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Approximate solutions to a nonintegrable problem of propagation of elliptically polarised waves in an isotropic gyrotropic nonlinear medium, and periodic analogues of multisoliton complexes

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Abstract. **Using the linearization method, we obtain approximate solutions to a one-dimensional nonintegrable problem of propagation of elliptically polarised light waves in an isotropic gyrotropic medium with local and nonlocal components of the Kerr nonlinearity and group-velocity dispersion. The consistent evolution of two orthogonal circularly polarised components of the field is described analytically in the case when their phases vary linearly during propagation. The conditions are determined for the excitation of waves with a regular and 'chaotic' change in the polarisation state. The character of the corresponding nonlinear solutions, i.e., periodic analogues of multisoliton complexes, is analysed.**

Keywords: *cubic nonlinearity, spatial and frequency dispersion, linear and nonlinear gyrotropy, nonlinear Schrödinger equation, elliptical polarisation, polarisation chaos, periodic analogue of a multisoliton complex.*

1. Introduction

Propagation of a plane elliptically polarised wave through an isotropic gyrotropic medium with Kerr nonlinearity and group-velocity dispersion is described by the nonlinear Schrödinger equation (NSE) [1–5]:

$$\frac{\partial A_\pm}{\partial z} - \frac{\mathrm{i}k_2}{2}\frac{\partial^2 A_\pm}{\partial t^2} + \mathrm{i}[\mp\rho_0 + (\sigma_1/2 \mp \rho_1)\,|A_\pm|^2 + (\sigma_1/2 + \sigma_2)\,|A_\mp|^2]A_\pm = 0. \quad (1)$$

Here $A_\pm(z,t)$ are the slowly varying amplitudes of two orthogonal circularly polarised components of the field; ω is the frequency; t is the time in the intrinsic (running) coordinate system; $k_2 = \partial^2 k/\partial\omega^2$ is a constant characterising the second-order group velocity dispersion; and k is the wave number. The parameters $\sigma_1 = 4\pi\omega^2\chi^{(3)}_{xyxy}/(kc^2)$ and $\sigma_2 = 2\pi\omega^2\chi^{(3)}_{xxyy}/(kc^2)$ are given by two independent components of the tensor of the local cubic nonlinearity $\chi^{(3)}(\omega;-\omega,\omega,\omega)$, and $\rho_{0,1} = 2\pi\omega^2\gamma_{0,1}/c^2$ – by pseudoscalar constants $\gamma_{0,1}$ of linear and nonlinear gyration. The latter take into account the spatial nonlocality of the linear and nonlinear responses of the medium in the case of relatively slow changes in the amplitudes of the field components on the scale of the order of the wavelength in the propagation direction of the z axis [2, 4].

At an arbitrary value of σ_2 and with account for the nonlocal component in the nonlinear polarisation, system (1) is nonintegrable [6–9] and a number of numerical [1–3] and analytical particular [4, 5, 10–12] solutions are known. Thus, Makarov and Petrov [4], on the assumption of a linear relationship between A_+ and A_-, found particular solutions (1) in the form of soliton pairs. Under the condition of the formation of waveguides of the same profile for circularly polarised components of the field, the authors of papers [5, 10] obtained the solutions, in which $|A_\pm|$ are proportional to the Jacobi elliptic functions [13], and $\arg\{A_\pm\}$ are linearly dependent on z. In [11, 12] under the same assumptions, the authors found exact and approximate solutions, where $\arg\{A_\pm\}$ depend nonlinearly on t and linearly on z. In this case, both elliptically polarised cnoidal waves and waves with an aperiodic change in the polarisation state, which resemble the polarisation chaos, can propagate in a medium. Note that the integrable NSE systems in the conditions of the formation of waveguides of the same profile have similar solutions describing the propagation of waves whose phases are not dependent [14] or dependent [15] on time. However, for integrable NSE systems, there are also other solutions, i.e. multisoliton complexes [16–19]. Their analogues for the nonintegrable problem (1) are unknown. The aim of this work is the search for periodic analogues of multisoliton complexes, which may also be relevant for a number of other problems described by the NSE system of type (1).

In this paper, using the approach [12] we have analysed one of the families of approximate solutions to the nonintegrable problem (1), in which the phases of circularly polarised components of the field of an elliptically polarised wave vary linearly during propagation. We have found the conditions for the excitation of waves with a regular and 'chaotic' change in the polarisation state. We have analysed the character of the corresponding currently unknown nonlinear solutions, i.e., periodic analogues of multisoliton complexes [16–19].

2. Approximate solution

As in [5], we use the procedure of separation of variables, by setting

$$A_\pm(z,t) = r_\pm(t)\exp(\mathrm{i}\kappa_\pm z), \quad (2)$$

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where $\kappa_\pm$ are the propagation constants, and $r_\pm(t)$ are the real functions. By substituting (2) into (1), we obtain the system of ordinary differential equations:

$$\frac{d^2 r_\pm}{dt^2} = -\frac{\partial U}{\partial r_\pm} = \frac{1}{k_2}[2\Delta\kappa_\pm + (\sigma_1 \mp 2\rho_1) r_\pm^2 + (\sigma_1 + 2\sigma_2) r_\mp^2] r_\pm, \quad (3)$$

where $\Delta\kappa_\pm = \kappa_\pm \mp \rho_0$. System (3) can also be regarded as a system of equations describing the change in the Cartesian coordinates of a material point of unit mass with a potential energy

$$U(r_-, r_+) = -\frac{1}{4k_2}[4\Delta\kappa_+ r_+^2 + 4\Delta\kappa_- r_-^2 + (\sigma_1 - 2\rho_1) r_+^4 + (\sigma_1 + 2\rho_1) r_-^4 + 2(\sigma_1 + 2\sigma_2) r_+^2 r_-^2]. \quad (4)$$

Coordinates $\tilde{r}_-, \tilde{r}_+$ of the position of the equilibrium point are found from the system of algebraic equations

$$[2(k_\pm \mp \rho_0) + (\sigma_1 \mp 2\rho_1)\tilde{r}_\pm^2 + (\sigma_1 + 2\sigma_2)\tilde{r}_\mp^2]\tilde{r}_\pm = 0. \quad (5)$$

The search for approximate solutions to (1) is reduced to obtaining the system of linear equations for small deviations $\xi_\pm = r_\pm - \tilde{r}_+$ from nonzero coordinates of the equilibrium position,

$$\tilde{r}_\pm^2 = \frac{(\sigma_1 \pm 2\rho_1)\Delta\kappa_\pm - (\sigma_1 + 2\sigma_2)\Delta\kappa_\mp}{2(\rho_1^2 + \sigma_1\sigma_2 + \sigma_2^2)}, \quad (6)$$

and to finding its solutions, which, even after linearization of the problem, retain, as in [12], the dependence of the obtained solution on the nonlinear parameters of the medium. Minimum (6) exists if $(\sigma_1 \mp 2\rho_1)/k_2 < 0$, $\rho_1^2 + \sigma_1\sigma_2 + \sigma_2^2 < 0$ and $\tilde{r}_\pm^2 > 0$.

In the vicinity of this minimum, the degenerate particular solution to (1) exists if $\Delta\kappa_+ = \Delta\kappa_- = \Delta\kappa^{(d)}$ [5]:

$$A_\pm^{(d)}(z,t) = v\sqrt{\frac{-k_2(\sigma_2 \mp \rho_1)}{(\rho_1^2 + \sigma_1\sigma_2 + \sigma_2^2)}}\,\mathrm{dn}(vt,\mu) \times \exp\{iz[\pm\rho_0 + k_2 v^2(2-\mu^2)/2]\}. \quad (7)$$

Here, $\mathrm{dn}(vt,\mu)$ is the elliptic Jacobi function [13] with the modulus $0 \leqslant \mu \leqslant 1$ and $v = \{2\Delta\kappa^{(d)}/[k_2(2-\mu^2)]\}^{1/2}$. Violation of the condition $\Delta\kappa_+ = \Delta\kappa_-$ will result in the fact that solutions (2) of the problem will leave the family of solutions (7) and moreover the class of degenerate solutions. Figure 1 shows, in grayscale, a map of the distribution of $U(r'_-, r'_+)$, where $r'_\pm = r_\pm\sqrt{\sigma_2/\rho_0}$, in the vicinity of the equilibrium point (6) at $\Delta\kappa_+/\rho_0 = \Delta\kappa_-/\rho_0 = \Delta\kappa^{(d)}/\rho_0 = 3.767$ (Fig. 1a) and $\Delta\kappa_+/\rho_0 = 5.4$, $\Delta\kappa_-/\rho_0 = 3.5$ (Fig. 1b). The bold lines show the dependences $r'_+(r'_-)$, corresponding to a particular solution (7) at the same value of $\Delta\kappa^{(d)}/\rho_0$ and $\mu = 0.95$.

Following the standard procedure of linearization $\xi_\pm = r_\pm - \tilde{r}_\pm \ll \tilde{r}_\pm$ we obtain a system of equations

$$\frac{d^2\xi_\pm}{dt^2} - \frac{2}{k_2}[(\sigma_1 \mp 2\rho_1)\tilde{r}_\pm^2\xi_\pm + (\sigma_1 + 2\sigma_2)\tilde{r}_+\tilde{r}_-\xi_\mp] = 0. \quad (8)$$

The origin of the coordinates of the systems ξ'_-, ξ'_+ and ζ'_-, ζ'_+ is shifted to the point defined by the radius vector $\{\tilde{r}'_-, \tilde{r}'_+\}$ ($\tilde{r}'_\pm = \tilde{r}_\pm\sqrt{\sigma_2/\rho_0}$, $\xi'_\pm = \xi_\pm\sqrt{\sigma_2/\rho_0}$, $\zeta'_\pm = \xi'_\pm\cos\alpha \mp \xi'_\mp\sin\alpha$).

By changing the variables $\zeta_\pm = \xi_\pm\cos\alpha \mp \xi_\mp\sin\alpha$, where

$$\alpha = \arctan\{[(\sigma_1 - 2\rho_1)\tilde{r}_+^2 - (\sigma_1 + 2\rho_1)\tilde{r}_-^2]/[2(\sigma_1 + 2\sigma_2)\tilde{r}_+\tilde{r}_-] - \mathrm{sign}[(\sigma_1 + 2\sigma_2)\tilde{r}_+\tilde{r}_-]\{[(\sigma_1 - 2\rho_1)\tilde{r}_+^2 - (\sigma_1 + 2\rho_1)\tilde{r}_-^2]^2(\sigma_1 + 2\sigma_2)^{-2}\tilde{r}_+^{-2}\tilde{r}_-^{-2}/4 + 1\}^{1/2}\}, \quad (9)$$

we obtain two equations describing small harmonic oscillations of a material point near the equilibrium position:

$$\frac{d^2\zeta_\pm}{dt^2} + \Omega_\pm^2\zeta_\pm = 0, \quad (10)$$

$$\Omega_\pm^2 = -k_2^{-1}\{(\sigma_1 - 2\rho_1)\tilde{r}_+^2 + (\sigma_1 + 2\rho_1)\tilde{r}_-^2 \pm \sqrt{[(\sigma_1 - 2\rho_1)\tilde{r}_+^2 - (\sigma_1 + 2\rho_1)\tilde{r}_-^2]^2 + 4(\sigma_1 + 2\sigma_2)^2\tilde{r}_+^2\tilde{r}_-^2}\}. \quad (11)$$

Finally, we obtain the expressions for the amplitudes $r_\pm(t)$:

$$r_\pm(t) = \tilde{r}_\pm + \zeta_\pm^{(0)}\cos\alpha\cos(\Omega_\pm t + \phi_\pm) \pm \zeta_\mp^{(0)}\sin\alpha\cos(\Omega_\mp t + \phi_\mp). \quad (12)$$

As was expected, dispersion, gyrotropy and nonlinearity still play a role in the solution of (12) because k_2, $\rho_{0,1}$ and $\sigma_{1,2}$

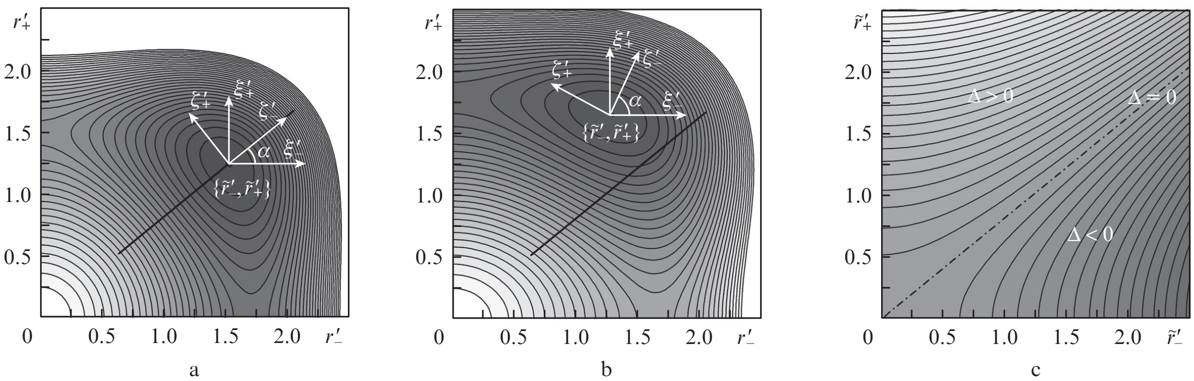

Figure 1. Maps of distributions $U(\tilde{r}'_-, \tilde{r}'_+)$ in grayscale for (a) $\Delta\kappa_+/\rho_0 = \Delta\kappa_-/\rho_0 = \Delta\kappa^{(d)}/\rho_0 = 3.767$, (b) $\Delta\kappa_+/\rho_0 = 5.4$, $\Delta\kappa_-/\rho_0 = 3.5$ and $\Delta = (\Delta\kappa_+ - \Delta\kappa_-)/\rho_0$ and (c) variation in $\{\tilde{r}'_-, \tilde{r}'_+\}$ during the change in the position of the equilibrium point specified by the radius vector $\sigma_1/\sigma_2 = -3$, $\rho_1/\sigma_2 = 0.2$. The bold lines show the dependences $r'_+(r'_-)$, corresponding to a particular solution (7) for the same value of $\Delta\kappa^{(d)}/\rho_0$.

specify the values of $\tilde{r}_\pm$, α and $\Omega_\pm$. The dependences of the modulus of the amplitudes $A_\pm$ on the running time have the character of beats of two harmonic components at frequencies $\Omega_\pm$ near the points $\tilde{r}_\pm$.

Particular solutions (7) do not have a linear analogue ($\mu \to 0$), since in this limit $\mathrm{dn}(\nu t,\mu) \to 1 - (\mu^2/2)\cos\Omega_- t$ and the oscillation amplitude $A_\pm$ near the points $\tilde{r}_\pm$ tends to zero. However, the frequency of these oscillations coincides with the frequency of in-phase oscillations Ω_- and therefore solution (7) can still be regarded as a nonlinear asymptotic of the in-phase approximate solution (12), which is degenerate in its eigenvalues. At the same time among particular solutions found in [5], there are no analogues of both the anti-phase solution (12), in which $r_\pm(t)$ oscillate at a frequency Ω_+, and of the solutions of a general character in the form of beats $r_\pm(t)$ at frequencies $\Omega_\pm$. Figure 2 shows the dependence of the normalised frequencies $\Omega'_\pm\{\tilde{r}'_-, \tilde{r}'_+\} = \sqrt{k_2/\rho_0}\,\Omega_\pm\{\tilde{r}'_-, \tilde{r}'_+\}$ and the angle $\alpha\{\tilde{r}'_-, \tilde{r}'_+\}$ on the position of the equilibrium point $\{\tilde{r}'_-, \tilde{r}'_+\}$. It is easy to see that the frequencies of normal oscillations can be significantly different, and the angle virtually varies from 0 to $\pi/2$. Because the components of the field $r_\pm$ never vanish, their spectra have a constant component. Therefore, the frequencies of normal oscillations $\Omega_\pm$ and contributions of oscillations at these frequencies into the components $r_\pm$, given by the angle α (12), determine the widths of the spectra of the resulting beats. These widths may also be significantly different for the components $r_\pm$.

In the general case, the approximate solutions (12) are linear asymptotics of new periodic (at commensurable normal frequencies) solutions, undergoing a transition, by increasing their period, to the analogues of multisoliton complexes [16–19], what drastically differs them from the particular solutions found in [5].

Similar approximate solutions can be constructed also in those cases when the minima $U(r_-,r_+)$ (4) lie on the axes $r_\pm$, e.g.,

$$\tilde{r}_+ = 0, \quad \tilde{r}_- = [-2\Delta\kappa_-/(\sigma_1 + 2\rho_1)]^{1/2}. \tag{13}$$

Solutions of this type will remain dependent only on the part of the nonlinear parameters. In the vicinity of point (13), the quadratic form (4) becomes diagonal, and the coordinates $\xi_\pm$ become normal modes. Due to this, the components $r_\pm$ always oscillate at different frequencies, which are readily obtained through second derivatives of the potential energy $U(r_-,r_+)$ at the equilibrium point (13):

$$\omega_+^2 = \frac{2}{k_2}\left(\frac{\sigma_1 + 2\sigma_2}{\sigma_1 + 2\rho_1}\Delta\kappa_- - \Delta\kappa_+\right), \quad \omega_-^2 = \frac{4\Delta\kappa_-}{k_2}. \tag{14}$$

Note that the conditions for the existence of minimum (13) are reduced to the requirements $\omega_\pm^2 > 0$, $\tilde{r}_-^2 > 0$, and they are different from the conditions for the existence of minima (6), for which we obtained the approximate equation (10) and normal frequencies (11); therefore, from (10) and (11) one cannot obtain the oscillation equation in the vicinity of (13) and frequencies (14), and moreover, the latter cannot be expressed through the coordinates of minimum (13) as in (11).

Without going into detail about the approximate solution (3) in the vicinity of the equilibrium point (13), we note only that for the corresponding particular solutions obtained in [5], the amplitudes of the components $A_\pm$ are proportional to the elliptic Jacobi functions $\mathrm{cn}(\gamma t,\mu)$ and $\mathrm{dn}(\gamma t,\mu)$ [13]. In the linear limit ($\mu \to 0$) their scale factor γ and eigenvalues are given by [5]

$$\gamma\big|_{\mu\to 0} = \frac{2(\rho_1^2 + \sigma_1\sigma_2 + \sigma_2^2)}{(3\rho_1^2 + \sigma_1\sigma_2 - \sigma_2^2)}(\Delta\kappa_- - \Delta\kappa_+)\big|_{\mu\to 0},$$

$$\Delta\kappa_+\big|_{\mu\to 0} = -\frac{\gamma^2 k_2(\rho_1^2 - \rho_1\sigma_1 - \sigma_2^2 - 2\rho_1\sigma_2)}{2(\rho_1^2 + \sigma_1\sigma_2 + \sigma_2^2)}, \tag{15}$$

$$\frac{\Delta\kappa_-}{\Delta\kappa_+}\bigg|_{\mu\to 0} = \frac{(\rho_1 + \sigma_2)(\sigma_1 + 2\rho_1)}{\rho_1\sigma_1 + \sigma_2^2 + 2\rho_1\sigma_2 - \rho_1^2},$$

and the deviation amplitudes $r_\pm - \tilde{r}_\pm$ tend to zero. Frequencies of their oscillations in the vicinity of point (13) are equal to γ and 2γ, respectively. By directly substituting (15) into (14) we can easily see that at $\mu \to 0$ the factor γ coincides with ω_+, whereas $2\gamma \neq \omega_-$. This means that in the vicinity of minimum (13), approximate solutions are linear asymptotics of the currently unknown periodic analogues of multisoliton complexes that differ them from particular solutions found in [5].

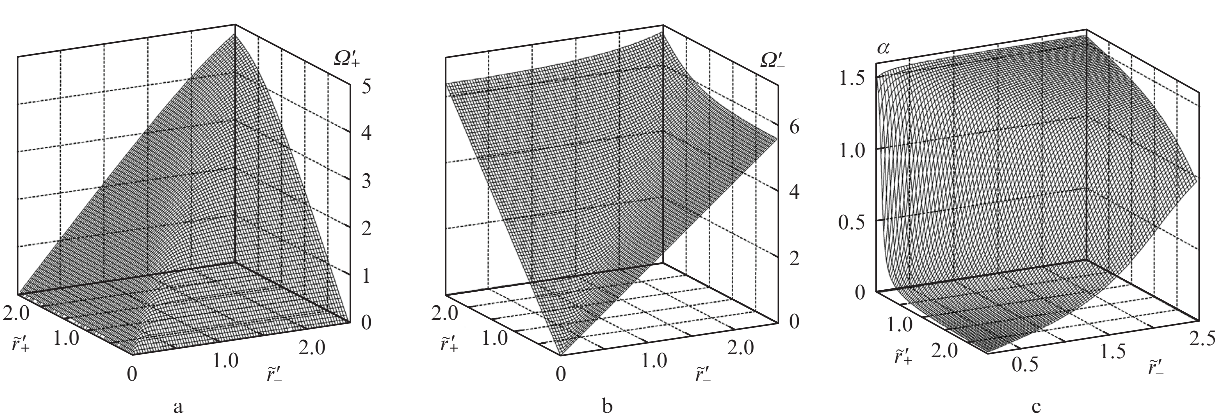

Figure 2. Changes in the frequencies of (a) antiphase, $\Omega'_+(\tilde{r}'_-, \tilde{r}'_+)$, and (b) in-phase, $\Omega'_-(\tilde{r}'_-, \tilde{r}'_+)$, normal oscillations and (c) the angle $\alpha(\tilde{r}'_-, \tilde{r}'_+)$ at a fixed position of the equilibrium point defined by the radius vector $\{\tilde{r}'_-, \tilde{r}'_+\}$. The parameters of the medium are the same as in Fig. 1.

3. Evolution of the polarisation state of a propagating wave

The dependence of the moduli of the amplitudes $A_{\pm}$ on the running time becomes periodic in the case of commensurable normal frequencies $\Omega_{\pm}$ or in the case of excitation of only one normal oscillation. Evolution of the polarisation state of the waves corresponding approximate solutions found can be described by the Stokes parameters [20] related to the complex amplitudes $A_{\pm}$ by the expressions

$$S_0(z,t) = \frac{1}{2}[|A_+(z,t)|^2 + |A_-(z,t)|^2] = \frac{1}{2}(\tilde{r}_+^{\,2} + \tilde{r}_-^{\,2})$$
$$+ \zeta_+^{(0)}(\tilde{r}_+\cos\alpha - \tilde{r}_-\sin\alpha)\cos(\Omega_+ t + \phi_+)$$
$$+ \zeta_-^{(0)}(\tilde{r}_+\sin\alpha + \tilde{r}_-\cos\alpha)\cos(\Omega_- t + \phi_-), \qquad (16)$$

$$S_1(z,t) = \mathrm{Re}\{A_+(z,t)A_-^*(z,t)\} = \cos[(\kappa_+ - \kappa_-)z]$$
$$\times[\tilde{r}_+\tilde{r}_- - \zeta_+^{(0)}(\tilde{r}_+\sin\alpha - \tilde{r}_-\cos\alpha)\cos(\Omega_+ t + \phi_+)$$
$$+ \zeta_-^{(0)}(\tilde{r}_+\cos\alpha + \tilde{r}_-\sin\alpha)\cos(\Omega_- t + \phi_-)], \qquad (17)$$

$$S_2(z,t) = \mathrm{Im}\{A_+(z,t)A_-^*(z,t)\} = \sin[(\kappa_+ - \kappa_-)z]$$
$$\times[\tilde{r}_+\tilde{r}_- - \zeta_+^{(0)}(\tilde{r}_+\sin\alpha - \tilde{r}_-\cos\alpha)\cos(\Omega_+ t + \phi_+)$$
$$+ \zeta_-^{(0)}(\tilde{r}_+\cos\alpha + \tilde{r}_-\sin\alpha)\cos(\Omega_- t + \phi_-)], \qquad (18)$$

$$S_3(z,t) = \frac{1}{2}[|A_-(z,t)|^2 - |A_+(z,t)|^2] = \frac{1}{2}(\tilde{r}_-^{\,2} - \tilde{r}_+^{\,2})$$
$$- \zeta_+^{(0)}(\tilde{r}_+\cos\alpha + \tilde{r}_-\sin\alpha)\cos(\Omega_+ t + \phi_+)$$
$$- \zeta_-^{(0)}(\tilde{r}_+\sin\alpha - \tilde{r}_-\cos\alpha)\cos(\Omega_- t + \phi_-). \qquad (19)$$

In this case, the normalised Stokes parameters $s_{x,y,z}(z,t) = S_{1,2,3}/S_0$ determine the Cartesian coordinates of the end of the unit vector $\boldsymbol{s}$, which moves along the Poincare sphere when the coordinate and/or time change [20]. Because the longitude $\Phi = \arctan(s_y/s_x) = (\kappa_+ - \kappa_-)z$ of the end of the vector $\boldsymbol{s}$ is proportional to z, it would be natural to analyse the change in the polarisation for situations when the running time t varies only due to z. As in the cases considered in [12], the change in the polarisation is periodic only for a periodic variation in s_z and a consistent variation in Φ. The first requirement coincides with the periodicity condition for $|A_{\pm}|$, and the second one reduces to the condition of commensurability of the frequency of s_z variation with $(\kappa_+ - \kappa_-)$.

The character of the change in the polarisation state of the approximate solution (12) is illustrated in Fig. 3, which shows the trajectory of the motion of the end of the Stokes vector on the Poincare sphere, when the first and second conditions of periodicity (in-phase and antiphase solutions) and only the second condition (the case of the beats) are fulfilled. In the last of the three considered situations, despite the well-marked

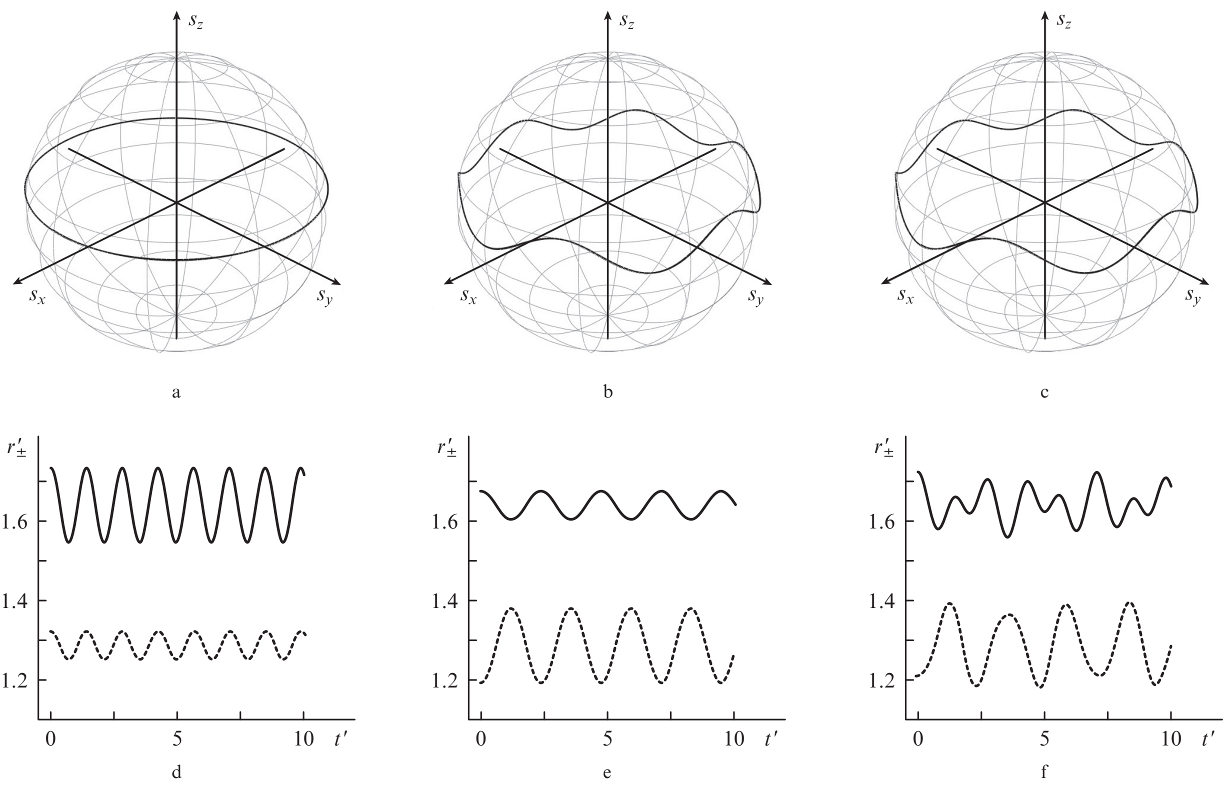

Figure 3. Trajectories of motion of the end of the Stokes vector along the surface of the Poincare sphere at $\Delta\kappa_+/\rho_0 = 5.4$, $\Delta\kappa_-/\rho_0 = 3.5$, $t = 2.751032z\sqrt{\rho_0 k_2}$ and $\phi_{\pm} = 0$ for (a) in-phase (at $\zeta'_{+0} = 0.1$ and $\zeta'_{-0} = 0$), (b) antiphase (at $\zeta'_{-0} = 0.1$ and $\zeta'_{+0} = 0$) and (c) aperiodic (at $\zeta'_{+0} = 0.1$ and $\zeta'_{-0} = 0.05$) solutions ($\zeta'_{\pm 0} = \zeta_{\pm 0}\sqrt{\sigma_2/\rho_0}$). The dependences of $r'_{\pm}(t)$, corresponding to the regimes mentioned, are shown by solid and dashed curves (d, e, f) ($t' = t\sqrt{\rho_0/k_2}$). The parameters of the medium are the same as in Fig. 1.

beats in dependences $r'_{\pm}(t')$ in Fig. 3d, the motion of the end of the vector $\boldsymbol{s}$ on the surface of the Poincare sphere remains almost periodic. This is easily explained by the character of the dependence s_z on the amplitudes $\zeta_{\pm}^{(0)}$, which in view of their smallness has the form

$$s_z(z,t) = \frac{\tilde{r}_-^2 - \tilde{r}_+^2}{\tilde{r}_-^2 + \tilde{r}_+^2}\Big[1 + 4\zeta_+^{(0)}\tilde{r}_+\tilde{r}_- \frac{\tilde{r}_+\sin\alpha + \tilde{r}_-\cos\alpha}{\tilde{r}_+^4 - \tilde{r}_-^4}$$

$$\times\cos(\Omega_+ t + \phi_+) - 4\zeta_-^{(0)}\tilde{r}_+\tilde{r}_- \frac{\tilde{r}_+\cos\alpha - \tilde{r}_-\sin\alpha}{\tilde{r}_+^4 - \tilde{r}_-^4}\cos(\Omega_- t + \phi_-)\Big]. \quad (20)$$

It follows from (20) that the character of variations in the polarisation state in cases of in-phase and antiphase oscillations is determined by the quantities $\tilde{r}_-\cos\alpha - \tilde{r}_+\sin\alpha$ and $\tilde{r}_+\cos\alpha + \tilde{r}_-\sin\alpha$, respectively. When the values of the parameters corresponding to $\tilde{r}_-\cos\alpha - \tilde{r}_+\sin\alpha \cong 0$ are used in the calculations, the component s_z for the in-phase solutions should be virtually constant (Fig. 2a). For the same reason, $\zeta_-^{(0)}$ determines the character of changes in s_z in the general case (Fig. 2c).

However, in the latter situation, in the case of sufficiently long evolution the end of the vector $\boldsymbol{s}$, as in [12], will completely fill the part of the surface of the Poincare sphere, limited by the inequalities $s_{z\,\min} \leqslant s_z \leqslant s_{z\,\max}$ ($s_{z\,\min}$ and $s_{z\,\max}$ are the minimum and maximum values of s_z), and there arises the regime that resembles polarised 'chaos'.

4. Conclusions

We have obtained approximate (linearization method in the vicinity of equilibrium points) solutions of the nonintegrable problem of propagation of plane elliptically polarised light waves in an isotropic gyrotropic medium with local and nonlocal components of the Kerr nonlinearity and group-velocity dispersion. We have described analytically consistent evolution of two orthogonal circularly polarised components of the field in cases where their phases vary linearly with distance. It is shown that approximate solutions are asymptotics of currently unknown periodic analogues of multisoliton complexes [16–19], which are radically different from particular solutions found earlier in [5]. We have determined the conditions of excitation (restrictions on the initial conditions and parameters of the medium) of light waves with a regular and 'chaotic' change in the polarisation state.

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