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Effect of temperature on the shape of spatial quasi-periodic oscillations of the refractive index of alkali atoms in an optically dense medium with a closed excitation contour of Δ type

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Abstract. **A theory of a closed excitation contour (Δ system) of a three-level atom in an optically dense medium is constructed with allowance for temperature. The spatial quasi-periodic oscillations of the refractive index in the system under study are shown to damp with increasing temperature. The range of temperatures at which these oscillations are most pronounced is found.**

Keywords: *three-level atom, Δ contour of excitation, temperature dependence, spatial oscillations of refractive index.*

1. Introduction

It is known that interaction of a three-level system with a two-frequency laser field in the Λ configuration leads, due to the presence of quantum interference, to the emergence of a new superposition state, which no longer interacts with the laser radiation. This phenomenon was called coherent population trapping (CPT) [1, 2]. When a three-level atom is excited by a probe and a strong fields, a CPT-related effect, i.e. electromagnetically induced transparency (EIT), comes to the fore [3–5]. The essence of EIT is that quantum interference under certain conditions makes a medium virtually transparent with respect to the probe field. The CPT and EIT properties have made these effects widely applicable in the field of quantum frequency standards [6], optical magnetometers [7–9], deceleration and storage of light [10–13], lasers without inversion [14–17] and for production of materials with a controllable photonic bandgap [18, 19] and a negative refractive index [20].

One of new directions in the investigation of three-level systems is the study of a closed interaction contour (so-called Δ system) between the exciting fields. In such systems, the nonabsorbing superposition state can be controlled by changing the algebraic sum of the initial phases of the fields (hereinafter, phases) acting on an atomic system; in this case, the CPT effect can be either completely eliminated or reconstructed. The ability to control the superposition state properties has been studied in atomic [21–25] and solid-state systems [26]. At the same time, there arises a possibility of changing the optical properties of the medium, in particular the coefficients of refraction and absorption, which may lead to the appearance of a refractive index extremum at a given phase and to the emergence of a negative absorption region (gain region) [27].

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Another interesting feature of the Δ system, which is formed by three-level atoms representing an optically dense medium and excited simultaneously by three-frequency laser radiation, are spatial quasi-periodic oscillations of the refractive index. This feature was first demonstrated in [28]. Interest in such a nontrivial phenomenon is due to the fact that in the future it can be used to produce controllable photonic bandgap materials, the bandgap being controlled by manipulating laser fields that are incident on an atomic ensemble. However, the authors of paper [28] considered a relatively narrow temperature range (30–100 μK) because temperatures below 100 μK and above 30 μK allow one to neglect, respectively, the influence of the atom motion on the absorption spectrum width and the recoil of the atom duting the emission of a photon.

In this paper we investigate the influence of the motion of atoms on the behaviour of the refractive index, found in [28]. We construct a theory of interaction of three-frequency laser radiation, forming a closed excitation contour, with a three-level atom in an optically dense medium taking into account the motion of the atoms. The highest possible temperatures are found at which spatial quasi-periodic oscillations of the refractive index manifest themselves.

2. Theory

2.1. General form of the quantum kinetic equation and the radiative transfer equation

Consider the interaction of laser radiation with ^{87}Rb atomic vapours confined in a gas cell at some temperature (Fig. 1a). The laser field is quasi-resonant to the D_1 line of the ^{87}Rb atom. The energy level diagram of the D_1 line can be described by a three-level model in which the two lower sublevels $|1\rangle$ and $|2\rangle$ correspond to the hyperfine splitting of the $^2S_{1/2}$ state, while the upper level $|3\rangle$ – to the $^2P_{1/2}$ state.

Coherent laser fields with Rabi frequencies $\Omega_1(x)$ and $\Omega_2(x)$, propagating along the x axis, are applied to the optical transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$. We consider the case of an optically dense medium; therefore, the damping of the fields along the x coordinate should be taken into account. The concentration of the atoms is assumed such that no more than one atom can fit to the optical radiation wavelength (so that the collective effects [29] are neglected in the scattering of radiation). The transition $|1\rangle \leftrightarrow |2\rangle$ is subjected to the action of the microwave field with a Rabi frequency U, which is phase matched with the laser fields (Fig. 1a). Its attenuation in a medium can be neglected, because the $|1\rangle \leftrightarrow |2\rangle$ transition is a magnetic dipole and scatters radiation by four orders of magnitude weaker than the optical transitions. Thus, the so-called Δ system of excitation of atoms is realised [27], in which one

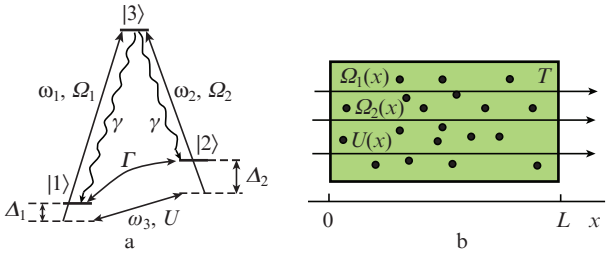

Figure 1. (a) Energy-level diagram of atoms in a closed excitation contour (Δ system) and (b) propagation of three-frequency radiation in a gas cell filled with ^{87}Rb vapour and buffer gas; ω_1 and ω_2 are the optical frequencies, ω_3 is the microwave frequency, Ω_1 and Ω_2 are the Rabi frequencies of the optical fields, U is the Rabi frequency of the microwave field, 2γ is the decay rate of the population of the excited level, Γ is the decay rate of the coherence between the ground states, Δ_1 and Δ_2 are one-photon detunings, L is the length of the gas cell, and T is the temperature.

can observe the CPT effect. We consider the laser intensity to be constant in time and assume that the spectral width of the optical and microwave fields is less than the natural widths of atomic levels. This fact allows us to treat the fields as monochromatic.

To describe the interaction of laser radiation with atoms, use is made of the density matrix $\tilde{\rho}_{mn}(\boldsymbol{v},\boldsymbol{r})$. Quantum kinetic equations for the density matrix in the Wigner representation have the form [30]

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}\nabla\right)\tilde{\rho}_{mn} = -\frac{\mathrm{i}}{\hbar}\sum_{j}^{3}[H_{mj}\tilde{\rho}_{jn} - \tilde{\rho}_{mj}H_{jn}] + R_{mn} + S_{mn}. \quad (1)$$

Here, H is the Hamiltonian; R is the relaxation matrix; S is the collision integral; $\boldsymbol{v}$, $\boldsymbol{r}$ are the velocity and coordinate of the atoms; and m, $n = 1,2,3$. The Hamiltonian can be represented as $H = H_0 + H_{\mathrm{int}}$, where

$$H_0 = \sum_{i=1}^{3}\varepsilon_i |i\rangle\langle i| \quad (2)$$

is the Hamiltonian in the absence of the laser field; ε_i is the energy of the atomic levels; and H_{int} describes the interaction of the quantum system with the laser field. In the resonance approximation and in the plane wavefront approximation

$$H_{\mathrm{int}} = \hbar\Omega_1\exp[-\mathrm{i}(\omega_1 t - k_1 x + \varphi_1)]|3\rangle\langle 1|$$
$$+ \hbar\Omega_2\exp[-\mathrm{i}(\omega_2 t - k_2 x + \varphi_2)]|3\rangle\langle 2|$$
$$+ \hbar U\exp[-\mathrm{i}(\omega_3 t - k_3 x + \varphi_3)]|2\rangle\langle 1| + \mathrm{h.c.}, \quad (3)$$

where k_i are the wavenumbers; and φ_i are the initial phases of the fields, with $i = 1,2,3$.

We will express the rapidly oscillating factor $\tilde{\rho}_{mn} = \rho_{mn}\exp(\mathrm{i}\omega_j t)$ through the off-diagonal elements of the density matrix (atomic coherences) (1) with $\tilde{\rho}_{nn} = \rho_{nn}$ and will use the rotating wave approximation. As a result, we obtain a system of differential equations for slowly varying amplitudes of atomic coherences $\rho_{mn}(\boldsymbol{v},\boldsymbol{r})$ and populations $\rho_{nn}(\boldsymbol{v},\boldsymbol{r})$ of the levels. Below, we are interested in the steady-state solution, and that is why the time derivatives of the amplitudes of the density matrix elements on the left side of the system are equated to zero.

To increase the time of coherent interaction of ^{87}Rb atoms with the laser fields, a buffer gas (usually inert gases, nitrogen or methane) is added to the gas cell. Active atoms are scattered by buffer ones. This leads to a reduction in their mean free path and to a sharp decrease in atomic polarisation relaxation during collisions with the wall, which in turn causes resonance narrowing [31]. The addition of a buffer gas allows for a substantial narrowing of CPT resonances, up to a few tens of Hz [32].

We will use the plane wavefront approximation and assume the relaxation of the wall to be small. Then, we can neglect edge effects and reduce the problem to a one-dimensional one, in which the optical fields and the density matrix vary only along the x axis.

The damping of optical fields in a medium is described by truncated wave equations, which in the steady-state case can be integrated over the x coordinate. For the Rabi frequencies the radiative transfer equations have the form [28]:

$$\Omega_1(x) = \Omega_1(0) + \mathrm{i}q_1\int_0^x \rho_{13}(x')\,\mathrm{d}x', \quad (4)$$

$$\Omega_2(x) = \Omega_2(0) + \mathrm{i}q_2\int_0^x \rho_{23}(x')\,\mathrm{d}x', \quad (5)$$

where

$$\rho_{mn}(x) = \int_{-\infty}^{\infty}\rho_{mn}(v_x,x)\,\mathrm{d}v_x$$

is the reduced density matrix;

$$q_1 = \frac{2\pi n_{\mathrm{Rb}}\mu_{13}^2\omega_{13}^{\mathrm{at}}}{c\hbar}; \quad q_2 = \frac{2\pi n_{\mathrm{Rb}}\mu_{23}^2\omega_{23}^{\mathrm{at}}}{c\hbar};$$

$\mu_{13,23}$ are the matrix elements of the optical transitions; $\omega_{13,23}^{\mathrm{at}}$ are the frequencies of the optical transitions; n_{Rb} is the concentration of ^{87}Rb atoms; c is the velocity of light in vacuum; and v_x is the projection of the atom velocity on the x axis. In the numerical integration, in (4) and (5) the density matrix is assumed constant over the partitioning interval $[x, x + \mathrm{d}x]$, which allows us to neglect the term $v_x\partial\rho_{mn}/\partial x$ in (1) and reduce the system of differential equations for the density matrix to the system of algebraic equations. Thus, we believe that the evolution of the density matrix in space is only affected by the damping of the laser fields.

2.2. Collision integral and spontaneous relaxation

Consider the collision integral S, which describes the escape of ^{87}Rb atoms from a specific velocity group and their arrival to this group from other velocity groups (Fig. 2). This occurs due to collisions of metal atoms with buffer gas atoms and cell walls, as well as due to collisions of active atoms with each other. In elastic collisions (with frequencies ν^{ee}, ν^{gg}) the atom changes the velocity component $v_x \leftrightarrow v_x'$, while keeping the internal energy constant. In the case of inelastic collisions (with frequencies ν^{31}, ν^{32}, $\nu^{\mathrm{gg}'}$), not only the velocity component changes, but also the energy states undergo a quantum transition. The spontaneous relaxation matrix R describes the decay of atomic states without changing the velocity due to the interaction with the field of vacuum (Fig. 2, the decay rates γ_{31}, γ_{32}).

In inelastic collisions the internal energy of the atom cannot increase due to this collision by a value greater than kT.

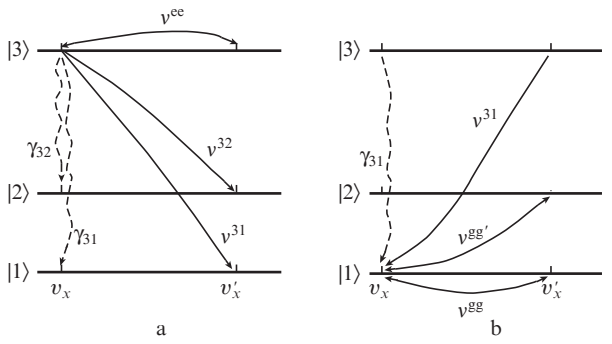

Figure 2. Channels for increasing and decreasing populations of (a) $|3\rangle$ and (b) $|1\rangle$ levels of atoms with a velocity projection v_x during collisions (solid arrows) and spontaneous relaxation (dashed arrows).

Therefore, at room temperature, the collisional excitation of the atom from the $|1\rangle$ and $|2\rangle$ levels to the $|3\rangle$ level is impossible, because the energy of such a transition is $\hbar\omega_{1,2} \approx 1$ eV, and the thermal energy at room temperature is $kT \approx 0.026$ eV. In collisions only the decay of the population from the $|3$ level is possible. Quite opposite is the case of the microwave transition $|1\rangle \leftrightarrow |2\rangle$, the energy of which is equal to $\hbar\omega_3 = 28$ meV. The thermal energy is sufficient to excite such a transition, and therefore the collision is accompanied by mixing of the populations between $|1\rangle$ and $|2\rangle$ levels with frequency $\nu^{gg'}$. As can be seen from Fig. 2a, the total collision frequency ν of active atoms is defined as $\nu = \nu^{ee} + \nu^{32} + \nu^{31}$. Similarly, from Fig. 2b we may write $\nu = \nu^{gg} + \nu^{gg'}$. It is also easy to verify that the relations

$$\frac{\nu^{31}}{\nu^{32}} = \frac{\gamma_{31}}{\gamma_{32}} = \frac{\mu_{13}}{\mu_{23}} \tag{6}$$

hold true.

Let us consider separately collisional and relaxation terms for the populations of each level; for brevity, $v_x \equiv v$:

$$R_{11} = \gamma_{31}\rho_{33}(v,x), \tag{7}$$

$$S_{11} = -\nu\rho_{11}(v,x) + \int A^{gg}(v,v')\rho_{11}(v',x)\,dv'$$

$$+ \int A^{gg'}(v,v')\rho_{22}(v',x)\,dv' + \int A^{31}(v,v')\rho_{33}(v',x)\,dv'; \tag{8}$$

$$R_{22} = \gamma_{32}\rho_{33}(v,x), \tag{9}$$

$$S_{22} = -\nu\rho_{22}(v,x) + \int A^{gg}(v,v')\rho_{22}(v',x)\,dv'$$

$$+ \int A^{gg'}(v,v')\rho_{11}(v',x)\,dv' + \int A^{32}(v,v')\rho_{33}(v',x)\,dv'; \tag{10}$$

$$R_{33} = -(\gamma_{31} + \gamma_{32})\rho_{33}(v,x), \tag{11}$$

$$S_{33} = -\nu\rho_{33}(v,x) + \int A^{ee}(v,v')\rho_{33}(v',x)\,dv'. \tag{12}$$

Here the integration is performed over all the velocity groups from which the atoms can arrive to this velocity group. The kernels $A^{ij}(v,v')$ of the integrals determine the frequency of the transitions $|i\rangle \to |j\rangle$ with a change in velocity $v' \to v$ during collisions. When the masses of the buffer gas atoms and ^{87}Rb atoms are approximately equal, we can apply the model of strong collisions, in which the velocity distribution of atoms after collisions does not depend on the velocity v' before the collision, and the change in velocity is comparable with its value [33–36]. In this case, the kernel can be represented as $A^{ij}(v,v') = \nu^{ij}M(v)$, where $M(v) = (\sqrt{\pi}v_t)^{-1}\exp(-v^2/v_t^2)$ is the Maxwellian distribution over the velocity projection; $v_t = \sqrt{2kT/m_{Rb}}$ is the most probable velocity; and m_{Rb} is the atomic mass of Rb.

Now let us consider the relaxation and collisional terms for the amplitudes of the atomic coherences. The decay rate of the low-frequency coherence, $\rho_{12}(v,x)$ is defined by the following physical processes:

1) the collision frequency $\nu_{12}^{vv'}$, at which the coherence is not destroyed, but the atom changes its velocity (mainly this occurs due to collisions with the buffer gas atoms);

2) the collision frequency ν_{12}^{q}, at which the coherence is destroyed (mainly this occurs due to the collisions of the active atoms with the cell walls and with each other); and

3) the decay caused by the interaction with the field of vacuum, Γ_{12}.

The sum of these collision frequencies yields the total collision frequency $\nu = \nu_{12}^{vv'} + \nu_{12}^{q}$.

Because the concentration of the buffer gas is much greater than the concentration of working atoms and $\nu_{12}^{vv'} \gg \nu_{12}^{q}$, we assume below that $\nu \approx \nu_{12}^{vv'}$. Then we have

$$R_{12} = -\Gamma_{12}\rho_{12}, \tag{13}$$

$$S_{12} = -(\nu_{12}^{vv'} + \nu_{12}^{q})\rho_{12}(v,x) + \nu_{12}^{vv'}M(v)\int\rho_{12}(v',x)\,dv'. \tag{14}$$

Here, similar to the kernels for the populations, the kernel of the collision integral is $\nu_{12}^{vv'}M(v)$.

For coherent transitions in the optical range

$$R_{13,23} = -\Gamma_{13,23}\rho_{13,23}, \tag{15}$$

$$S_{13,23} = -\nu\rho_{13,23}(v,x) + \nu_{13,23}^{vv'}M(v)\int\rho_{13,23}(v',x)\,dv'. \tag{16}$$

Here, $\nu_{13,23}^{vv'}$ are the collision frequencies, at which the coherence is not destroyed, but the atom changes its velocity. In contrast to the low-frequency coherences, optical coherences are characterised by the inverse relationship $\nu_{13,23}^{vv'} \ll \nu_{13,23}^{q}$, because the probability of their decay in each collision is close to unity.

2.3. Explicit form of the equations for the density matrix

By substituting in equation (1) the explicit forms of the Hamiltonians (2) and (3), $\tilde{\rho}_{mn} = \rho_{mn}\exp(i\omega_j t)$, $\tilde{\rho}_{nn} = \rho_{nn}$ [we are interested here in the steady-state solution $(\partial\rho_{mn}/\partial t = 0)$], assuming the matrix density in the partitioning interval $(\partial\rho_{mn}/\partial x = 0)$ to be spatially homogeneous, and substituting the relaxation and collision terms (7)–(16), we obtain the explicit form of the steady-state equations for the density matrix in the layer of the medium $[x, x + dx]$ (for brevity, we omit hereafter the dependence on the coordinate x in the arguments of the Rabi frequencies and density matrix elements):

$$0 = iU^*\rho_{12}(v) - iU\rho_{21}(v) + i\Omega_1^*\rho_{13}(v) - i\Omega_1\rho_{31}(v)$$

$$+\gamma_{13}\rho_{33}(v) - \nu\rho_{11}(v) + M(v)\Big\{\nu^{gg}\int\rho_{11}(v')\,dv' +$$

$$+\nu^{gg'}\int\rho_{22}(v')\,dv'+\nu^{31}\int\rho_{33}(v')\,dv'\Big\}, \quad (17)$$

$$0=-iU^*\rho_{12}(v)+iU\rho_{21}(v)+i\Omega_2^*\rho_{23}(v)-i\Omega_2\rho_{32}(v)$$
$$+\gamma_{23}\rho_{33}(v)-\nu\rho_{22}(v)+M(v)\Big\{\nu^{gg}\int\rho_{22}(v')\,dv'$$
$$+\nu^{gg'}\int\rho_{11}(v')\,dv'+\nu^{32}\int\rho_{33}(v')\,dv'\Big\}, \quad (18)$$

$$0=-i\Omega_1^*\rho_{13}(v)+i\Omega_1\rho_{31}(v)-i\Omega_2^*\rho_{23}(v)+i\Omega_2\rho_{32}(v)$$
$$-(\gamma_{13}+\gamma_{23})\rho_{33}(v)-\nu\rho_{33}(v)+\nu^{ee}M(v)\int\rho_{33}(v')\,dv', \quad (19)$$

$$0=[i(\Delta_2-\Delta_1)-\Gamma'_{12}]\rho_{12}(v)+iU(\rho_{11}-\rho_{22})(v)$$
$$+i\Omega_2^*\exp(i\Phi_0)\rho_{13}(v)-i\Omega_1\exp(i\Phi_0)\rho_{32}(v)$$
$$+\nu_{12}^{vv'}M(v)\int\rho_{12}(v')\,dv', \quad (20)$$

$$0=[i(k_1v-\Delta_1)-\Gamma'_{13}]\rho_{13}(v)+i\Omega_1(\rho_{11}-\rho_{33})(v)$$
$$+i\Omega_2\exp(-i\Phi_0)\rho_{12}(v)-iU\exp(-i\Phi_0)\rho_{23}(v)$$
$$+\nu_{13}^{vv'}M(v)\int\rho_{13}(v')\,dv', \quad (21)$$

$$0=[i(k_2v-\Delta_2)-\Gamma'_{23}]\rho_{23}(v)+i\Omega_2(\rho_{22}-\rho_{33})(v)$$
$$+i\Omega_1\exp(i\Phi_0)\rho_{21}(v)-iU^*\exp(i\Phi_0)\rho_{13}(v)$$
$$+\nu_{23}^{vv'}M(v)\int\rho_{23}(v')\,dv'. \quad (22)$$

Here, $\Gamma'_{mn}=\Gamma_{mn}+\nu$ is the rate of the coherence decay, modified due to collisions; $\Phi_0=\varphi_1-\varphi_2-\varphi_3$ is the total phase of the closed excitation contour at the medium input; and $\Delta_j=\omega_j-\omega_{3j}^{at}$ $(j=1,2)$ are the single-photon detunings of the laser fields from the $|1\rangle\to|3\rangle$ and $|2\rangle\to|3\rangle$ transitions. Due to the Doppler effect, the detuning of the laser radiation frequency from the frequency of the corresponding transition is shifted by $-k_jv$, which is reflected in the equations for optical coherences (21) and (22). This leads to an inhomogeneous broadening of the absorption line. The Doppler effect for the microwave field can be neglected because of the smallness of the wavenumber; therefore, the frequency shift of the detuning is neglected in the equation for the microwave coherence (20).

The solution of integral equations (17)–(22) for the density matrix $\rho_{mn}(v,x)$ is a challenging task. In order to reduce the equations to algebraic ones, one needs to make use of the reduced density matrix by integrating the equation in v. It can be easily done when the fields are sufficiently small and weakly distort the Maxwellian velocity distribution of the atoms. In this case, the frequency of collisions of the atoms should be much higher than the Rabi frequency of the fields, $\nu\gg\Omega_i, U$. In this approximation the collision terms in equations (17), (18) and (20) will lead to the Maxwellization of the velocity distribution of the populations $\rho_{11,22}(v,x)$ of the lower levels and low-frequency coherences $\rho_{12}(v,x)$. Therefore, they can be presented in the form [35]

$$\rho_{nn}(v,x)=M(v)\rho_{nn}(x),\ n=1,2, \quad (23)$$

$$\rho_{12}(v,x)=M(v)\rho_{12}(x). \quad (24)$$

Equations (21) and (22) do not contain a Maxwellian term and cannot be represented as in (23) and (24). In the equation for the populations $\rho_{33}(v,x)$ of the excited level, the collision term can be compared with field terms because the optical fields are small and the total population is $\rho_{33}(v,x)\ll\rho_{11,22}(x)$, which does not allow $\rho_{33}(v,x)$ to be represented in the form of (23) and (24), but makes it possible to produce an adiabatic elimination of this quantities in equations (21) and (22). In order to integrate (21) and (22) in v, we should separate the variables, by solving a system of two equations with respect to $\rho_{13}(v,x)$ and $\rho_{23}(v,x)$. Note that in the absence of the microwave field the U variables would be separated automatically. The remaining equations (17)–(20) can be integrated directly.

As a result, the equations for the reduced density matrix

$$\rho_{mn}(x)=\int\rho_{mn}(v,x)\,dv$$

have the form:

$$0=iU^*\rho_{12}-iU\rho_{21}+i\Omega_1^*\rho_{13}-i\Omega_1\rho_{31}+\gamma'_{13}\rho_{33}$$
$$+\nu^{gg'}(\rho_{22}-\rho_{11}), \quad (25)$$

$$0=-iU^*\rho_{12}+iU\rho_{21}+i\Omega_2^*\rho_{23}-i\Omega_2\rho_{32}+\gamma'_{23}\rho_{33}$$
$$+\nu^{gg'}(\rho_{11}-\rho_{22}), \quad (26)$$

$$0=-i\Omega_1^*\rho_{13}+i\Omega_1\rho_{31}-i\Omega_2^*\rho_{23}+i\Omega_2\rho_{32}-(\gamma'_{13}+\gamma'_{23})\rho_{33}, \quad (27)$$

$$0=[i(\Delta_2-\Delta_1)-\Gamma'_{12}+\nu_{12}^{vv'}]\rho_{12}+iU(\rho_{11}-\rho_{22})$$
$$+i\Omega_2^*\exp(i\Phi_0)\rho_{13}-i\Omega_1\rho_{32}\exp(i\Phi_0)\rho_{32}, \quad (28)$$

$$0=\Gamma'_{13}\rho_{13}+W_1^{13}(\Delta_1\Delta_2)[i\Omega_1\rho_{11}+i\Omega_2\exp(-i\Phi_0)\rho_{12}+\nu_{13}^{vv'}\rho_{13}]$$
$$+W_2^{13}(\Delta_1\Delta_2)[i\Omega_2\exp(-i\Phi_0)\rho_{22}+i\Omega_1\rho_{21}+\nu_{23}^{vv'}\rho_{23}], \quad (29)$$

$$0=\Gamma'_{23}\rho_{23}+W_1^{23}(\Delta_1\Delta_2)[i\Omega_2\rho_{22}+i\Omega_1\exp(i\Phi_0)\rho_{21}+\nu_{23}^{vv'}\rho_{23}]$$
$$+W_2^{23}(\Delta_1\Delta_2)[i\Omega_1\exp(i\Phi_0)\rho_{11}+i\Omega_2\rho_{12}+\nu_{13}^{vv'}\rho_{13}]. \quad (30)$$

Here, $\gamma'_{13}=\gamma_{13}+\nu^{31}$ and $\gamma'_{23}=\gamma_{23}+\nu^{32}$ are the rates of the population decay modified due to collisions;

$$W_1^{13}(\Delta_1\Delta_2)$$
$$=\int_{-\infty}^{\infty}\frac{\Gamma'_{13}M(v)[i(kv-\Delta_2)-\Gamma'_{23}]\,dv}{[i(kv-\Delta_2)-\Gamma'_{23}][i(kv-\Delta_1)-\Gamma'_{13}]+UU^*}; \quad (31)$$

$$W_2^{13}(\Delta_1\Delta_2) = \int_{-\infty}^{\infty} \frac{\mathrm{i}U\Gamma'_{13}\,M(v)\,\mathrm{d}v}{[\mathrm{i}(kv-\Delta_2)-\Gamma'_{23}][\mathrm{i}(kv-\Delta_1)-\Gamma'_{13}]+UU^*}; \quad (32)$$

$$W_1^{23}(\Delta_1\Delta_2) = \int_{-\infty}^{\infty} \frac{\Gamma'_{23}\,M(v)[\mathrm{i}(kv-\Delta_1)-\Gamma'_{13}]\,\mathrm{d}v}{[\mathrm{i}(kv-\Delta_2)-\Gamma'_{23}][\mathrm{i}(kv-\Delta_1)-\Gamma'_{13}]+UU^*}; \quad (33)$$

$$W_2^{23}(\Delta_1\Delta_2) = \int_{-\infty}^{\infty} \frac{\mathrm{i}U^*\Gamma'_{23}\,M(v)\,\mathrm{d}v}{[\mathrm{i}(kv-\Delta_2)-\Gamma'_{23}][\mathrm{i}(kv-\Delta_1)-\Gamma'_{13}]+UU^*}. \quad (34)$$

The integrands of the coefficients $W_j^{mn}(\Delta_1,\Delta_2)$ determine the strength of interaction of the optical fields with a specific velocity group of the atoms. It is seen that for $U=0$, the coefficients $W_2^{mn}(\Delta_1,\Delta_2)=0$ and $W_1^{m3}(\Delta_1,\Delta_2)=W_1^{m3}(\Delta_m)$ become dependent on one of the detunings, and their real part undergoes a transition into the Voigt contour. We assume in expressions (31)–(34) that the wavenumbers of the optical fields are approximately equal to each other ($k_1 \approx k_2 = k$). Solving equations (25)–(30) for the density matrix together with equations (4) and (5) for the fields passing through an optically dense medium, we obtain the distribution of the density matrix and Rabi frequencies of the fields along the length of the gas cell.

3. Results and discussion

We will use ^{87}Rb and ^{84}Kr as a working gas and a buffer gas, respectively. Their masses are approximately equal, and thus the model of strong collisions works well for this pair. We will consider the low-intensity fields (up to several mW cm^{-2}), corresponding to the Rabi frequencies $\sim 10^5$–10^6 s^{-1}. At ^{84}Kr concentrations roughly equal to 10^{18} cm^{-3} and temperatures above 0.1 K, the condition $v \gg \Omega_i, U$ is met, and so these fields do not introduce significant distortions in the Maxwellian velocity distributions of the populations of the ground states and low-frequency coherences, which justifies the assumptions of (23) and (24). At temperatures below 0.1 K, the homogeneous width of the excited level is greater than the Doppler broadening and the effect of the field is no longer selective in velocity; therefore, the Maxwellian distribution in (23) and (24) is not distorted.

The action of the microwave field U creates a closed excitation contour, which makes it possible to obtain spatial quasi-periodic oscillations of the refractive index, as shown in [28] for stationary atoms. We introduce the parameter $\zeta = (\gamma/2+\nu)/(kv_t)$, which describes the ratio of the Lorentzian and Doppler half-widths of the excited level. Note that the Lorentzian half-width is determined by a natural half-width $\gamma/2$ of the excited level and by a collisional homogeneous broadening ν. In the case of low (down to 0.1 K) temperatures, $\zeta > 1$. At the resonant tuning of the laser carrier frequencies ($\Delta_1 = \Delta_2 = 0$), all the ^{87}Rb atoms in the region of the laser beam action effectively interact with it. Physically, this situation corresponds to a cloud of cold ^{87}Rb atoms in a magneto-optical trap in which collisions occur only between working atoms. Such clouds can be cooled down to temperatures below 1 mK. Figures 3a and 3b illustrate the law of the intensity decay $I_1(x) = |\Omega_1(x)|^2/|\Omega_1(0)|^2$ of one of the optical fields for the temperature of 1 mK and 30 μK of the atoms. At a relative phase $\Phi_0 = \pi/4$, the EIT state of the medium cannot exist and the energy is transferred between the laser fields, which is demonstrated in Figs 3a and 3b by a spatial decrease in the laser field intensity. The quasi-periodic field intensity in space induces quasi-periodic oscillations of the refractive index (Figs 3d and 3e). As the fields propagate through the medium, the relative phase relaxes to zero and the medium undergoes a transition into the EIT state, which is demonstrated by a horizontal portion of the intensity decay curve in Figs 3a and 3b.

At room temperatures and above, $\zeta < 1$. A part of ^{87}Rb atoms becomes nonresonant with the laser fields due to the Doppler shift, which leads to the disappearance of the spatial oscillations of the refractive index (Fig. 3f). Physically, this case corresponds to the ^{87}Rb atomic vapour in the gas cell in the presence of a buffer gas. The system does not have time to undergo a transition into the EIT state, as in the case of cold atoms, and the radiation is completely absorbed (Fig. 3c). This increased absorption is due to the fact that at high temperatures the rate of spontaneous decay of the excited level $|3\rangle$ significantly increases as a result of collisions of the working atoms with the buffer gas, the walls and each other.

In addition, it is worth noting that even at fairly low concentrations of the atoms, the refractive index enhancement is accompanied by vanishing absorption [37].

4. Conclusions

We have studied a closed excitation contour (Δ system) of three-level atoms in an optically dense medium with account for the motion of the atom due to nonzero temperature. We have derived quantum kinetic equations describing such a system. On the basis of the solution of these equations we have found that the emerging spatial quasi-periodic oscillations of the refractive index begin to damp with increasing temperature. We have determined the range of temperatures at which these oscillations are most pronounced.

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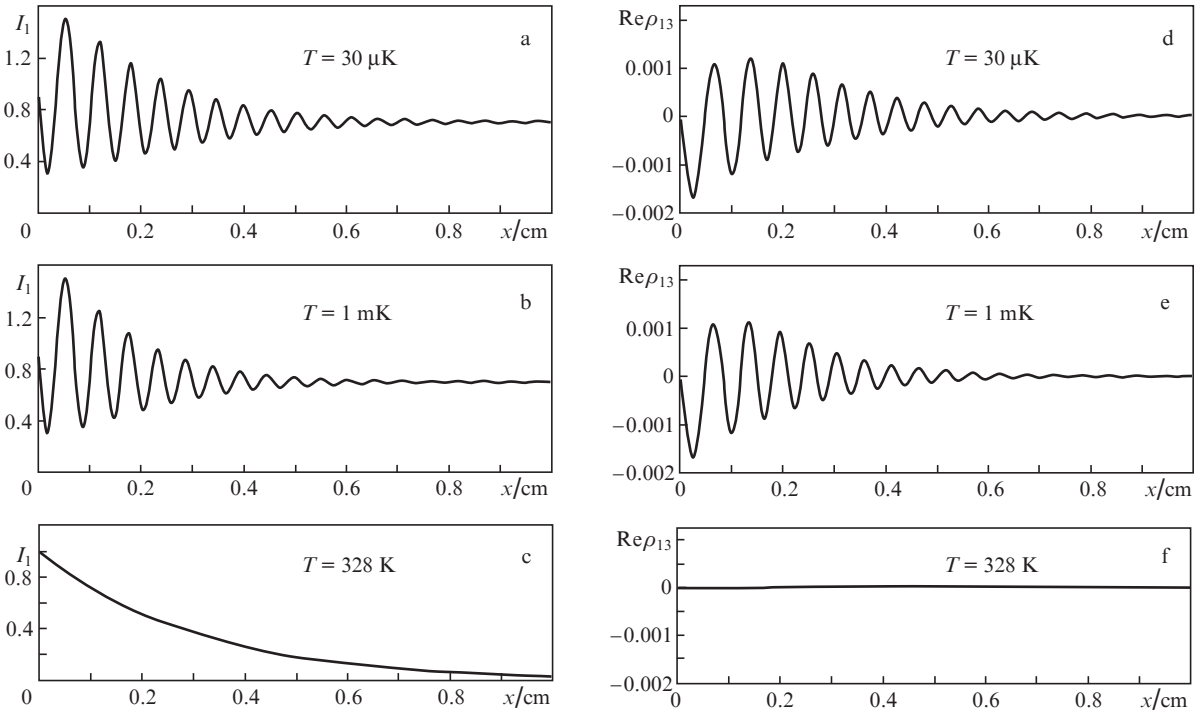

Figure 3. (a–c) Intensity I_1 of the field on the $|1\rangle \leftrightarrow |3\rangle$ transition and (d–f) real part of the coherence determining refraction as a function of the coordinate x at a fixed concentration for low temperatures [(a, d) ^{87}Rb atoms in an optical trap, (b, e) ^{87}Rb atoms in a magneto-optical trap] and high temperatures [(c, f) a cell filled with vapours of a working ^{87}Rb gas and a buffer ^{84}Kr gas]. At the entrance to the medium, $\Omega_1 = \Omega_2 = 5 \times 10^5$ s^{-1}, $U = 2 \times 10^3$ s^{-1}, relative phase of $\Phi_0 = \pi/4$, field detunings of $\Delta_1 = \Delta_2 = 0$, $n_{\mathrm{Rb}} = 4 \times 10^{11}$ cm^{-3} and $n_{\mathrm{Kr}} = 5 \times 10^{18}$ cm^{-3}.

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