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Development of a prototype compact fibre frequency synthesiser for mobile femtosecond optical clocks

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Abstract. **A prototype compact fibre frequency synthesiser based on a femtosecond erbium fibre laser and an original hybrid highly nonlinear fibre is developed and preliminarily studied. This synthesiser will ensure an extremely low relative instability of synthesised frequencies (down to 10^{-17}) with the use of a corresponding optical standard and will be used in mobile optical clocks. The realised frequency stabilisation principle makes the synthesiser universal and allows it to transfer the frequency stability of various types of optical standards to the synthesised radio- and optical frequencies.**

Keywords: *femtosecond frequency synthesiser, optical clocks, femtosecond fibre laser, hybrid highly nonlinear fibre.*

1. Introduction

A femtosecond frequency synthesiser is the main unit of clockworks in modern optical quantum clocks [1 – 4], also called femtosecond optical clocks (FOCs). This synthesiser is used for direct division of the optical frequency of atomic oscillators (optical frequency standards), which provides the possibility for direct synthesis of standard radiofrequencies and formation of time marks with the stability and accuracy of the optical standard. In practice, the FOC accuracy is limited by errors introduced by the clockwork (frequency synthesiser instability) and by the fundamental factors related to the quantum nature of atomic oscillators.

The main component of any femtosecond frequency synthesiser designed to transfer the stability and accuracy of optical frequency standards to the RF range is a femtosecond mode-locked master oscillator with an active stabilisation system. This laser must emit a stable comb of equidistant optical frequencies (corresponding to the longitudinal modes of the laser), whose parameters can be determined with an extremely high accuracy upon stabilisation to an optical frequency standard.

First femtosecond synthesisers developed in the very beginning of 2000s were made using solely solid-state femtosecond lasers based on discrete elements, mainly titanium-sapphire [5, 6], chromium-forsterite [7], and ytterbium-doped crystals [8]. Under proper laboratory conditions, these laser systems, in principle, allow one to completely use the potential of modern optical standards in transfer of their stability to FOCs. However, due to principal design constraints, these systems cannot be used for creating mobile FOCs. This is related both to a large size of these laser systems, to their high energy consumption and low efficiency (first of all, this concerns titanium-sapphire and chromium-forsterite systems), and to the problems with long-term maintenance of a precision alignment of discrete laser optics under the conditions of external disturbance upon outdoor autonomous mobile use. At the same time, the realisation of mobile FOCs will allow one to considerably extend the application field of optical quantum clocks. In particular, one of the most topical applied problems that can be solved with the use of mobile FOCs is an increase in the accuracy of global navigation satellite systems. For example, to achieve a positioning accuracy of ~10 cm, the long-term instability of reference radiofrequencies in on-board synchronisation systems of global navigation sattelite systems must be significantly decreased, down to $\sim 10^{-16}$ [9]. This can be done only by replacing the currently used microwave frequency standards with RF synthesisers by mobile FOCs (i.e., by optical frequency standards with femtosecond frequency synthesisers).

At present, the most promising approach to creation of mobile FOCs is the use of fibre optics as a base for femtosecond frequency synthesisers. The principal possibility of using fibre laser systems for these purposes has already been demonstrated in several works [10 – 14]. Fibre femtosecond lasers are distinguished by extreme compactness, low energy consumption, and high efficiency. In addition, these laser systems are not so sensitive to external disturbances, which can cause optical misalignment, because the use of discrete aligned optical units in them can be reduced to a minimum or even excluded. For on-board satellite systems, it seems most reasonable to use laser systems based on erbium-doped fibres since their radiation resistance is higher than that of erbium fibres. The working wavelength range of erbium fibre lasers is more convenient for varying the system parameters and for creating all-fibre schemes. To date, the published works on femtosecond frequency synthesisers based on erbium fibre laser systems were not aimed at creating specifically mobile devices. A known variant of compact FOCs using a fibre laser system and an optical standard was developed based on a compact methane frequency standard (He – Ne/CH_4), whose relative instability did not exceed 10^{-14} [4, 14].

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The aim of the present work is to create of a prototype compact energetically efficient fibre synthesiser of optical frequencies based on a low-power femtosecond erbium fibre laser system in combination with a compact optical frequency standard and to study the limiting characteristics of this prototype in order to demonstrate the possibility of creating mobile FOCs with extremely low instability.

2. Concept of femtosecond frequency synthesiser

The concept of femtosecond frequency synthesiser for mobile FOCs that we use as a base is illustrated in Fig. 1.

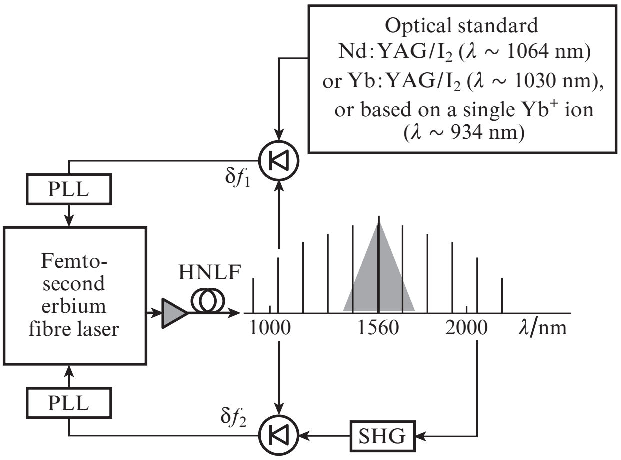

Figure 1. Structure and stabilisation principle of a femtosecond frequency synthesiser for mobile FOCs: (HNLF) highly nonlinear fibre; (PLL) phase-locked loop; (SHG) second harmonic generator; (δf_1) RF beating (difference frequency) between the optical frequency standard reference and the closest short-wavelength line of the femtosecond optical frequency comb; (δf_2) RF beating (difference frequency) between the optical frequency standard reference and the corresponding second harmonic of the long-wavelength line of the femtosecond optical frequency comb. (There exists a possible variant $\delta f_1 = \delta f_{\text{offset}}$, when the frequency-doubled long-wavelength lines of the optical frequency comb are mixed with the short-wavelength lines with the help of an $f-2f$ interferometer).

A femtosecond erbium fibre master oscillator generates a comb of equidistant optical frequencies with a width of several tens of nanometres near 1560 nm. After amplification of the femtosecond radiation power by an erbium fibre-optic amplifier, this comb is multiply broadened in a special highly nonlinear fibre (HNLF) due to various nonlinear effects (self-phase modulation, four-wave mixing, etc. [15]). In principle, the frequency of each comb line can be determined (with accuracy to phase) because it is determined by two particular parameters – the fundamental pulse repetition rate of the master oscillator (intermode frequency spacing) f_{rep} and the so-called comb offset frequency f_{offset} [1–3]. The first parameter is controlled by changing the optical length of the master oscillator cavity, and the second parameter is controlled by changing the intracavity dispersion [16].

For precision frequency synthesis, it is necessary to lock a given femtosecond optical frequency comb to one or another optical frequency standard by one or another method. As such a standard for creating mobile FOCs, one can use a secondary optical frequency standard based on a molecular-iodine-stabilised Nd:YAG [17] or Yb:YAG [18] lasers with frequency doubling developed in the Institute of Laser Physics SB RAS. This frequency standard is the best for realisation of precision mobile femtosecond metrology systems because it is very compact, reliable, and energetically efficient. The extremely narrow nonlinear saturated absorption resonances at the hyperfine structure components of molecular iodine (forbidden dipole transitions near λ = 532 and 515 nm) make it possible to minimise the long-term frequency instability to $\sim 10^{-15}$ [19]. The Nd:YAG/I$_2$ and Yb:YAG/I$_2$ standards represent simultaneously two optical reference lines each – at the fundamental laser wavelengths ($\sim$1064 and $\sim$1030 nm, respectively) and at the second harmonic wavelengths corresponding to the clock transition ($\sim$532 and $\sim$515 nm, respectively).

This specific feature of the considered standards, in principle, allows one to create several variants of femtosecond frequency synthesisers differing by the method of locking of the optical frequency comb to the frequency standard. It seems that the optimal variant for creating mobile FOCs based on femtosecond erbium fibre lasers is the use of only the long-wavelength reference line of the optical frequency standard (Fig. 1). The closest short-wavelength component of the femtosecond comb is phase-locked to the long-wavelength reference line of the used standard (due to phase autotuning of the frequency of a separate comb line to the frequency of a standard line). The complete stabilisation of all the comb components can be then achieved by two methods, namely, by detection with a nonlinear interferometer ('$f-2f$ interferometer' as in [10–12]) and successive phase locking of the offset frequency f_{offset} to the reference radiofrequency generated by the synthesiser itself or directly by phase locking of the second harmonic of the long-wavelength component of the femtosecond frequency comb to the long-wavelength reference of the optical standard. The second method is structurally simpler because it does not require an accurately aligned nonlinear interferometric scheme, but it cannot be realised without a rather high intensity and coherence of the separate long-wavelength line of the comb. Thus, the proposed concept ensures a flexibility of the synthesiser design. In addition, the synthesised femtosecond comb can also be locked to other promising optical frequency standards of the near IR region, for example, to a standard developed in the Institute of Laser Physics based on single Yb$^+$ ion, in which a clock laser operates at a wavelength of $\sim$934 nm (short-wavelength edge of a broadened femtosecond frequency comb) [20]. The expected long-term frequency instability of this standard is $\sim 10^{-17}$ [21].

The main problem related to the realisation of the proposed synthesiser type is that the femtosecond frequency comb must be considerably broadened – to cover an optical octave. The comb must cover (with a safety margin) the wavelength range 1000–2000 nm. In addition, the intensity and coherence of the spectral components at the edges of this supercontinuum must be sufficiently high for reliable detection of RF beating of these components with the long-wavelength reference of the used optical frequency standard. For the maximum efficiency of the digital system of phase-locked loop (PLL), the signal-to-noise ratio in the RF beating spectrum must be, as a rule, no lower than 25 dB (within a 100 kHz band). The problem is also complicated by specific constraints on the weight–size and energy parameters of the mobile system (the system must be operable at a moderate optical power typical for femtosecond erbium fibre lasers without high-power amplifiers).

Thus, the main steps of our investigations were the development of a special HNLF in order to solve the problem of creation of a functioning prototype synthesiser, the development of systems for active stabilisation of this synthesiser, and the determination of its limiting characteristics from the viewpoint of the possibility to transfer the stability of optical frequency standards to the RF region.

3. Hybrid HNLF

The spectral broadening or supercontinuum generation in optical HNLFs is, as a rule, related to the Kerr nonlinearity efficiency. Generally speaking, the broadening degree is determined not only by nonlinear effects, but also by the dispersion properties of the optical fibre. In particular, one of the spectral supercontinuum generation regimes is realised in the anomalous dispersion region and is related to the formation of high-order solitons [22]. The pulse evolution in this regime is characterised by a complex dynamics: solitons can separate into several individual components, and the self-scattering becomes more intense up to the formation of Raman solitons [23]. The quantum nature of the Raman effect and the instability caused by amplification of noises in the region of anomalous dispersion [24, 25] suggest a low spectral coherence. This scenario is possible only in the case if the optical HNLF length noticeably exceeds the dispersion length. The latter, under the conditions typical for our problem (femtosecond pulse duration, $\lambda \sim 1560$ nm, quartz fibre) can be from several tens of centimetres to several meters. Figure 2 shows the experimental erbium laser spectrum broadened in a HNLF ~5 m long with anomalous dispersion at the centre pump wavelength (the parameters of the used fibre and pump radiation are given in the capture to Fig. 2). One can clearly see an asymmetry of the spectrum with respect to the centre wavelength, which is caused by the Raman self-scattering: the long-wavelength part of the spectrum contains more power than the short-wavelength region.

Another spectral supercontinuum generation regime is the classical spectral broadening due to nonlinear effects, first of all, due to self-phase modulation. This regime is realised in the regions of both anomalous and normal dispersion, and it is the dispersion characteristics that to a large extent determine the shape and the broadening range of the spectrum [26].

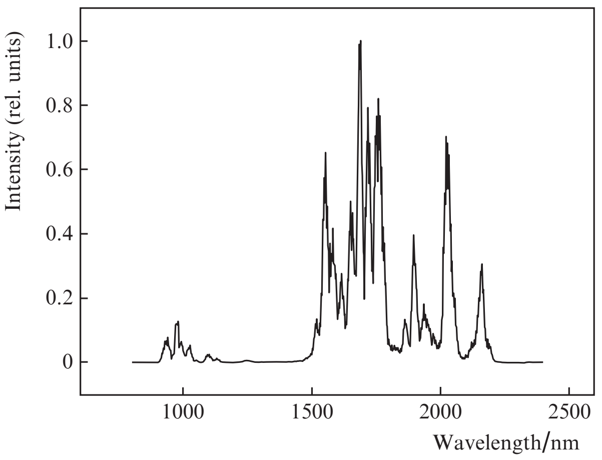

Figure 2. Supercontinuum spectrum in a HNLF ~5 m long. Nonlinear fibre parameter $\gamma = 10$ W^{-1} km^{-1}, dispersion parameter $D = 0.67$ ps nm^{-1} km^{-1} for $\lambda = 1560$ nm (centre pump radiation wavelength), average power $P \approx 145$ mW, pulse duration $t_0 \approx 150$ fs.

The generation of a supercontinuum with an octave width is not the sole condition for obtaining desired RF beatings between the optical frequency standard reference and the femtosecond optical frequency comb. The necessity to minimise noises (including Raman noises) imposes restrictions on the length of the nonlinear fibre that broadens the spectrum. In such experiments, beatings with a satisfactory signal-to-noise ratio were obtained for supercontinuums broadened in HNLFs shorter than one meter [11, 27] or about one meter long [28, 29]. In the present work, we consider a supercontinuum generation system based on an erbium fibre laser with the following characteristic: the average power $P \sim 100-200$ mW, the pulse duration $t_0 \sim 100-200$ fs, and the pulse repetition rate ~100 MHz. We use HNLFs doped with germanium oxide with the nonlinear parameter $\gamma = 10$ W^{-1} km^{-1} and a chromatic dispersion of $-0.6-0.7$ ps nm^{-1} km^{-1} at the wavelength $\lambda = 1560$ nm; the dispersion varies depending on the fibre core diameter (the outer fibre diameter is 110–122 μm). For this scheme, a theoretical estimate of the minimal HNLF length at which the spectrum can be broadened to an octave due to self-phase modulation yields tens of centimetres provided that chromatic dispersion is absent. However, the width of experimental spectra, for example, for 50-cm HNLFs with the mentioned nonlinear parameter is considerably smaller. Since this difference is caused by chromatic dispersion, the spectral broadening efficiency can be increased by controlling the dispersion profile of the fibre. A successful example of this optimisation can be HNLFs with variable dispersion [30], but a typical length at which the dispersion considerably changes in these fibres is tens of meters, which is one–two orders of magnitude large than the HNLF length optimal for our purpose. To solve this problem, we studied hybrid HNLFs [31, 32] consisting of two or more segments with different dispersion profiles. The principle of such a fibre is demonstrated in Fig. 3. Hybrid fibres provide wide possibilities for controlling dispersion parameters at small (to 1 m) lengths.

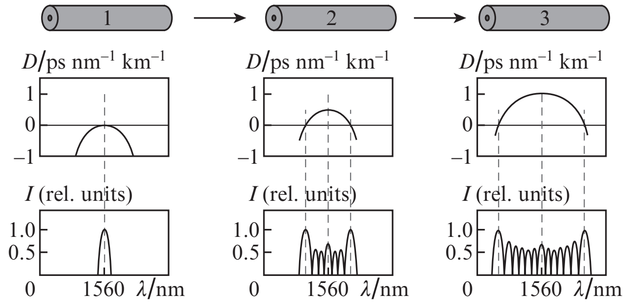

Figure 3. Concept of a hybrid HNLF: variations in the dispersion profile of the fibre and evolution of the ultrashort pulse spectrum from segment 1 to segment 3.

The dispersion parameters of hybrid HNLFs optimal for achieving the maximum width of spectral supercontinuums were searched by numerical simulation. The evolution of an individual pulse was calculated using the nonlinear Schrödinger equation and the split-step Fourier method [15]. The connection schemes and the dispersion profiles of individual segments of hybrid fibres that can provide efficient supercontinuum generation in the range 900–2200 nm were found in [32].

This spectral range must be covered for locking the femtosecond optical frequency comb to references of optical standards.

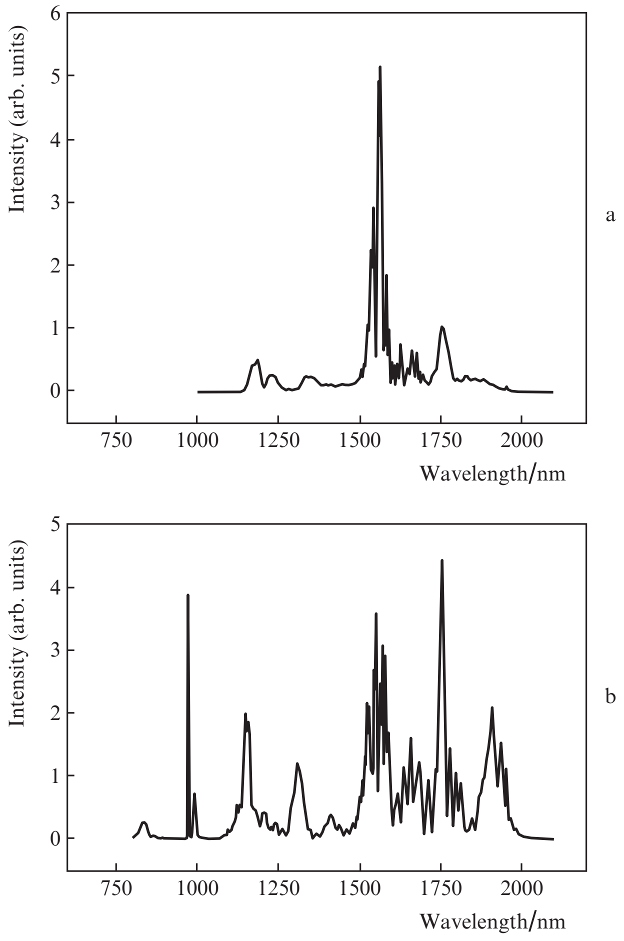

Figure 4. Supercontinuum generation in a HNLF shorter than 1 m (γ = 10 W^{-1} km^{-1}, λ = 1560 nm, $P \approx$ 140 mW, $t_0 \approx$ 150 fs): spectra broadened in a single fibre 50 cm long (dispersion parameter D = –0.54 ps nm^{-1} km^{-1}) (a) and in a hybrid fibre consisting of two segments with lengths of 30 and 50 cm (D = –0.54 and 0.67 ps nm^{-1} km^{-1}, respectively) (b).

Figure 4 presents the experimental results for a single fibre and a hybrid fibre consisting of two segments (corresponding to segments 1 and 2 in Fig. 3). The parameters of the first hybrid fibre segment are identical to the parameters of the single fibre, because of which the beginning of the pulse evolution in both fibres is the same. The centre wavelength lies in the region of low (in absolute value) normal dispersion, which ensures the dominance of self-phase modulation over dispersion and, hence, efficient spectral broadening. As the pulse propagates in the fibre, the spectral wings reach the regions with increasing absolute values of dispersion, and the spectral broadening becomes weaker. The dispersion profile of the second segment of the hybrid fibre is close to parabolic, namely, the centre wavelength lies in the anomalous dispersion region and the two zero points of the profile are close to the wings of the spectrum. The advantage of this hybrid fibre configuration is that the change in its dispersion profile from one segment to another correlates with the pulse spectrum evolution (Fig. 3) and the energy transfer from the spectrum centre to the periphery with pulse propagation is not reduced by the negative effect of dispersion. The comparison of the broadened spectra convincingly demonstrates the following advantage of hybrid fibres: in the case of a broadening HNLF shorter than one meter, the supercontinuum covers the desired range (1000–2000 nm).

The hybrid fibres used in this work were produced by the MCVD technique in cooperation with the Fibre Optics Research Centre, Russian Academy of Sciences. The fibre segments were spliced using a Fujikura FSM-100P fusion splicer. Optical losses in the splice region did not exceed 0.1 dB.

4. Synthesiser stabilisation system

The scheme of the experimental setup developed to study the maximum achievable frequency stability of the proposed femtosecond synthesiser is shown in Fig. 5.

As a femtosecond master laser in the synthesiser for mobile FOCs, we use a passively mode-locked erbium fibre laser. The laser has a hybrid linear-ring cavity similar to the configuration proposed by us in [33]. The mode-locking regime in the laser is achieved due to nonlinear polarisation evolution [34]. The laser cavity consists of two parts, an all-fibre ring part and a short linear section with several bulk optical

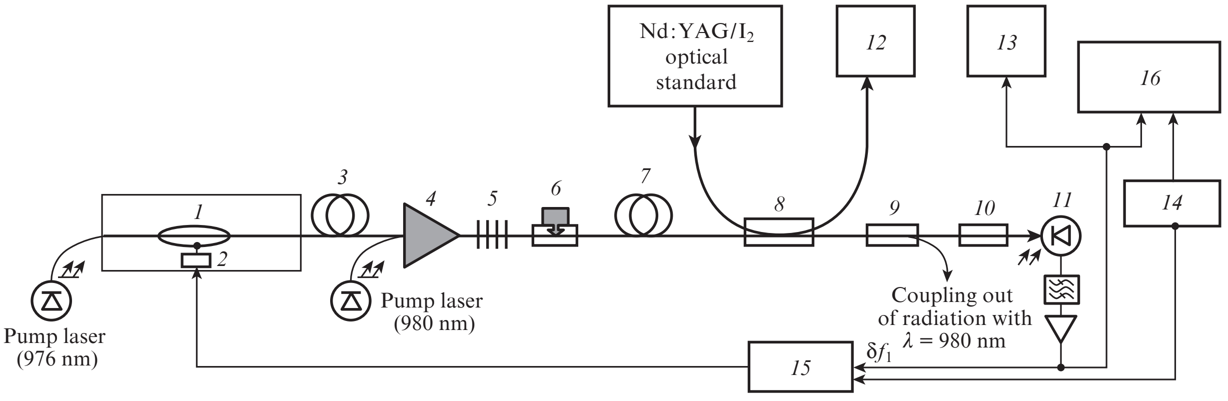

Figure 5. Scheme of the experimental setup for measuring the frequency instability introduced by a synthesiser for FOCs:
(*1*) femtosecond erbium fibre laser; (*2*) piezo translator; (*3*) dispersion compensating fibre; (*4*) erbium fibre-optic amplifier; (*5*) fibre Bragg grating at λ = 980 nm; (*6*) polarisation controller; (*7*) hybrid HNLF; (*8*) fused fibre coupler; (*9*) spectral multiplexer; (*10*) interference filter; (*11*) photodiode; (*12*) optical spectrum analyser; (*13*) RF spectrum analyser; (*14*) hydrogen optical standard; (*15*) phase–frequency detector; (*16*) frequency counter.

elements. The cavity length is controlled using piezoceramic translators (*2*), which moves the mirrors in the linear part of the cavity. The main characteristics of the master oscillator are as follows: the centre wavelength ~1560 nm, the spectral width ~30 nm, the pulse duration ~150 fs, the pulse repetition rate ~100 MHz, and the average output power ~15 mW. The femtosecond laser is pumped by a fibre-coupled diode laser with a wavelength of 976 nm and a power of ~400 W.

The laser was added with a fibre-optic power amplifier (*4*) created in cooperation with the Institute of Automation and Electrometry, Siberian Branch of the Russian Academy of Sciences. The amplifier is designed according to the classical EDFA scheme [35] based on an erbium-doped fibre (nLight Liekki Er80-8/125), which is concurrently pumped at a wavelength of ~980 nm. The unabsorbed pump radiation returns to the amplifier by a fibre Bragg grating (*5*) written in the output waveguide of the amplifier. The fibre-optical path between the femtosecond master laser and the hybrid HNLF (*7*) also contains a segment of a dispersion compensating fibre (Corning MetroCor), whose length was experimentally selected so that the pulse duration at the input of fibre (*7*) was optimal for spectral broadening. The average output amplifier power reaches ~200 mW.

The radiation of the comb broadened to an optical octave enters a spectrally selective fibre-optic mixer, in which the frequency of the long-wavelength ($\lambda \sim 1064$ nm) reference of the Nd:YAG/I_2 standard is mixed with the frequency of the closest short-wavelength line of the optical frequency comb. The mixer consists of a fused fibre coupler (*8*), a spectral multiplexer (*9*), and an interference filter (*10*). At the exit of the mixer, a photodiode detects an RF beating signal with the frequency δf_1 (difference frequency). This frequency can vary from zero to $f_{\rm rep}/2$ and is determined by the parameters of the femtosecond master oscillator cavity (optical length and dispersion).

The short-wavelength line of the optical frequency comb is stabilised with respect to the optical standard frequency by phase control of the frequency δf_1. The main element of the electronic PLL system is a digital phase–frequency detector (PFD) (*15*). In the PFD, the frequencies and phases of two RF signals (reference and stabilised) are compared. As a reference signal in this experiment, we use the signal of a commercial hydrogen frequency standard (*14*) (Ch1-75) with a relative instability of 10^{-14} per 100 s. When the frequencies of the input signals are mismatched, an error signal is formed at the detector output. After filtration and amplification, this signal is fed to actuating devices, which tune the frequency (and phase) of the stabilised signal. The listed elements form the PLL circuit. Appropriate filters and amplifiers (integrators) placed at the PFD output are used to form two feedback loops, fast and slow. An actuating device for the slow loop is a piezoceramic stack with a large motion path, which moves the intermediate mirror and thus compensates slow drifts of the optical frequency (caused by thermal and mechanical relaxation processes in the laser cavity units). The maximum sensitivity of this regulator is ~1 MHz V^{-1}. The effective band of the disturbance compensation with the slow loop is shorter than 100 Hz; the optical frequency tuning range in this case considerably exceeds $f_{\rm rep}$, which makes it possible to compensate the frequency drift during a long time even without thermal stabilisation of the laser cavity. For compensation of fast disturbances and phase locking of the frequency δf_1, i.e., for complete stabilisation of the frequency of the short-wavelength line of the optical frequency comb with respect to the optical standard, we use fast piezoceramics with a short motion path and a high resonance frequency (exceeding 100 kHz). To realise the maximum compensation band using this ceramics, we developed a massive low-quality-factor construction from bronze alloy (monolithic mount), on which we glued (without any consoles) a ceramic sample with a small mirror, which terminates the linear part of the cavity. As a result, the maximum effective compensation band in the PLL circuit with the fast ceramics reaches ~30 kHz and is limited mainly by decreasing sensitivity and an excessively large phase incursion at high frequencies (see the measured frequency characteristics in Fig. 6). The maximum sensitivity of the fast piezoceramic translator is ~50 kHz V^{-1}.

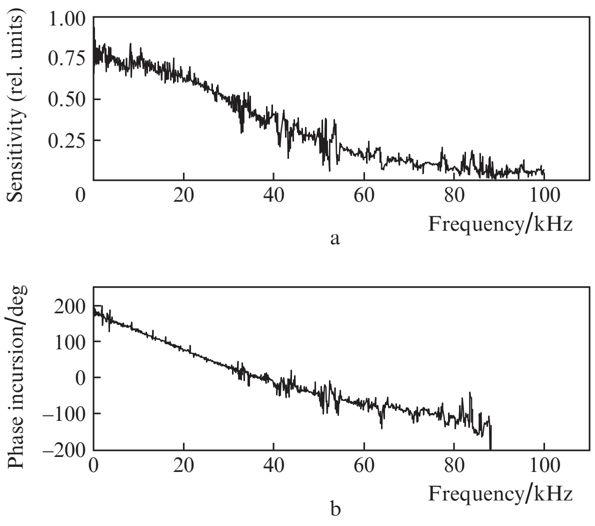

Figure 6. Frequency characteristics of the piezo translator in the fast loop of the PLL circuit.

The developed PFD block is build based on digital elements, namely, an input comparator (AD96685), a counter/divider (MC10H131M), and a microcircuit with the PFD function (AD9901). In addition, the output circuit contains filters and amplifiers forming proper amplitude–frequency characteristics of feedback loops. The high response speed of the used modern digital elements allows us to achieve a wide compensation band of the PLL system without using analogous mixer or hybrid analogue-digital PFDs [36].

To exclude the possibility of exit outside the piezoceramic translator motion range and provide the possibility of unlimited operation time of the PLL system, we additionally developed a system for thermal stabilisation of the femtosecond fibre master oscillator cavity, which maintained a required laser temperature (in a thermostatted box) with an accuracy to 0.1 °C.

5. Measurement results and discussion

One of the main goals of our study was to measure the characteristics limiting the stability of the prototype mobile femtosecond synthesiser for FOCs. We determined the random frequency error, which characterises the efficiency of the active stabilisation system of the synthesiser, with respect to the optical standard frequency (relative frequency instability expressed via the Allan parameter), i.e., determined the limiting possibilities of the system in the transfer of the optical standard stability and accuracy to synthesised frequencies.

For primary adjustment and start-up of the PLL system, we measured the RF spectrum of beatings (δf_1) between the reference frequency of the optical standard (Nd:YAG/I$_2$) and the closest ($\lambda \sim 1064$ nm) optical frequency of the synthesiser. For this purpose, the signal from the photodiode used in the spectrally selective fibre-optic mixer was sent to a broadband Rohde&Schwarz FSP spectrum analyser. When adjusting the synthesiser, the main attention was paid to such key parameters of the beating signal spectrum as the line width and shape, the frequency drift rate, and the signal-to-noise ratio. The measure RF spectrum of beatings between the reference frequency of the optical standard and the closest optical frequency of the synthesiser are shown in Fig. 7 for a free-running regime and with the operating PLL system.

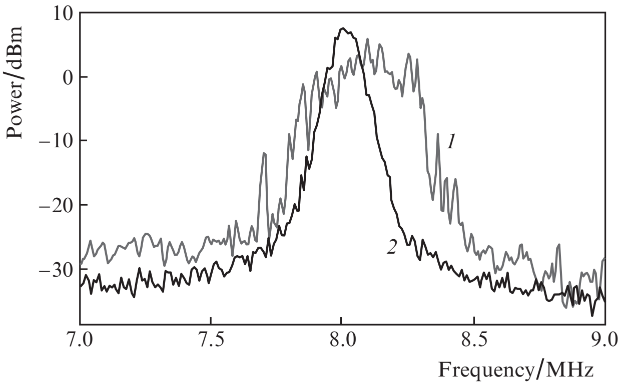

Figure 7. RF spectra of the beating signal δf_1 in the free-running synthesiser regime (when the PPL system is inactive) (*1*) and in the case when the PPL system is switched on (*2*). The analyser resolution is 10 kHz.

The measurement results revealed that, in the free-running synthesiser operation regime, the beating signal undergoes very strong frequency disturbances – the RF spectrum of the signal is broadened by an acoustic jitter to $\sim$1 MHz and, as a result, has an irregular (non-Lorentzian) shape [curve (*1*) in Fig. 7]. In addition, one observes a strong frequency drift with a rate up to 0.5 MHz s^{-1} due to thermal and mechanical relaxation processes in the cavity of the femtosecond master oscillator of the synthesiser. Nevertheless, the signal-to-noise ratio for beatings (determined as the ratio of the maximum signal power to the noise threshold) reaches approximately 30 dB, which is sufficient for reliable operation of PLL blocks in the synthesiser. Switching on of the PLL system efficiently stabilises the beating frequency, which is evidenced by the RF spectrum [curve (*2*) in Fig. 7]. The line width decreases to several kilohertz. The line itself takes a more regular (close to Lorentzian) shape, and the frequency drift disappears (compensated by the slow PLL loop). In addition, to qualitatively estimate the operation efficiency of the PLL system, we measured the noise spectrum at the output of an additional frequency detector (Fig. 8). This was done using an analogue-digital transducer connected to a computer, which quickly performed the Fourier-transform of the signal. Thus, the qualitative pattern of the spectral distribution of the amplitude of frequency–phase disturbances in the synthesiser was visualised. As is seen from the preliminarily obtained data, in the phase-locking regime [curve (*2*) in Fig. 8], the noise is suppressed in the frequency band to $\sim$25 kHz. Subsequently, this band was extended to $\sim$30 kHz.

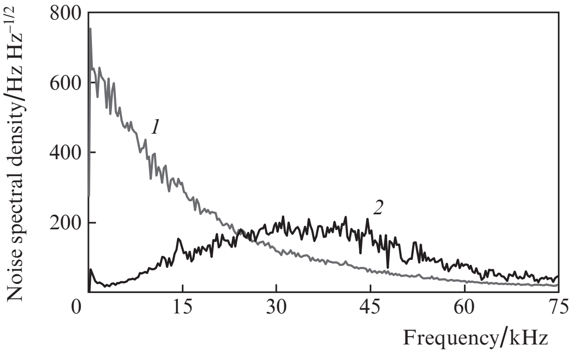

Figure 8. Noise spectra at the output of the frequency detector in the free synthesiser regime (when the PPL system is inactive) (*1*) and in the case when the PPL system is switched on (*2*).

At the final stage of the preliminary investigations, we performed precision long-term measurements of the beating signal frequency δf_1 (difference between the optical standard frequency and the closest optical frequency of the synthesiser) in the phase-locking regime (with the PLL system switched on). The measurements were performed using a precision Pendulum CNT-91 frequency counter, whose reference input was fed by a signal from the same hydrogen frequency standard whose signal was sent to the reference input of the PFD. The calibrated relative random error of the RF measurement by the mentioned frequency counter allows one to measure the long-term instability of megahertz RF frequencies at a level of $\sim 10^{-13}$ (for averaging times no shorter than 100 s).

As a result, we measured the absolute values of the beating signal frequency for a time exceeding 5000 s (single measurement time 1 s) (Fig. 9). Using a specialised AlaVar 5.0 software, we calculated the Allan parameter (Allan standard deviation) for relative random deviations from the reference RF (8.0 MHz) and plotted its time dependence [curve (*1*) in Fig. 10]. The obtained data show that the relative instability of the beating signal frequency δf_1 does not exceed $\sim 5.4\times10^{-10}$ per 100 s, $\sim 1.9\times10^{-10}$ per 500 s, and $\sim 1.2\times10^{-10}$ per 1000 s.

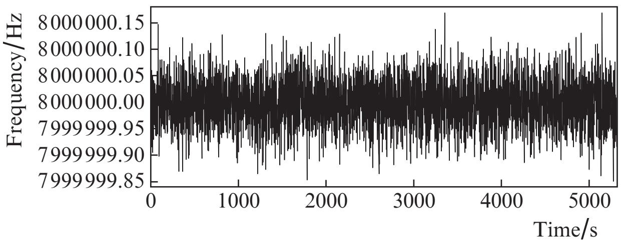

Figure 9. Time dependence of the absolute values of the beating signal frequency δf_1 (single measurement time is 1 s).

The instability of the synthesiser optical frequency ($\sim 2.82\times10^{14}$ Hz) is composed of its random deviations from the optical standard frequency, which must be followed by the phase-locked closest line of the femtosecond frequency comb with a present constant shift (in our case, equal to 8.9 MHz), and of the instability of the optical standard itself. It is the accuracy of phase locking to the reference signal (stability of frequency beatings) that is determined by the operation quality of the PLL system, i.e., by its intrinsic error. The recal-

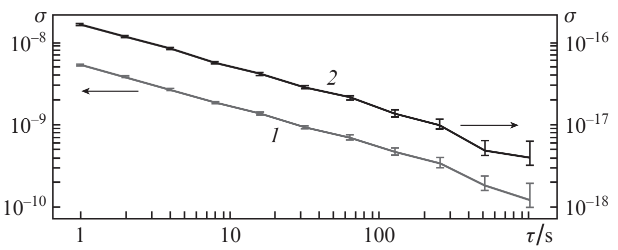

Figure 10. Allan parameter for relative random deviations of the RF beating signal δf_1 from the reference radiofrequency (*1*) and for relative random deviations of the synthesiser optical frequency from the reference optical standard frequency to which the synthesiser frequency is phase-locked (*2*).

culation of the Allan parameter for relative random deviations of the closest optical frequency of the synthesiser from the frequency of the reference optical standard (to which phase locking takes place) yields values corresponding to curve (*2*) in Fig. 10: $\sim 1.7\times10^{-17}$ per 100 s, $\sim 5.1\times10^{-18}$ per 500 s, and $\sim 4.0\times10^{-18}$ per 1000 s. It is these values that characterise the relative random error introduced by the PLL system of the synthesiser when locking its frequency to the optical standard frequency.

Thus, the developed PLL system of the synthesiser can, in principle, transfer the long-term stability of all currently available optical frequency standards to the synthesised optical and radiofrequencies without its deterioration. In the future, to realise the completely functional prototype of a mobile femtosecond synthesiser and a FOC based on this synthesiser, the experimental setup will be added by the second PLL circuit for stabilisation of a long-wavelength edge of the optical frequency comb to an optical standard (see Fig. 1). The final results will be published in a separate paper.

6. Conclusions

A prototype of a compact femtosecond frequency synthesiser based on a femtosecond erbium fibre laser system is preliminarily experimentally studied. A specially developed hybrid HNLF used in the synthesiser provides efficient generation of an octave-spanning optical frequency comb with relatively high intensity and coherence at the spectral edges even at a moderate femtosecond radiation power and a small (decimetre) fibre length. It is demonstrated that the PLL systems developed for the synthesiser can transfer the long-term stability of optical frequency standards to synthesised frequencies. The minimum achievable frequency instability can reach $\sim 10^{-17}$ (per averaging times shorter than 100 s).

The compactness, stabilisation reliability, and ability to operate at a relatively low power of optical signals (without additional amplifiers and high-power pump sources) make the proposed laser system (synthesiser) a promising candidate for application as a base for creating mobile FOCs.

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