrid mode is the same in a regular optical waveguide as well. When the TE- or TM- eige mode with a linear polarisation falls from a regular domain of the integrated optical waveguide to an irregular domain, it becomes the mode with a coupled state having an alternating imaginary shift in the propagation constant.

In the case of a 'horn' waveguide, the nonhorizontal part of the interface (wedge) varies with z, but is independent of y. Hence,

$$\beta_y = 0,\quad \beta_z = \beta,\quad \frac{\partial h}{\partial y} = 0,\quad \frac{\partial h}{\partial z} = \tan\theta, \tag{17}$$

where θ is the inclination angle of the nonhorizontal plane of the horn (wedge-shaped domain) relative to the horizontal plane yz.

In this case, Eqns (7), (8) take the form

$$\frac{\mathrm{d}^2E_z^0}{\mathrm{d}x^2} + k_0^2(\varepsilon\mu - \beta^2)E_z^0 = 0,$$
$$\frac{\mathrm{d}^2H_z^0}{\mathrm{d}x^2} + k_0^2(\varepsilon\mu - \beta^2)H_z^0 = 0, \tag{18}$$

and relationships (9)–(12) are as follows:

$$H_y^0 = -\frac{\mathrm{i}k_0\varepsilon}{\chi^2}\frac{\mathrm{d}E_z^0}{\mathrm{d}x},\quad E_x^0 = -\frac{\mathrm{i}k_0\beta}{\chi^2}\frac{\mathrm{d}E_z^0}{\mathrm{d}x}, \tag{19}$$

$$E_y^0 = \frac{\mathrm{i}k_0\mu}{\chi^2}\frac{\mathrm{d}H_z^0}{\mathrm{d}x},\quad H_x^0 = -\frac{\mathrm{i}k_0\beta}{\chi^2}\frac{\mathrm{d}H_z^0}{\mathrm{d}x}. \tag{20}$$

In view of expressions (18)–(20) in the waveguide of 'horn' type the boundary conditions at the nonhorizontal interface are fulfilled separately for different modes, but each mode acquires a rotation of the polarisation plane.

For the TM-modes we have

$$H^{\tau(0)} = H_y^{(0)},\quad E_z^{\tau(0)} = \left(\frac{\partial h}{\partial z}E_x^{(0)} + E_z^{(0)}\right)\left[1+\left(\frac{\partial h}{\partial z}\right)^2\right]^{-1},$$

that is

$$E_z^{\tau(0)} = \frac{\tan\theta E_x^{(0)} + E_z^{(0)}}{1+\tan^2\theta}.$$

Similarly, for the TE-modes

$$E_y^{\tau(0)} = E_y^{(0)},\quad H_z^{\tau(0)} = \left(\frac{\partial h}{\partial z}H_x^{(0)} + H_z^{(0)}\right)\left[1+\left(\frac{\partial h}{\partial z}\right)^2\right]^{-1},$$

that is

$$H^{\tau(0)} = \frac{\tan\theta H_x^{(0)} + H_z^{(0)}}{1+\tan^2\theta}.$$

Similar equations for the case of the TGWLL were derived in our papers [1–5], and so we will only make a brief comment. In this case the nonhorizontal part of the interface (the upper profile of the lens) varies with arguments y and z $[x = h(y,z)]$, which substantially complicates all stages of investigation of such waveguide structure (for details refer to [1–3, 5, 7–10]).

4. System of equations describing transformation of guided modes

Now consider the TM-mode. The general solutions for three nonzero components of the electromagnetic field of the TM-mode with the allowance made for the boundary conditions at infinity can be written in an explicit form

$$E_z^{\mathrm{c}} = A_{\mathrm{c}}\exp(-\gamma_{\mathrm{c}}x), \tag{21}$$

$$E_z^{\mathrm{s}} = A_{\mathrm{s}}\exp(\gamma_{\mathrm{s}}x), \tag{22}$$

$$E_z^{\mathrm{f}} = A_{\mathrm{f}}^{+}\exp(\mathrm{i}\chi_{\mathrm{f}}x) + A_{\mathrm{f}}^{-}\exp(-\mathrm{i}\chi_{\mathrm{f}}x), \tag{23}$$

$$E_z^{\text{layer}} = A_{\text{layer}}^{+}\exp(\mathrm{i}\chi_{\text{layer}}x) + A_{\text{layer}}^{-}\exp(-\mathrm{i}\chi_{\text{layer}}x). \quad (24)$$

Here $\chi_j = +k_0\sqrt{\varepsilon_j - \beta^2}$; $\gamma_j^2 = -\chi_j^2$; j = s, layer, f, c; and H_y and E_x are obtained by substituting (21)–(24) into relationships (19), (20).

By substituting these expressions to the boundary conditions for a waveguide of 'horn' type we obtain in an explicit form the system of algebraic equations with respect to unknown coefficients A_s, A_f^{+}, A_f^{-}, A_{layer}^{+}, A_{layer}^{-}, A_c, which after simple transformations can be reduced to the equivalent system of linear algebraic equations of lower dimensionality:

$$[\mathrm{i}\tan\theta k_0\beta\gamma_c(\varepsilon_{\text{layer}} - \varepsilon_c) + \chi_c^2\varepsilon_{\text{layer}}]$$
$$\times[A_{\text{layer}}^{+}\exp(\mathrm{i}\chi_{\text{layer}}a_3) - A_{\text{layer}}^{-}\exp(-\mathrm{i}\chi_{\text{layer}}a_3)]$$
$$= \mathrm{i}\varepsilon_c\chi_{\text{layer}}\gamma_c[A_{\text{layer}}^{+}\exp(\mathrm{i}\chi_{\text{layer}}a_3) + A_{\text{layer}}^{-}\exp(-\mathrm{i}\chi_{\text{layer}}a_3), \quad (25)$$

$$\frac{k_0\varepsilon_f}{\chi_f}[A_f^{+}\exp(\mathrm{i}\chi_f a_2) - A_f^{-}\exp(-\mathrm{i}\chi_f a_2)]$$
$$= \frac{k_0\varepsilon_{\text{layer}}}{\chi_{\text{layer}}}[A_{\text{layer}}^{+}\exp(\mathrm{i}\chi_{\text{layer}}a_2) - A_{\text{layer}}^{-}\exp(-\mathrm{i}\chi_{\text{layer}}a_2)], \quad (26)$$

$$A_f^{+}\exp(\mathrm{i}\chi_f a_2) + A_f^{-}\exp(-\mathrm{i}\chi_f a_2)$$
$$= A_{\text{layer}}^{+}\exp(\mathrm{i}\chi_{\text{layer}}a_2) + A_{\text{layer}}^{-}\exp(-\mathrm{i}\chi_{\text{layer}}a_2), \quad (27)$$

$$\frac{k_0\varepsilon_f}{\chi_f}[A_f^{+}\exp(\mathrm{i}\chi_f a_1) - A_f^{-}\exp(-\mathrm{i}\chi_f a_1)]$$
$$= -\frac{\mathrm{i}k_0\varepsilon_s\gamma_s}{\chi_s^2}[A_f^{+}\exp(\mathrm{i}\chi_f a_1) + A_f^{-}\exp(-\mathrm{i}\chi_f a_1)]. \quad (28)$$

From these expressions it follows that at a nonzero angle of inclination of the additional waveguide layer the energy is redistributed over the components of the magnetic field $\boldsymbol{H}$.

Similarly, for the TE-mode general solutions can be obtained for three nonzero components of the electromagnetic field in terms of unknown coefficients B_s, B_f^{+}, B_f^{-}, B_{layer}^{-}, B_{layer}^{+}, B_c. Finally, the boundary conditions for the tangential field components of the TE-mode are reduced to the system of linear algebraic equations with the dimensionality of four:

$$B_{\text{layer}}^{+}(\chi_c^2 - \mathrm{i}\gamma_c\chi_{\text{layer}})\exp(\mathrm{i}\chi_{\text{layer}}a_3)$$
$$+ B_{\text{layer}}^{-}(-\chi_c^2 - \mathrm{i}\gamma_c\chi_{\text{layer}})\exp(-\mathrm{i}\chi_{\text{layer}}a_3) = 0, \quad (29)$$

$$B_f^{+}\chi_{\text{layer}}\exp(\mathrm{i}\chi_f a_2) - B_f^{-}\chi_{\text{layer}}\exp(-\mathrm{i}\chi_f a_2)$$
$$- B_{\text{layer}}^{+}\chi_f\exp(\mathrm{i}\chi_{\text{layer}}a_2) + B_{\text{layer}}^{-}\chi_f\exp(-\mathrm{i}\chi_{\text{layer}}a_2) = 0, \quad (30)$$

$$B_f^{+}\exp(\mathrm{i}\chi_f a_2) + B_f^{-}\exp(-\mathrm{i}\chi_f a_2)$$
$$- B_{\text{layer}}^{+}\exp(\mathrm{i}\chi_{\text{layer}}a_2) - B_{\text{layer}}^{-}\exp(-\mathrm{i}\chi_{\text{layer}}a_2) = 0, \quad (31)$$

$$B_f^{+}(\chi_s^2 + \mathrm{i}\chi_f\gamma_s)\exp(\mathrm{i}\chi_f a_1)$$
$$+ B_f^{-}(-\chi_s^2 + \mathrm{i}\chi_f\gamma_s)\exp(-\mathrm{i}\chi_f a_1) = 0. \quad (32)$$

In the case of the TM-mode, the energy is not redistributed between the components of the electromagnetic field.

Similar equations for the TGWLL have been derived in our previous papers (see, for example, [1–3, 5, 10]). Since the upper profile of the lens transforms with varying arguments y and z, at all investigation stages of this waveguide structure one should take into account the dependence $h(y, z)$.

5. Comparison of calculation results of employment of the two methods for describing waveguiding structures of 'horn' and TGWLL types

In the method of comparative waveguides the explicit inclined boundary conditions in a waveguide of 'horn' type are substituted for approximate horizontal conditions [23, 24]. In describing such waveguides we will employ the method of comparative waveguides in order to find analytical and numerical distinctions between the waveguide modes propagating in them. In this method, all equations for the components of the electromagnetic field of waveguide TM-modes coincide with Eqns (18)–(20) which are valid for the method of adiabatic modes. Thus, the general solutions of these equations satisfying the boundary conditions at infinity coincide with solutions (21)–(24). In the method of comparative waveguides, irregular parts of the waveguide are substituted for a set of regular parts with different heights [23, 24] so that in (25)–(28) the summands proportional to $\partial h/\partial z$ are equal to zero. Hence, the boundary conditions for the electromagnetic field of TM-modes will be equivalent [23, 24] to a uniform system of linear algebraic equations coinciding with (25)–(28) if we formally set $\tan\theta = 0$.

A distinction between the methods of adiabatic waveguide modes and comparative waveguides is that boundary conditions (25)–(28) include the summands proportional to the tangent of the inclination angle of the upper plane of 'horn'. These summands are responsible for differences in dispersion curves and in transformations of field waveguide modes propagating in a smoothly irregular waveguide of 'horn' type or in a TGWLL. Because in such a waveguide these contributions determine differences in the dispersion relationships only for TM-modes, we will show by numerical examples how their dispersion characteristics differ.

The uniform system of linear algebraic equations (29)–(32) which is equivalent to the system of boundary conditions in the method of adiabatic modes $[\hat{M}_{\text{TM}}(\beta,\theta)\boldsymbol{A}(\beta,\theta) = 0]$ has a nontrivial solution if the compatibility condition $\det\hat{M}_{\text{TM}}(\beta,\theta) = 0$ is satisfied. In the method of comparative waveguides a similar system of linear algebraic equations has the form $\hat{M}_{\text{TM}}(\beta,0)\boldsymbol{A}(\beta,0) = 0$ and admits a nontrivial solution if the compatibility condition $\det\hat{M}_{\text{TM}}(\beta,0) = 0$ is satisfied, which in the integrated optics is called the dispersion relationship.

Figure 2 demonstrates quantitative differences in distributions of the phase retardation coefficient $\beta(z)$ for quasi-waveguide modes of the waveguide of 'horn' type whose upper edge is inclined at the angle $\theta = 5°$ (for clearness) in the methods of adiabatic modes and comparative waveguides. The parameters of waveguide media are given for a laser radiation wavelength $\lambda = 0.9$ μm. Similar differences are observed in the TGWLL for all rays which are parallel to the z axis.

For TE-modes, the tangential boundary conditions in the method of adiabatic modes do not depend explicitly on the angle of inclination θ and are equivalent to the uniform system of linear algebraic equations (29)–(32) in the form

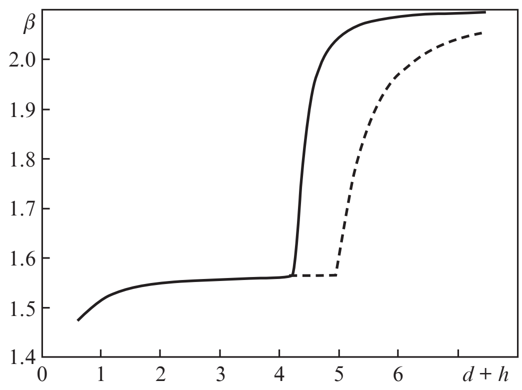

Figure 2. Dispersion curves for the TM_0-mode calculated by the method of comparative waveguides (solid curve) and by the method of adiabatic modes (dashed curve) while varying the thickness of the first waveguide layer d from 0.61 to 4.0 and thickness h of the second waveguide layer from 0 to 3.5 (both parameters are given in units of λ).

$\hat{M}_{\rm TE}(\beta)\boldsymbol{B}(\beta) = 0$. They coincide with the tangential boundary conditions in the method of comparative waveguides [23, 24]. The system of linear algebraic equations common for both the methods has a nontrivial solution if the compatibility condition $\det\hat{M}_{\rm TE}(\beta) = 0$, which also does not depend explicitly on the inclination angle θ, is satisfied.

Note that formulae (21)–(24) are not sufficient for calculating the vertical distribution of the electromagnetic field of the quasi-waveguide TM-mode. For calculating a complete electromagnetic field for the corresponding mode it is necessary to employ formula (2) which comprises a phase lag $\varphi(z)$. The latter is calculated by the method of numerical integration of the tabulated value $\beta(z)$.

Some results of comparison of the comparative waveguide method with the method of adiabatic modes with explicit inclined boundary conditions for the TGWLL are presented in our papers (see, for example, [1, 3, 5, 7, 10]). Recall that the dispersion relationship for the TGWLL $\det\hat{M}(\beta) = 0$ has the form of a nonlinear partial differential equation with respect to h and an algebraic equation with respect to the vector field $\boldsymbol{\beta}$:

$$F_{\rm disp}\left(\beta,\beta_y,\beta_z;h,\frac{\partial h}{\partial y},\frac{\partial h}{\partial z};n_{\rm s},n_{\rm f},n_{\rm layer},n_{\rm c};d\right) = 0.$$

Here the matrix itself $\hat{M}(\beta)$ and its determinant $\det\hat{M}(\beta)$ depend on the real-value parameter $\beta \in [n_{\rm s}, n_{\rm layer}]$. The algorithm for calculating the dispersion dependence in the zero-order approximation of the method of adiabatic modes is described in [4–9].

From the expressions for the longitudinal field components E_z, H_z of corresponding modes in the zero-order approximation with respect to δ one can calculate the solutions for E_y, H_x и E_x, H_y in the zero order with respect to δ. Thus, all components of vertical distribution for the quasi-waveguide modes $\boldsymbol{E}(x;y,z)$ and $\boldsymbol{H}(x;y,z)$ are calculated in the zero-order approximation with respect to δ for arbitrary horizontal coordinates (y,z) with the prescribed thickness distribution (profile) $h(y,z)$ and for the arbitrary x coordinate.

The method of adiabatic waves was compared with the Southwell method by using the matrix model of comparative waveguides [4–9, 11]. This model is obtained by substituting the tangential conditions for their horizontal approximations in which the summands comprising $\partial h/\partial y$, $\partial h/\partial z$ turn to zero and the dispersion relationship $\det\hat{M}(\beta) = 0$ becomes a transcendental algebraic equation with respect to h and $\beta = \beta_z$ and for all (y,z) coincides with the dispersion relationship for a regular comparative waveguide. We especially stress that the theoretical model used by Southwell has no 'hybridisation' of waveguiding modes. It is not surprising because instead of strict tangential boundary conditions in the method of comparative waveguides, their horizontal projections are used, i.e., approximate boundary conditions admitting separate description of TE- and TM-modes. An algorithm for calculating the vertical field distribution in the matrix model of comparative waveguides and in the zero-order approximation of the model of adiabatic modes is also described in [4–10]. Note that the uniform system of linear algebraic equations is solved by using the Tikhonov method of regularisation:

$$F((\boldsymbol{A},\boldsymbol{B})_k) = \left\| \hat{M}\left(\beta,\beta_y,\beta_z;h,\frac{\partial h}{\partial y},\frac{\partial h}{\partial z}\right)(\boldsymbol{A},\boldsymbol{B})_k^T \right\|^2$$

$$+\alpha_1\left(\|(\boldsymbol{A},\boldsymbol{B})_k^T\|^2 - I_{\rm inc}\right)^2 + \alpha_2\|(\boldsymbol{A},\boldsymbol{B})_k^T - (\boldsymbol{A},\boldsymbol{B})_{k-1}^T\|^2 \to \min.$$

In Fig. 3 one can see the electromagnetic field distribution in the rear focal plane of the TGWLL (the normalised focal length of the lens is $s = F/R = 1.5$, where $F = 1.5R$ and $R = 0.5$ cm). The calculations have been performed in the approximation of the method of adiabatic modes (the zero-order approximation).

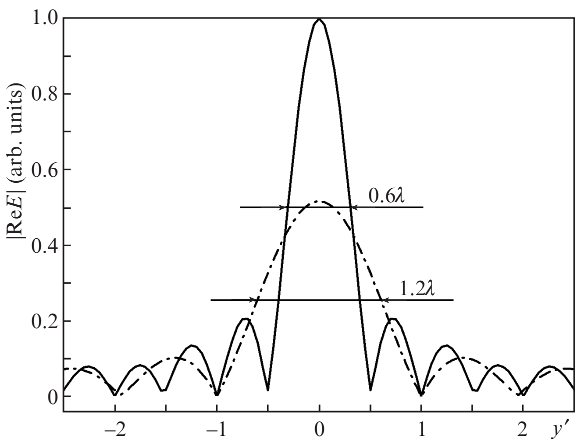

Figure 3. Field distribution in the rear focal plane for the focal length $F = 1.5R$ (y' is given in units of λ). Solid curve corresponds to the case of taking into account 99% of the lens aperture; dot-and-dash curve corresponds to taking into account 40% of the aperture.

The results obtained show that a super-resolution is possible while using the TGWLL, which undoubtedly will improve, for example, the characteristics of the integrated optical spectral analyser [7, 10, 19]. For obtaining curves in Fig. 3, the dispersion curves have been preliminarily calculated by both methods and ray tracing in the plane yz has been performed. The calculations were performed for the TGWLL with the following parameters: normalised focal lens length $s = 1.5$, normalised lens radius $r = 1$ and thickness of regular waveguide layer $d = 1.0665$ (in units of λ). The refractive index of the substrate (SiO_2) was $n_{\rm s} = 1.470$, the refractive index of the first (regular) waveguide layer (glass of the type Corning 7059) was $n_{\rm f} = 1.565$, the refractive index of the second waveguide layer (Ta_2O_5) of the TGWLL with variable thickness $h(y, z)$ was $n_{\text{layer}} = 2.100$ and the refractive index of the covering layer (air) was $n_c = 1.000$. The parameters of waveguide media are given for the laser IR radiation wavelength $\lambda = 0.9$ μm.

6. Conclusions

In the present work, some important results are demonstrated which indicate a difference between theoretical and numerical descriptions of guided and quasi-guided modes of smoothly irregular waveguide structures of horn types and the TGWLL by two methods: the zero-order approximation of the method of adiabatic modes and the method of comparative waveguides.

It was established that already the zero-order approximation of the method of adiabatic modes introduces noticeable distinctions, which are revealed in describing both the fields of quasi-waveguide modes and the dispersion relationships. The zero-order approximation of the method of adiabatic modes provides the transformation of linear polarisations of guided modes for a smoothly irregular waveguide of horn type without 'hybridisation'; in the case of the TGWLL such transformation leads to coupling of quasi-waveguide modes. Novelty of the method of adiabatic modes is that the approximate solution of the electrodynamic problem obeys the 'inclined' boundary conditions at the interface between two media with the allowance made for the nonhorizontal planes tangent to a nonplane interface of a smoothly irregular waveguide structure. This results in new equations for coupled vector quasi-waveguide adiabatic modes, and leads to mixing of two linear polarisations of an irregular multilayer waveguide and, as a consequence, to appearance of the mode with a new mixed state which is specific in rotation of the polarisation plane.

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