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# Simulation of the generation of the characteristic X-ray emission under vacuum heating of cluster electrons by a femtosecond laser pulse

O.F. Kostenko

***Abstract.*** **We have developed a model for the generation of $K_\alpha$ radiation and the formation of hot electrons under vacuum heating by a femtosecond laser pulse near the surface of spherical clusters. The simulation results correspond to measurements at a cluster diameter of less than or of the order of the wavelength for the cases of p-polarised laser radiation and radiation incident along the normal to the surface. We discuss a significant decrease in the conversion of the laser energy into the energy of $K_\alpha$ emission with decreasing wavelength from 1.24 to 0.4 μm, which is observed at an intensity of $2\times10^{17}$ W cm$^{-2}$.**

***Keywords:*** *vacuum heating of electrons, clusters, generation of $K_\alpha$ radiation.*

## 1. Introduction

Energy absorption in the interaction of p-polarised femtosecond non-relativistically intense laser pulses with a dense plasma on the surface of a solid target may be caused by vacuum heating of electrons, if the amplitude of their oscillation due to the surface-perpendicular component of the laser field exceeds the characteristic size of the inhomogeneity of the plasma density near its critical density [1, 2]. In this case, electrons are dragged by the laser field from the plasma into the vacuum for a quarter of the field cycle, and their further movement is determined by the incident and reflected laser fields, and so by the electrostatic self-consistent field [1]. The main part of the electrons is then sent back to the plasma during the cycle of the laser field, gaining energy of the order of the oscillation energy. Because the electric field inside a sharply bounded overdense plasma is small, a further exchange of energies of these electrons with the field is negligible. Electrons penetrate the cold material behind the plasma, causing impact ionisation of the K-shell of atoms. One of the channels of the de-excitation of atoms is the emission of photons of characteristic X-rays on 2p–1s transitions. The analytical model of vacuum electron heating [1] satisfactorily describes the data on the measured $K_\alpha$ yield from massive targets with a flat surface [3, 4].

Local enhancement of the laser field near the surface of the structures that are smaller than or of the order of the wavelength increases the yield of hard X-rays, which is observed in experiments with nanospheres [5], spheres [6, 7], nanotubes [8] and nanorods [9] deposited on the surface of targets and in the formation of periodic structures on these surfaces [10, 11]. In particular, an increase in the $K_\alpha$ yield and temperature of hot electrons was investigated as a function of the size of spherical particles in the form of a hexagonally packed single layer on a flat surface of a silicon target [6].

Simulation of the $K_\alpha$ yield from a copper target, taking into account laser field energy absorption by hot electrons generated at the surface of the plasma clusters according to the mechanism of vacuum heating, showed a strong dependence of this yield on the ratio of the cluster size and the wavelength [12]. Comparison of simulation results [13] and measurements [6] is difficult because the approximate analytical model for the generation of $K_\alpha$ photons by electrons in solids dramatically reduces the accuracy of calculations even for such a light element as titanium [14] and, moreover, does not take into account the anisotropy of the $K_\alpha$ yield. In this paper, we calculate the $K_\alpha$ yield from a silicon target coated with spherical clusters by using an analytical model [15], generalised to the case of oblique incidence of electrons. The developed model of $K_\alpha$ generation under vacuum heating of electrons is verified by comparing the results of calculations and measurements [16] of the $K_\alpha$ yield from a massive iron target with a flat surface. The average electron energy is calculated as the ratio of the energy absorbed by the electrons on the surface of spherical clusters to the number of these electrons, and is compared with the results of measurements of the temperature of hot electrons [6].

The yield of $K_\alpha$ photons with an energy of 1.74 keV from a flat silicon target, measured at an intensity of $2\times10^{17}$ W cm$^{-2}$ and a laser wavelength of 0.4 μm [6], is 40 times less than that of $K_\alpha$ photons with an energy of 6.4 keV from a flat iron target, measured at a wavelength of 1.24 μm, intensity of $1.9\times10^{17}$ W cm$^{-2}$ and other similar experimental parameters [16]. In this case, the conversion coefficient of the laser energy into the $K_\alpha$ energy (into a 2π solid angle) decreases from $8\times10^{-5}$ to $8\times10^{-7}$, i.e., by 100 times. In this paper we also discuss the relationship of this decrease with a decrease in the efficiency of vacuum electron heating by short-wavelength laser radiation.

## 2. Model of vacuum electron heating

According to the model of vacuum electron heating [1], the electric field $E_{\rm os}\sin(\omega t)$, applied perpendicularly to the surface of a sharply bounded dense plasma at $t > 0$, pulls out electrons, which are then sent back to the plasma at $t > \pi/(2\omega)$. The moment of electron emission $0 < t_{\rm s} < \pi/(2\omega)$ and the moment of its return $t$ are related by the equation

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$$\sin\tau - \sin\tau_s - (\tau - \tau_s)\cos\tau_s + \frac{1}{2}(\tau - \tau_s)^2\sin\tau_s = 0,$$

where $\tau_s = \omega t_s$ and $\tau = \omega t$. The concentration and velocity of returning electrons at the plasma boundary are described by the expressions

$$n_e(\tau) = \frac{2n_0}{(\tau-\tau_s)^2}, \quad v_e(\tau) = v_{os}[\cos\tau - \cos\tau_s + (\tau-\tau_s)\sin\tau_s], \quad (1)$$

where $n_0 = m\omega^2/(4\pi e^2)$ is the critical electron concentration; $v_{os} = eE_{os}/(m\omega)$; and $e$ and $m$ are the absolute value of the charge and mass of the electron.

The surface density of the energy absorbed by the electrons during the time $\pi/(2\omega) < t < 5\pi/(2\omega)$ is determined by numerical integration of the electron energy flux [1]:

$$W_a = \frac{m}{2\omega}\int_{\pi/2}^{5\pi/2} n_e(\tau)v_e^3(\tau)\mathrm{d}\tau = \eta N\frac{mv_{os}^2}{2}, \quad (2)$$

where $N = E_{os}/(4\pi e)$ is the surface concentration of electrons emitted by the time $\pi/(2\omega)$; $\eta = 1.57$. The surface concentration of the electrons returning to the plasma within the specified time is

$$n = \frac{1}{\omega}\int_{\pi/2}^{5\pi/2} n_e(\tau)v_e(\tau)\mathrm{d}\tau = \gamma N, \quad (3)$$

where $\gamma = 0.77$.

## 3. Model of generation of $K_\alpha$ radiation

An electron with an initial energy $\mathcal{E}_0$, falling on a flat target at an angle $\chi$, generates

$$\mathrm{d}n_K = \omega_K p_\alpha n_a \sigma_K(\mathcal{E})\mathrm{d}s$$

$K_\alpha$ photons on a path of length d$s$ at a distance $s\cos\chi$ from the surface, where $\sigma_K(\mathcal{E})$ is the cross section of impact ionisation of the K shell by an electron with energy $\mathcal{E}(\mathcal{E}_0, s)$; $n_a$ is the concentration of atoms; $\omega_K$ is the probability of radiative de-excitation; and $p_\alpha$ is the probability of $K_\alpha$ fluorescence yield. In this case, from the target at an angle $\alpha_0$,

$$\mathrm{d}n_{em} = \mathrm{d}n_K\frac{\mathrm{d}\Omega}{4\pi}\exp\left(-\frac{s\cos\chi}{l_a\cos\alpha_0}\right)$$

photons are emitted into a solid angle d$\Omega$, where $l_a$ is the absorption length. Electron energy losses are described by the function $S_p(\mathcal{E})$:

$$\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}s} = -S_p(\mathcal{E}).$$

The path length at which the electron energy decreases from $\mathcal{E}_0$ to $\mathcal{E}$ is

$$s(\mathcal{E}_0, \mathcal{E}) = \int_{\mathcal{E}}^{\mathcal{E}_0}\frac{\mathrm{d}\mathcal{E}_1}{S_p(\mathcal{E}_1)}.$$

The total number of photons emitted by the electron per unit solid angle from a massive target at an angle $\alpha_0$ is given by

$$N_{em}(\mathcal{E}_0, \chi) = \frac{\omega_K p_\alpha n_a}{4\pi}\int_{\mathcal{E}_K}^{\mathcal{E}_0}\frac{\sigma_K(\mathcal{E})}{S_p(\mathcal{E})}\mathrm{d}\mathcal{E}\exp\left[-\frac{s(\mathcal{E}_0,\mathcal{E})\cos\chi}{l_a\cos\alpha_0}\right], \quad (4)$$

where $\mathcal{E}_K$ is the ionisation potential of the K-shell. It is understood that the massive target is thicker than $s(\mathcal{E}_0, \mathcal{E}_K)\cos\chi$.

## 4. Simulation of generation of $K_\alpha$ radiation under vacuum heating of electrons by a femtosecond laser pulse near the surface of a flat target

Under vacuum heating of electrons by a p-polarised laser field of nonrelativistic intensity near a flat surface of a dense plasma, which is modelled by a permittivity $\varepsilon_p \to -\infty$, the amplitude of the electric field perpendicular to the surface can be approximated by the expression

$$E_{os} = \alpha E_L\sin\theta, \quad (5)$$

where

$$\alpha = \frac{\sqrt{1+8\beta}-1}{2\beta}; \quad \beta = \frac{a_L\eta}{2\pi}\frac{\sin^3\theta}{\cos\theta}; \quad a_L = \frac{eE_L}{m\omega c};$$

$E_L = (8\pi I_L/c)^{1/2}$ is the amplitude of the laser field; $I_L$ is the intensity of the laser pulse; $\theta$ is the angle of radiation incidence; and $c$ is the velocity of light [1, 4]. Equation (5) is obtained under the assumption that the reflection coefficient with respect to the field can be expressed as $(1-f)^{1/2}$, where $f = \beta\alpha^3$ is the coefficient of laser radiation absorption by hot electrons.

Electrons with energy $\mathcal{E}_0 = mv_e^2/2$, falling perpendicularly to the target surface ($\chi = 0$) per unit area during a laser cycle, result in the emission of

$$n_{ph}(a_L) = \frac{1}{\omega}\int_{\pi/2}^{5\pi/2} n_e(\tau)v_e(a_L,\tau)N_{em}(\mathcal{E}_0(a_L,\tau),0)\mathrm{d}\tau \quad (6)$$

photons per unit solid angle. Here the electron velocity $v_e$ (1) is determined by the field (5), and their energy $\mathcal{E}_0$ is proportional to

$$\frac{mv_{os}^2}{2} = \frac{mc^2}{2}\alpha^2 a_L^2\sin^2\theta;$$

$$a_L^2(I_L) = \frac{2e^2}{\pi m^2 c^5}I_L\lambda^2.$$

By taking into account the dependence of the intensity on the radius and time and neglecting its change along the laser beam axis, we obtain the total number of photons generated by a laser pulse per unit solid angle in a given direction:

$$N_K = \frac{\omega}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}t\frac{2\pi}{\cos\theta}\int_0^{\infty} rn_{ph}(I_L(r,t))\mathrm{d}r.$$

In the case of a Gaussian intensity distribution

$$I_L(\varkappa) = I_0\exp(-\varkappa), \quad \varkappa = t^2/t_0^2 + r^2/r_0^2 \quad (7)$$

the number of photons is

$$N_K = \frac{r_0^2 t_0}{\cos\theta}\int_0^{\varkappa_{max}}\sqrt{\varkappa}\,\omega n_{ph}(I_L(\varkappa))\mathrm{d}\varkappa, \quad (8)$$

where $\omega n_{ph}$ is determined by formula (6). The upper limit of integration in $\varkappa$ is found from the condition

$$\max_{\pi/2 \leqslant \tau \leqslant 5\pi/2}[\mathcal{E}_0(I_{\min},\tau)] = \mathcal{E}_{\rm K}, \tag{9}$$

according to which the maximum electron energy at the minimum intensity $I_{\min} = I_{\rm L}(\varkappa_{\max})$ is equal to the ionisation potential.

The $K_\alpha$ yield from a massive iron target was calculated for a laser pulse with a wavelength of 1.24 μm under the experimental parameters corresponding to [16]. Electron energy losses in iron, $S_{\rm p}(\mathcal{E})$, were calculated using the ESTAR database [17]. The ionisation cross section of the K-shell, $\sigma_{\rm K}(\mathcal{E})$, was found from the analytical expression [18]. In the calculations we used the following parameters: probabilities $\omega_{\rm K} = 0.34$ [19] and $p_\alpha = 0.882$ [20], concentration of atoms $n_{\rm a} = 8.5\times10^{22}$ cm$^{-3}$, ionisation potential $\mathcal{E}_{\rm K} = 7.11$ keV and absorption length $l_{\rm a} = 18.6$ μm of $K_\alpha$ radiation with energy 6.4 keV [21], which correspond to iron under normal conditions.

Calculations by formula (8) lead to the values of the $K_\alpha$ yield that are close to those measured at laser pulse energies $\mathcal{E}_{\rm p} = 14-27$ mJ (Fig. 1), which corresponds to the intensities $I_0 = \mathcal{E}_{\rm p}/(\pi^{3/2}r_0^2t_0) = (1.45-2.8)\times10^{17}$ W cm$^{-2}$.

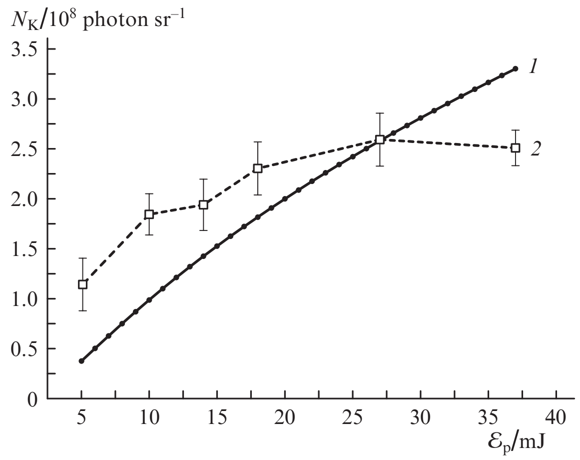


**Figure 1.** $K_\alpha$ yield from an iron target at an angle $\alpha_0 = 45°$ as a function of laser pulse energy ($\lambda = 1.24$ μm; p-polarisation; $\theta = 45°$; and diameter of the focal spot and laser pulse duration at half intensity, respectively, 10 μm and 80 fs). Curve (*1*) is the calculation by formula (8) with $\varkappa_{\max}$ determined from condition (9), and curve (*2*) is the results of measurements of the $K_\alpha$ yield [16] recalculated per laser shot.

## 5. Simulation of generation of $K_\alpha$ radiation under vacuum heating of electrons by a laser pulse near the surface of spherical clusters

Under vacuum heating of electrons by the laser field near the surface of a dense ionised cluster the electromagnetic field of a scattered wave can be described by the expression $\boldsymbol{E}_{\rm s} = r_{\rm s}\boldsymbol{E}_{\rm s0}$, $\boldsymbol{B}_{\rm s} = r_{\rm s}\boldsymbol{B}_{\rm s0}$, where $\boldsymbol{E}_{\rm s0}$ and $\boldsymbol{B}_{\rm s0}$ are the electric and magnetic fields of the wave scattered by a sphere with a permittivity $\varepsilon_{\rm p} \to -\infty$, and $r_{\rm s} < 1$ is a coefficient taking into account the absorption of laser energy by hot electrons. This approach is similar to that discussed in the previous section in the case of flat plasma [12, 13].

If the laser field is described by a plane wave $\boldsymbol{E}_{\rm L} = \boldsymbol{e}_x E_{\rm L}\exp({\rm i}k_0z - {\rm i}\omega t)$, polarised along the unit vector $\boldsymbol{e}_x$ and propagating along the $z$ axis, then the radial component of the total field $\boldsymbol{E} = \boldsymbol{E}_{\rm L} + \boldsymbol{E}_{\rm s}$ on the cluster surface is given by

$$E_r(r_{\rm s},\theta_1,\varphi_1,E_{\rm L},\rho) = -E_{\rm L}\exp(-{\rm i}\omega t)\frac{\cos\varphi_1}{\rho}$$
$$\times\sum_{n=1}^{\infty}{\rm i}^{n+1}(2n+1)P_n^1(\cos\theta_1)[j_n(\rho) + r_{\rm s}b_n^{\rm s}(\rho)h_n^{(1)}(\rho)],$$

where $\rho = k_0R$; $k_0 = \omega/c$; $R$ is the radius of the cluster; $j_n(\rho)$ and $h_n^{(1)}(\rho)$ are the spherical Bessel functions of the first and third kinds; and $P_n^1(\cos\theta_1)$ are the associated Legendre polynomials. In spherical coordinates the angle $\theta_1$ is measured from the direction of the wave vector $\boldsymbol{k}_0 \| \boldsymbol{e}_z$, and the angle $\varphi_1$ – from the direction of the polarisation vector $\boldsymbol{e}_x$. The expressions for the coefficients of the field expansion $\boldsymbol{E}_{\rm s0}$ in vector spherical wave functions have the form

$$a_n^{\rm s}(\rho) = -\frac{j_n(\rho)}{h_n^{(1)}(\rho)},\quad b_n^{\rm s}(\rho) = -\frac{[\rho j_n(\rho)]'}{[\rho h_n^{(1)}(\rho)]'}.$$

If $a_{\rm L} \ll \rho$, the electrons are emitted from the cluster to a distance that is small compared to its radius. The power absorbed by the electrons on the cluster surface is

$$P_{\rm e}(r_{\rm s},a_{\rm L},\rho) = \frac{\omega R^2}{2\pi}\int_0^{\pi}\sin\theta_1{\rm d}\theta_1\int_0^{2\pi}{\rm d}\varphi_1 W_{\rm a}(r_{\rm s},\theta_1,\varphi_1,a_{\rm L},\rho).$$

The surface density of the energy $W_{\rm a}$ (2), absorbed by electrons during the field cycle is given by

$$E_{\rm os} = |E_r(r_{\rm s},\theta_1,\varphi_1,E_{\rm L},\rho)|. \tag{10}$$

On the other hand, the power absorbed by the cluster is calculated by integrating the radial component of the energy flux density of the total field $\boldsymbol{S} = c{\rm Re}(\boldsymbol{E}\times\boldsymbol{B}^*)/(8\pi)$ over a sphere of large radius in the wave zone [22]:

$$P_W(r_{\rm s},a_{\rm L},\rho) = -\frac{m^2c^5}{4e^2}a_{\rm L}^2r_{\rm s}\sum_{n=1}^{\infty}(2n+1)$$
$$\times\{{\rm Re}[a_n^{\rm s}(\rho)] + {\rm Re}[b_n^{\rm s}(\rho)] + r_{\rm s}[|a_n^{\rm s}(\rho)|^2 + |b_n^{\rm s}(\rho)|^2]\}.$$

The coefficient $r_{\rm s}(a_{\rm L},\rho)$ is determined by solving the equation $P_{\rm e}(r_{\rm s},a_{\rm L},\rho) = P_W(r_{\rm s},a_{\rm L},\rho)$.

In calculating the $K_\alpha$ yield from a flat massive target coated by spherical clusters, the field on the upper hemisphere of the cluster, which determines the speed $v_{\rm e}$ (1) and energy $\mathcal{E}_0 = mv_{\rm e}^2/2$ of the electrons incident on the target was assumed equal to $E_{\rm os}(r_{\rm s}(a_{\rm L},\rho),\theta_1,\varphi_1,E_{\rm L},\rho)$ (10). Electrons, falling perpendicularly to the cluster surface per unit area during the laser cycle, cause the emission of

$$n_{\rm ph}(\theta_1,\varphi_1,a_{\rm L},\rho) = \frac{1}{\omega}\int_{\pi/2}^{5\pi/2}n_{\rm e}(\tau)v_{\rm e}(\theta_1,\varphi_1,\tau,a_{\rm L},\rho)$$
$$\times N_{\rm em}(\mathcal{E}_0(\theta_1,\varphi_1,\tau,a_{\rm L},\rho),\chi){\rm d}\tau \tag{11}$$

photons per unit solid angle. In a spherical coordinate system $(\chi,\varphi_2)$, associated with the upper hemisphere of the cluster, the angle $\chi$ is measured from the normal to the target surface, and is the angle of incidence of the electrons on the target. If the angle $\varphi_2$ is measured from the axis belonging to the plane of incidence of a p-polarised laser field, then

$$\cos\theta_1 = -(\sin\chi\cos\varphi_2\sin\theta + \cos\chi\cos\theta),$$

$$|\cos\varphi_1| = \frac{|\sin\theta\cos\chi - \sin\chi\cos\varphi_2\cos\theta|}{[1-(\sin\chi\cos\varphi_2\sin\theta+\cos\chi\cos\theta)^2]^{1/2}}, \quad (12)$$

which makes it possible to express the quantities $E_{\rm os}$ (10) and, consequently, $n_{\rm ph}$ (11) in variables $\chi$ and $\varphi_2$. Neglecting the change in intensity along the laser beam axis and taking into account its dependence on the radius and time (7) under the condition $R \ll r_0$, we obtain the number of photons per unit solid angle, which are generated by a laser pulse interacting with one cluster,

$$N_{\rm ph}(r,\rho) = R^2\int_0^{2\pi}{\rm d}\varphi_2\int_0^{\pi/2}{\rm d}\chi\sin\chi\frac{\omega}{2\pi}\int_{-\infty}^{\infty}{\rm d}t\,n_{\rm ph}(\chi,\varphi_2,I_{\rm L}(r,t),\rho).$$

The $\mathrm{K}_\alpha$ yield is

$$N_{\rm K}(\rho) = \frac{2\pi n_{\rm cl}}{\cos\theta}\int_0^{\infty}r{\rm d}rN_{\rm ph}(r,\rho),$$

where $n_{\rm cl}$ is the surface density of clusters on the target. In the case of densely packed clusters [$n_{\rm cl} \approx 1/(\pi R^2)$], by neglecting the contribution from the flat surface we obtain for the Gaussian pulse (7) the yield of photons per unit solid angle in a given direction

$$N_{\rm K}(\rho) = \frac{r_0^2 t_0}{\pi\cos\theta}\int_0^{\varkappa_{\max}}\sqrt{\varkappa}\,{\rm d}\varkappa$$

$$\times\int_0^{2\pi}{\rm d}\varphi_2\int_0^{\pi/2}\sin\chi{\rm d}\chi\omega n_{\rm ph}(\chi,\varphi_2,I_{\rm L}(\varkappa),\rho). \quad (13)$$

Here $\omega n_{\rm ph}$ is defined by expression (11). The upper limit of integration $\varkappa_{\max}$ is related with a condition

$$\max[\mathcal{E}_0(\chi,\varphi_2,\tau,I_{\rm L}(\varkappa_{\max}),\rho)] = \mathcal{E}_{\rm K}, \quad (14)$$

according to which the maximum energy of the electrons generated on the upper hemisphere of the cluster at a minimum intensity is equal to the ionisation potential.

The $\mathrm{K}_\alpha$ yield from a massive flat silicon target coated by closely packed spherical clusters was calculated for a 0.4-μm laser pulse at a peak intensity $I_0 = 2\times10^{17}$ W cm$^{-2}$ and other experimental parameters corresponding to [6] (see also [7, 23]). The energy losses of electrons in silicon, $S_{\rm p}(\mathcal{E})$, were calculated using the ESTAR database [17]. The ionisation cross section of the K-shell, $\sigma_{\rm K}(\mathcal{E})$, was determined by the analytical expression from paper [18]. In the calculations we used the following parameters: probabilities $\omega_{\rm K} = 0.05$ [19] and $p_\alpha = 0.974$ [20], concentration of atoms $n_{\rm a} = 5\times10^{22}$ cm$^{-3}$, ionisation potential $\mathcal{E}_{\rm K} = 1.84$ keV and absorption length $l_{\rm a} = 12.24$ μm of $\mathrm{K}_\alpha$ radiation with an energy of 1.74 keV [21], which correspond to silicon under normal conditions.

The calculation of the $\mathrm{K}_\alpha$ yield from a flat target by formula (8) at $\varkappa_{\max} = 3.37$, determined by the ionisation potential of silicon (9), gave a value that exceeds by approximately 20 times the measured value indicated in Fig. 2a for $\rho = 0$. Note that in the case of long-wavelength laser radiation with $\lambda = 1.24$ μm, the result of a similar calculation for a close-to-peak intensity $1.9\times10^{17}$ W cm$^{-2}$, which in Fig. 1 is equal to the pulse energy of 18 mJ, corresponds to the measured value with an accuracy of no worse than 30%. If we consider $\varkappa_{\max}$ at $\lambda = 0.4$ μm as a fitting parameter, the $\mathrm{K}_\alpha$ yield is described at small values of this parameter: 0.12 (p-polarisation, flat target), 0.18 (p-polarisation, target with clusters) and 0.13 ($\theta$ = 0, target with clusters) (Fig. 2). Therefore, in the case of short-wavelength radiation, the $\mathrm{K}_\alpha$ photons are generated at close-to-peak intensities $(1.7-2)\times10^{17}$ W cm$^{-2}$, and at $\lambda$ = 1.24 μm the minimum intensity determined by the ionisation potential of iron (9) is $I_{\min} = I_0\exp(-\varkappa_{\max}) = 2.8\times10^{15}$ W cm$^{-2}$ at $\varkappa_{\max} = 4.26$.

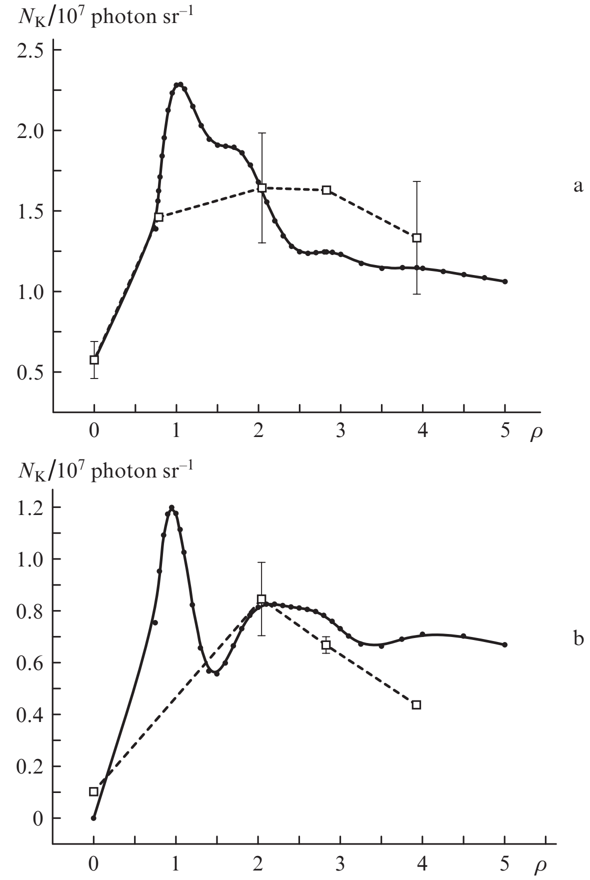


**Figure 2.** $\mathrm{K}_\alpha$ yield from a flat (at $\rho = 0$) silicon target coated with spherical clusters in the cases of (a) p-polarised ($\theta = 45°$) radiation and (b) radiation normally incident to the surface ($\theta = 0$). Curves are the calculations by formulas (8) (at $\rho = 0$) and (13) for the values of $\varkappa_{\max}$ indicated in the text. Open squares are the results of measurements [6] at $\lambda$ = 0.4 μm, $\mathcal{E}_{\rm p}$ = 12 mJ, laser pulse duration at half intensity of 100 fs, focal spot radius at 1/e$^2$ of intensity equal to 6 μm, and $\alpha_0 = 40°$.

The mechanism of vacuum heating is effective if the oscillation amplitude of hot electrons, $A_{\rm os} = v_{\rm os}/\omega$, exceeds the characteristic size of the inhomogeneity of the plasma density, $L_{\rm c}$, near the critical concentration of thermal electrons, $n_{\rm c}$ [1]. Because the oscillation amplitude is strongly dependent on the wavelength, $A_{\rm os} \propto I_{\rm L}^{1/2}\lambda^2$, and the scale of the density inhomogeneity is weakly dependent on it, $L_{\rm c} \propto I_{\rm L}^{4/27}\lambda^{-1/27}t^{29/27}$ [24], then at close durations of laser pulses the condition

$$A_{\rm os} > L_{\rm c} \quad (15)$$

can be fulfilled for long-wavelength radiation in the range of intensities from $I_{\min}$ to $I_0$ and only in a small range near $I_0$ for short-wavelength radiation. Indeed, the calculation of the size,

$$L_{\rm c} = n_{\rm c}/(\mathrm{d}N_{\rm e}/\mathrm{d}x)\big|_{N_{\rm e}=n_{\rm c}},$$

where $N_{\rm e}$ is the concentration of thermal electrons, by a one-dimensional hydrodynamic model [25] using the Virtual Laser Laboratory code [26] shows that when an aluminium target is irradiated by a laser pulse with high contrast [6], intensity $I_0 = 2\times10^{17}$ W cm$^{-2}$ and wavelength of 0.4 μm, the characteristic size of the density inhomogeneity, $L_{\rm c}$, is ~10 nm at the maximum intensity. At the same time, the oscillation amplitude of hot electrons becomes equal to a close value: $A_{\rm os} = 13$ nm. For long-wavelength radiation (1.24 μm) at the minimum intensity $I_{\min} = 2.8\times10^{15}$ W cm$^{-2}$, the oscillation amplitude is 15 nm, i.e., indeed, the $K_\alpha$ yield in this case is determined by the ionisation potential (9) rather than condition (15).

## 6. Calculation of the temperature of hot electrons

The average energy of electrons falling into the target coated by closely packed clusters was found as the ratio of the energy absorbed by the electrons on the upper hemispheres of clusters to the number of these electrons at a certain value of $\varkappa_{\max}$:

$$\mathcal{E}_{\rm av}(\rho) = \frac{\int_0^{\varkappa_{\max}}\sqrt{\varkappa}\,\mathrm{d}\varkappa\int_0^{2\pi}\mathrm{d}\varphi_2\int_0^{\pi/2}\sin\chi\,\mathrm{d}\chi\,W_{\rm a}(\chi,\varphi_2,a_{\rm L}(\varkappa),\rho)}{\int_0^{\varkappa_{\max}}\sqrt{\varkappa}\,\mathrm{d}\varkappa\int_0^{2\pi}\mathrm{d}\varphi_2\int_0^{\pi/2}\sin\chi\,\mathrm{d}\chi\,n(\chi,\varphi_2,a_{\rm L}(\varkappa),\rho)}.$$

Taking into account the dependences of the surface density of the absorbed energy, $W_{\rm a}$ (2), and the surface density of the electrons, $n$ (3), on the field $E_{\rm os}$ (10), we obtain

$$\mathcal{E}_{\rm av}(\rho) = \frac{\eta mc^2 a_{\rm L}^2(I_0)}{2\gamma\rho^2}\times$$
$$\frac{\int_0^{\varkappa_{\max}}\mathrm{e}^{-3\varkappa/2}\sqrt{\varkappa}\,\mathrm{d}\varkappa\int_0^{2\pi}\mathrm{d}\varphi_2\int_0^{\pi/2}\sin\chi\,\mathrm{d}\chi|\cos\varphi_1|^3\,\Sigma^3(\cos\theta_1,a_{\rm L}(\varkappa),\rho)}{\int_0^{\varkappa_{\max}}\mathrm{e}^{-\varkappa/2}\sqrt{\varkappa}\,\mathrm{d}\varkappa\int_0^{2\pi}\mathrm{d}\varphi_2\int_0^{\pi/2}\sin\chi\,\mathrm{d}\chi|\cos\varphi_1|\;\Sigma(\cos\theta_1\;a_{\rm L}(\varkappa)\;\rho)},\quad(16)$$

where

$$\Sigma(\cos\theta_1,a_{\rm L}(\varkappa),\rho) = \Bigg|\sum_{n=1}^{\infty}\mathrm{i}^n(2n+1)P_n^1(\cos\theta_1)$$
$$\times[j_n(\rho)+r_{\rm s}(a_{\rm L}(\varkappa),\rho)b_n^{\rm s}(\rho)h_n^{(1)}(\rho)]\Bigg|,$$

and $\cos\theta_1$ and $|\cos\varphi_1|$ are defined by (12). At normal incidence of laser radiation ($\theta = 0$), the expression for the mean energy (16) has the form

$$\mathcal{E}_{\rm av}(\rho) = \frac{\eta mc^2 a_{\rm L}^2(I_0)}{3\gamma\rho^2}$$
$$\times\frac{\int_0^{\varkappa_{\max}}\mathrm{e}^{-3\varkappa/2}\sqrt{\varkappa}\,\mathrm{d}\varkappa\int_0^1\mathrm{d}\xi\,\Sigma^3(-\xi,a_{\rm L}(\varkappa),\rho)}{\int_0^{\varkappa_{\max}}\mathrm{e}^{-\varkappa/2}\sqrt{\varkappa}\,\mathrm{d}\varkappa\int_0^1\mathrm{d}\xi\,\Sigma(-\xi,a_{\rm L}(\varkappa),\rho)}.\quad(17)$$

Sumeruk et al. [6] measured the temperature of hot electrons simultaneously with the $K_\alpha$ yield at normal incidence of laser radiation on the target caoted with clusters. The temperature was taken equal to that of bremsstrahlung hard X-rays from the target, which has an exponential spectrum. Figure 3 shows that the measured values of the temperature at different diameters of the clusters correspond to the values of the mean electron energy calculated by formula (17) at $\varkappa_{\max} = 0.13$ defined in the previous section by comparing the results of calculations and measurements of the $K_\alpha$ yield from a silicon target with clusters under normal incidence of laser radiation.

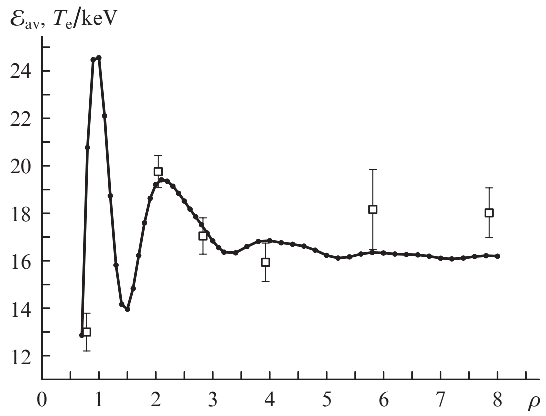


**Figure 3.** Mean electron energy calculated by formula (17) for $\varkappa_{\max}$ indicated in the text at $I_0 = 2\times10^{17}$ W cm$^{-2}$ and $\lambda = 0.4$ μm. Open squares are the measured values of the temperature $T_{\rm e}$ of hot electrons at different diameters of the clusters [6].

## 7. Conclusions

The calculation of the temperature of hot electrons supports the conclusion that under the experimental conditions [6] hot electrons and, therefore, $K_\alpha$ radiation are produced at close-to-peak intensities. The constructed model of generation of $K_\alpha$ radiation under vacuum heating of electrons by a femtosecond laser pulse near the surface of spherical clusters describes the measurements [6] at a cluster diameter of less than or of the order of the wavelength for both p-polarised radiation and radiation normally incident to the surface.

When flat targets are irradiated by femtosecond laser pulses with an intensity $2\times10^{17}$ W cm$^{-2}$, the observed decrease (by 100 times) in the coefficient of conversion of the laser energy into the energy of $K_\alpha$ radiation with decreasing wavelength from 1.24 to 0.4 μm [6, 16] cannot be explained only by a decrease in energy of hot electrons, which is proportional to $\lambda^2$, and by a decrease in probability of radiative de-excitation of atoms and an increase in self-absorption in silicon as compared with iron. In the case of short-wavelength laser radiation the efficiency of vacuum electron heating at the given intensity was limited by condition (15).

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