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Collective modes in cold paramagnetic gases

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***Abstract.* We have obtained a condition for the emergence of spin waves in paramagnetic gases Re$\hat{A} \gg$ Im$\hat{A}$, which is fulfilled only at temperatures of the order of 1 μK.**

Keywords: *spin waves, cold paramagnetic gases, Boltzmann equation.*

Until recently, the only propagating collective mode – sound waves – was observed in gases. Since the 1980s attempts have been made to detect spin waves in polarised gases, which, however, failed. The studies were conducted at temperatures of about 1 mK. Nevertheless, the use of laser cooling of gases to temperatures of the order of 1 μK changed the situation. Anomalously strong spin waves were observed in the rubidium vapour in the S-state at a temperature $T \approx$ 0.6 μK and concentration $n = 1.8 \times 10^{13}$ cm^{-3} [1]. Later, there appeared works, which investigated cold atoms Ga, In, I, etc. in the P-states, i.e., with a nonzero orbital angular momentum [2], but the spin waves were not observed. Recently, a group of researchers from the Massachusetts Institute of Technology has reported that their results may be indicative of the ferromagnetic phase transition in ultracold lithium vapour [3].

Theoretical description of spin waves in paramagnetic gases requires precise calculation of the collision integral in the Boltzmann equation, taking into account the spin degrees of freedom of the colliding atoms. In the related works the expressions for the collision integral are usually quite cumbersome, which makes it very difficult to study the dynamics of spin waves. To avoid this, use is made of model variants of the collision integral [4].

In our studies [5, 6] we showed that the theory of spin waves in cold paramagnetic gases whose temperature is higher than the degeneracy temperature, can be built only on the basis of the quantum Boltzmann collision integral without the inclusion of additional phenomenological terms in the kinetic equation. We calculated the exact Boltzmann collision integral in paramagnetic gases at low temperatures, taking into account the identity of colliding particles. It turned out that the dynamics of the magnetic moment in polarised and unpolarised gases is significantly different. Spin waves can propagate only in the polarised gas and are described by the Boltzmann equation, in which it is sufficient to retain only the 'losses'. To this end, the temperature must be low enough to satisfy the condition

$$\mathrm{Re}\hat{A} \gg \mathrm{Im}\hat{A}. \tag{1}$$

Here, $\hat{A}$ is the particle scattering amplitude, which depends on the internal degrees of freedom of an atom (in this case, on the spin projections) and is spherically symmetrical at low temperatures. Note that for clarity we use in the collision integral the scattering amplitude, rather than T-matrix, as it was in our previous studies [5, 6]. The relation between the scattering amplitude and T-matrix has the form [7]

$$\hat{A} = -4\pi^2 m_{\mathrm{red}} \hbar \hat{T},$$

where $m_{\mathrm{red}} = m/2$ is the reduced mass of colliding atoms (m is the mass of the atom). At sufficiently low temperatures the scattering becomes isotropic, the amplitude $\hat{A}$ is no longer dependent on the scattering angle, and the collision cross-section – on the energy.

The integral equation of the dynamics of the magnetic moment of the atom depends on the degree of spin polarisation of the gas M (ratio of the number of atoms polarised by the external magnetic field to their total number [1]) and for the component of the magnetic moment μ_{-1} has the form:

$$(\boldsymbol{kv} - \omega + \nu_1)\mu_{-1}(\boldsymbol{p},\omega) = \nu_2 \int w(\boldsymbol{p}_1)\mu_{-1}(\boldsymbol{p}_1,\omega)\,\mathrm{d}\boldsymbol{p}_1. \tag{2}$$

Here, $\mu_{-1}(\boldsymbol{p},\omega)$ is the Fourier transform of the component $\mu_{-1}(\boldsymbol{x}, t)$ of the magnetic moment; $w(\boldsymbol{p})$ is the Maxwellian distribution of atomic momenta $\boldsymbol{p}$ normalised to unity; ω is the spin-wave frequency; and $\boldsymbol{k}$ and $\boldsymbol{v}$ are the wave vector and the velocity of the atom. The quantities ν_1 and ν_2 (frequency dimensions) have the form [5, 6]

$$\nu_1 = \frac{\pi M n \hbar}{2m_{\mathrm{red}}}\left(\mathrm{Re}\,A_{\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2}} - \mathrm{Re}\,A_{\frac{1}{2},-\frac{1}{2};\frac{1}{2},-\frac{1}{2}}\right), \tag{3}$$

$$\nu_2 = \frac{\pi M n \hbar}{2m_{\mathrm{red}}}\mathrm{Re}\,A_{\frac{1}{2},-\frac{1}{2};-\frac{1}{2},\frac{1}{2}} = \frac{M}{2} n\bar{v}\lambda_{\mathrm{B}}\mathrm{Re}\,A_{\frac{1}{2},-\frac{1}{2};-\frac{1}{2},\frac{1}{2}}, \tag{4}$$

where n is the gas concentration; μ_{-1} and μ_1 are circularly polarised components of the magnetic moment vectors; $\lambda_{\mathrm{B}} = 2\pi\hbar/(m\bar{v})$ is the de Broglie wavelength of atoms; and $\bar{v}$ is the average thermal velocity of atoms. The pairs of indices of the scattering amplitude components are the projections of the spins of atoms before and after collisions in the coordinate system with the quantization axis directed along the magnetic field. From equation (2) we can obtain the dispersion law for spin waves. In the hydrodynamic limit, it has the form

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$$\omega - \omega_0 = -\frac{k^2\bar{v}^2}{3v_2}, \tag{5}$$

where

$$\omega_0 = \omega_{12} + v_1 - v_2 \tag{6}$$

(ω_{12} is the Zeeman splitting of the level in a magnetic field). Note that the quantity $\mathrm{Re}A_{\frac{1}{2},-\frac{1}{2};-\frac{1}{2},\frac{1}{2}}$ and therefore v_2 may have any sign. At low temperatures for fermions $A_{\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2}} = 0$, $A_{\frac{1}{2},-\frac{1}{2};\frac{1}{2},-\frac{1}{2}} = -A_{\frac{1}{2},-\frac{1}{2};-\frac{1}{2},\frac{1}{2}}$ and therefore $v_1 = v_2$.

In an unpolarised gas at low temperatures, the equation for the magnetic moment differs significantly from that discussed above. The fact is that if condition (1) is met in a polarised gas ($M \approx 1$) in the collision integral, it is sufficient to retain in the first approximation only the terms proportional to M (the remaining terms are small). In this case, we derive equation (2), which describes the propagating spin waves. When $M = 0$, we have only the terms which have not previously been taken into account, and the equation for the magnetic moment takes the form

$$\mathrm{i}(\boldsymbol{kv} - \omega)\mu_{-1}(\boldsymbol{p},\omega) = \frac{\pi n\hbar}{m_{\mathrm{red}}}\int\left(\mathrm{Im}\,A_{\frac{1}{2},-\frac{1}{2};\frac{1}{2},-\frac{1}{2}} + \mathrm{Im}\,A_{\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2}}\right)$$

$$\times[\mu_{-1}(\boldsymbol{p},\omega) - \mu_{-1}(\boldsymbol{p}_1,\omega)]w(\boldsymbol{p}_1)\,\mathrm{d}\boldsymbol{p}_1. \tag{7}$$

The left-hand side of the equation has an imaginary unit, and the collision integral contains only positive terms $\mathrm{Im}\hat{A}$. In accordance with this solution, the equations simply describe the diffusion of the magnetic moment.

Thus, spin waves in a paramagnetic gas can propagate only at sufficiently low temperatures, when condition (1) is fulfilled, and at induced polarisation of the gas.

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