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Resonant diffraction gratings for spatial differentiation of optical beams

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Abstract. **Diffraction of a two-dimensional optical beam from a resonant diffraction grating is considered. It is shown that at certain resonance parameters the diffraction grating allows for spatial differentiation and integration of the incident beam. The parameters of the diffraction grating for spatial differentiation of optical beams in the transmission geometry are calculated. It is shown that the differentiating diffraction grating allows the conversion of the two-dimensional beam into the two-dimensional Hermite–Gaussian mode. The presented results of numerical modelling are in good agreement with the proposed theoretical description. The use of the considered resonant diffraction gratings is promising for solving the problems of all-optical data processing.**

Keywords: *resonant grating, optical differentiation, spatial differentiation.*

1. Introduction

Optical devices implementing prescribed temporal and spatiotemporal transformations of optical signals are of great interest for a wide range of applications, including ultrafast all-optical data processing and analogue optical computation. The most important operations of the analogue processing of optical signals are the temporal and spatial differentiation. The temporal differentiation of an optical pulse is understood as the differentiation of the pulse envelope, and the spatial differentiation means the differentiation of the spatial profile of a light beam. For the temporal differentiation, multiple versions of Bragg gratings [1–8] and resonant diffraction gratings [9–12] have been proposed. The differentiation in these cases was implemented in both the reflection and the transmission geometry. The spatial differentiation was first considered in Ref. [13], where a phase-shifted Bragg grating was used, performing the operation of differentiation in the reflection geometry. Note that the Bragg grating does not allow the implementation of differentiation in the transmission geometry, which is an essential limitation of the approach, proposed in Ref. [13].

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In the present paper we derive the differential equation, providing a general description of the spatial transformation of the incident beam profile as a result of diffraction from a resonant diffraction grating. It is shown that at certain parameters of the resonance the diffraction grating allows the implementation of the differentiation and integration operations for the incident beam. For the first time the parameters of the diffraction grating are calculated providing the spatial differentiation of an optical beam in the transmission geometry. The possibility of controlling the relation between the differentiation quality and the transmitted signal amplitude by changing the geometric parameters of the grating is demonstrated. The conversion of a two-dimensional Gaussian beam into a two-dimensional Hermite–Gaussian beam is considered as an important practical application of the differentiating resonant grating.

2. Diffraction of a two-dimensional optical beam from a diffraction grating

Consider a two-dimensional optical beam incident on a diffraction structure at an angle θ_u. In the coordinate system xz, associated with the beam and rotated with respect to the grating coordinate system $x_{gr}z_{gr}$ by the angle θ_u (Fig. 1), the plane-wave expansion of the incident beam has the form

$$P_{inc}(x,z) = \int G(k_x)\exp\left(ik_x x - i\sqrt{k_0^2 n_{sup}^2 - k_x^2}\,z\right)dk_x, \quad (1)$$

where $k_0 = 2\pi/\lambda$ is the wavenumber; n_{sup} is the refractive index of the superstrate; k_x and $\sqrt{k_0^2 n_{sup}^2 - k_x^2} = k_z$ are the components of the wave vectors of the incident waves; and $G(k_x)$ is the angular spectrum of the beam having the width Δ_g ($|k_x| \leqslant \Delta_g$). The function $P_{inc}(x,z)$ in Eqn (1) corresponds to the component E_y of the electric field in the case of TE-polarisation, or to the component H_y of the magnetic field in the case of TM-polarisation.

As a result of the beam diffraction from the grating, the expressions of the reflected and transmitted field in the zeroth diffraction order take the form

$$P_{ref}(x_{ref},z_{ref}) = \int G(k_x)\,R(\tilde{k}_x)\exp\left(ik_x x_{ref} + i\sqrt{k_0^2 n_{sup}^2 - k_x^2}\,z_{ref}\right)dk_x,$$
$$P_{tr}(x_{tr},z_{tr}) = \int G(k_x)\,T(\tilde{k}_x)\exp\left(ik_x x_{tr} - i\sqrt{k_0^2 n_{sub}^2 - k_x^2}\,z_{tr}\right)dk_x, \quad (2)$$

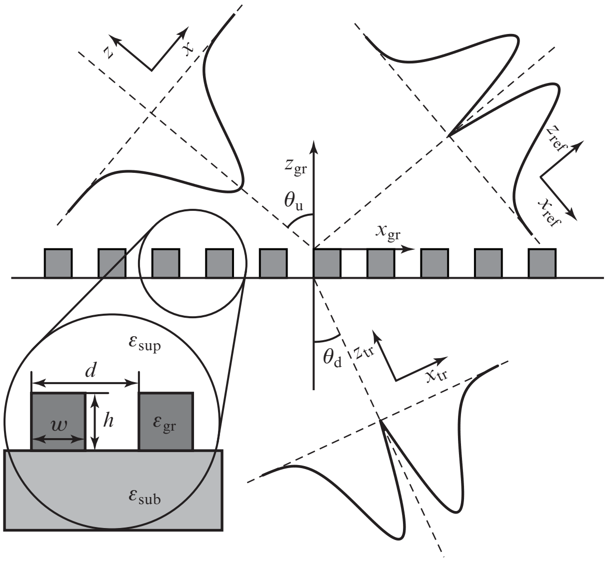

Figure 1. Geometry of optical beam diffraction from a diffraction grating (θ_d is the beam refraction angle).

where $R(\tilde{k}_x)$ and $T(\tilde{k}_x)$ are the complex reflection and transmission coefficients of the structure; $\tilde{k}_x = k_0 n_{sup}\sin(\theta + \theta_u) = k_x\cos\theta_u + k_z\sin\theta_u$ is the x component of the wave vector for the wave, incident on the grating at the angle $\theta + \theta_u$ in the coordinate system of the grating; and n_{sub} is the refractive index of the substrate material. Note, that expressions (2) are written in the coordinate systems, associated with the reflected and the refracted beam (Fig. 1). We assume that the origins of the coordinate systems xz, $x_{ref}z_{ref}$, and $x_{tr}z_{tr}$ coincide with the origin of the coordinate system $x_{gr}z_{gr}$.

Assuming the angular spectrum of the incident beam to be sufficiently narrow ($\Delta_g \ll k_0 n_{sup}$) we obtain

$$\tilde{k}_x \approx k_x\cos\theta_u + k_0 n_{sup}\sin\theta_u = k_x\cos\theta_u + k_{x0}, \tag{3}$$

where $k_{x0} = k_0 n_{sup}\sin\theta_u$. Consider the relation between the incident beam profile $P_{inc}(x,0)$ and the profiles of the reflected beam $P_{ref}(x_{ref},0)$ and the transmitted beam $P_{tr}(x_{tr},0)$. From expressions (1) and (2) it follows that the transformation of the incident beam profile $P_{inc}(x,0)$ into the profile $P_{tr}(x,0)$ or $P_{ref}(x,0)$ can be described in terms of passing the signal through a linear system with the transfer function, proportional to the reflection or transmission coefficient of the diffraction grating [13]:

$$\begin{aligned} H_{ref}(k_x) &= R(k_x\cos\theta_u + k_{x0}), \\ H_{tr}(k_x) &= T(k_x\cos\theta_u + k_{x0}). \end{aligned} \tag{4}$$

Note, that the transfer functions (4) are formally analogous to those describing the temporal transformation of an optical pulse [9–12].

3. Signal transformation by a resonant diffraction grating

Let us study the form of the transfer functions (4) in the vicinity of the spatial frequencies of waveguide resonances, associated with the excitation of eigenmodes in the grating. In the vicinity of a resonance the following approximate representations are valid for the reflection and transmission coefficients [10, 13–16]:

$$\begin{aligned} R(\tilde{k}_x) &\approx a_R + \frac{b_R}{\tilde{k}_x - k_{pole}} = a_R\frac{\tilde{k}_x - k_{zero}^{(R)}}{\tilde{k}_x - k_{pole}}, \\ T(\tilde{k}_x) &\approx a_T + \frac{b_T}{\tilde{k}_x - k_{pole}} = a_T\frac{\tilde{k}_x - k_{zero}^{(T)}}{\tilde{k}_x - k_{pole}}, \end{aligned} \tag{5}$$

where a_R, a_T are the nonresonance reflection and transmission coefficients; k_{pole} is the complex propagation constant for an eigenmode of the structure; b_R, b_T are the coefficients, describing resonance light scattering from the structure; and $k_{zero}^{(T)} = k_{pole} - b_T/a_T$, $k_{zero}^{(R)} = k_{pole} - b_R/a_R$ are zeros of the reflection and transmission coefficient, corresponding to the pole k_{pole}. Note, that representations (5) fail in the vicinity of $\theta_u = 0$ ($k_{x0} = 0$). Indeed, at $k_{x0} = 0$ the reflection and transmission coefficients of the sub-wavelength dielectric grating with the symmetric profile (Fig. 1) are even functions of the angle of incidence [17] and, therefore, functions of $\tilde{k}_x^2$ [13]. The case of normal incidence is not considered below.

With Eqns (5) taken into account, the transfer functions (4) can be presented in the form

$$H_{ref} = H_{tr} = H(k_x) = a\frac{k_x\cos\theta_u - (k_{zero} - k_{x0})}{k_x\cos\theta_u - (k_{pole} - k_{x0})}, \tag{6}$$

where $a = a_{R(T)}$.

Let us introduce the following notations: $g(x) = P_{inc}(x,0)$ for the field distribution in the incident beam; $f(x) = P_{tr}(x,0)$ [or $f(x) = P_{ref}(x,0)$] for the field distribution in the transmitted (or reflected) beam. Let $G(k_x)$ and $F(k_x)$ be the spectra of the functions $g(x)$ and $f(x)$, respectively. Then, according to Eqn (6), the spatial spectra of the input signal $G(k_x)$ and the output signal $F(k_x) = H(k_x)G(k_x)$ are related by the equation

$$\begin{aligned} &\mathrm{i}k_x F(k_x) - \mathrm{i}\frac{k_{pole} - k_{x0}}{\cos\theta_u}F(k_x) \\ &\quad = a\mathrm{i}k_x G(k_x) - a\mathrm{i}\frac{k_{zero} - k_{x0}}{\cos\theta_u}G(k_x). \end{aligned} \tag{7}$$

Applying the inverse Fourier transformation to the left-hand and right-hand sides of Eqn (7), we find that the reflected (or transmitted) output signal after the diffraction from a resonant diffraction grating is a solution of the first-order nonhomogeneous differential equation

$$\begin{aligned} &\frac{\mathrm{d}f(x)}{\mathrm{d}x} - \mathrm{i}\frac{k_{pole} - k_{x0}}{\cos\theta_u}f(k_x) \\ &\quad = a\frac{\mathrm{d}g(x)}{\mathrm{d}x} - a\mathrm{i}\frac{k_{zero} - k_{x0}}{\cos\theta_u}g(k_x). \end{aligned} \tag{8}$$

The solution to this equation can be easily found in the form:

$$f(x) = a\int_C^x \exp\left(\frac{x-\xi}{v}\right)\left[\frac{\mathrm{d}g(\xi)}{\mathrm{d}\xi} - \mathrm{i}(k_{zero} - k_{x0})g(\xi)\right]\mathrm{d}\xi, \tag{9}$$

where $v = \mathrm{i}\cos\theta_u/(k_{x0} - k_{pole})$ is the parameter, analogous to the time constant in the temporal differentiation of optical pulses [9, 11]; and $C = -\infty\cdot\mathrm{sgn}(\mathrm{Im}\,k_{pole})$. The presented expression for the lower limit of integration of C in Eqn (9) provides a decrease in the generated beam field at infinity [$f(\pm\infty) = 0$].

Note that the possibility of solving the differential equations of form (8) with a resonant diffraction grating is of great interest, because for 'optical' solution of such equations only rather complex systems with feedback have been proposed so far [18].

Let us show that under certain conditions for the coefficients a, b ($b = b_{R(T)}$) in Eqns (5), (9) the resulting signal is a derivative or an antiderivative of the initial signal. Indeed, under the condition that $k_{x0} = k_{\rm zero} = k_{\rm pole} - b/a \in \mathbb{R}$ we obtain

$$f(x) = a\int_C^x \exp\Big(\frac{x-\xi}{v}\Big)\frac{\mathrm{d}g(\xi)}{\mathrm{d}\xi}\mathrm{d}\xi$$

$$= a\Big[-v\frac{\mathrm{d}g(x)}{\mathrm{d}x} + v\int_C^x \exp\Big(\frac{x-\xi}{v}\Big)\frac{\mathrm{d}^2g(\xi)}{\mathrm{d}\xi^2}\mathrm{d}\xi\Big]$$

$$= -av\frac{\mathrm{d}g(x)}{\mathrm{d}x} - av^2\Big[\frac{\mathrm{d}^2g(x)}{\mathrm{d}x^2} - \int_C^x \exp\Big(\frac{x-\xi}{v}\Big)\frac{\mathrm{d}^3g(\xi)}{\mathrm{d}\xi^3}\mathrm{d}\xi\Big]. \quad (10)$$

Under the condition that $|v| \ll 1$, we have

$$f(x) \approx -av\frac{\mathrm{d}g(x)}{\mathrm{d}x}, \quad (11)$$

i.e., the resulting signal [$P_{\rm tr}(x,0)$ or $P_{\rm ref}(x,0)$] is proportional to the first derivative of the signal $g(x)$.

The operation of integration is implemented under the conditions $k_{x0} = {\rm Re}\,k_{\rm pole}$, $a = 0$. In this case

$$f(x) = \mathrm{i}b\int_C^x \exp\Big(\frac{x-\xi}{v}\Big)g(\xi)\mathrm{d}\xi$$

$$= \mathrm{i}b\Big[\Phi(x) + \frac{1}{v}\int_C^x \exp\Big(\frac{x-\xi}{v}\Big)\Phi(\xi)\mathrm{d}\xi\Big], \quad (12)$$

where

$$\Phi(x) = \int_C^x g(\xi)\mathrm{d}\xi.$$

Under the condition $|v| \gg 1$ we obtain

$$f(x) \approx \mathrm{i}b\Phi(x), \quad (13)$$

i.e., the resulting signal [($P_{\rm tr}(x,0)$ or $P_{\rm ref}(x,0)$] is proportional to the antiderivative of the initial signal $g(x)$.

It should be noted that the implementation of exact integration or differentiation is impossible. Indeed, in the case of integration the condition $|v| \to \infty$ corresponds to a system that cannot be implemented physically, while in the case of differentiation the condition $v \to 0$, according to Eqn (11), corresponds to the zero amplitude of the output signal. The variation in the $|v|$ value allows the compromise between the quality of differentiation (or integration) and the energy of the transmitted signal.

4. Differentiation of an optical beam

Consider the differentiation of an optical beam in the transmission geometry. As shown above, for this aim the presence of a zero of the transfer function (6) is necessary, corresponding to the zero transmission coefficient at the centre spatial frequency of the incident beam $k_{x0} = k_0 n_{\rm sup}\sin\theta_{\rm u}$. It is known that the condition of a zero in the transmission spectrum holds for a sub-wavelength dielectric grating (in this case the solution in the form of a propagating wave exists only for the zeroth-order diffraction) [12, 14–16]. Hence, for differentiation of the incident beam we will use a sub-wavelength dielectric diffraction grating.

To confirm the possibility of spatial differentiation we carried out the numerical modelling of light diffraction from the grating in the arrangement presented in Fig. 1. Analogous structures are widely used as narrow-band spectral filters [19]. In the calculations we used the permittivity values $\varepsilon_{\rm sup} = 1$ (the superstrate), $\varepsilon_{\rm gr} = 8.6207$ (the grating material), and $\varepsilon_{\rm sub} = 2.3535$ (the substrate material), which correspond to air, aluminium arsenide and silica. The parameters of the differentiating structure (the period d, the height h and the step width w) were obtained as a result of minimisation of $|v|$ under the condition of the presence of a zero in the transmission spectrum. Note that the calculation of the parameter v is reduced to the calculation of the pole $k_{\rm pole}$. To determine the latter, at each iteration the Padé approximation of the order [1/1] was constructed for the function $T(\tilde{k}_x)$ in the vicinity of the point $k_x = k_{x0}$. To calculate the function $T(\tilde{k}_x)$ we used the Fourier modal method as formulated in Refs [20, 21]. The calculation was performed for the beam with the wavelength λ = 1.064 μm, the spectral width being $\Delta_{\rm g} = 0.55$ μm^{-1}. The grating parameters obtained as a result of optimisation are given in the Fig. 2 caption. At these parameters $k_{\rm pole}$ = 1.4599 – 0.9843i μm^{-1} and $k_{\rm zero}$ = 1.8841 μm^{-1}. Thus, the spatial differentiation of the incident beam will be implemented at the centre spatial frequency $k_{x0} = k_{\rm zero}$ = 1.8841 μm^{-1}, corresponding to the incidence angle $\theta_{\rm u} = 0.3247$ rad.

Figure 2 shows the spatial transmission spectrum for the diffraction grating with the calculated parameters. According to Fig. 2a, the transfer function (6) in the vicinity of the value $k_x = 0$ demonstrates good agreement (up to a linear phase) with the transfer function of an ideal differentiating filter $H_{\rm dif}(k_x)$ = ik_x ($k_x|<\Delta_{\rm g}$). The linear phase causes the shift of the reflected beam (Goos–Hänchen effect) and does not affect the quality of differentiation. Figure 2b shows the amplitudes of the incident Gaussian beam $P_{\rm inc}(x,0) = \exp(-x^2/\sigma^2)$ (σ = 8.18.1 μm) and the transmitted beam $f(x) = P_{\rm tr}(x,0)$, as well as the absolute value of the analytically calculated derivative. The latter is presented with magnification, providing the equality of the maximal values of the analytic derivative absolute value and the amplitude of the transmitted beam. Figure 2b demonstrates high precision of differentiation, in particular, the absolute value of Pearson's correlation coefficient between the amplitude of the transmitted beam and the absolute value of the analytically calculated derivative exceeds 0.999.

The proposed diffraction structure can be considered as an analogue of the Fourier correlator, consisting of a pair of lenses with a spatial filter linear in amplitude, placed in the Fourier plane. The transfer function of such filter has the form $H_{\rm dif}(k_x) = k_x/\Delta_{\rm g}$ ($k_x|<\Delta_{\rm g}$). It is interesting to compare the energy efficiency of the calculated diffraction grating with the maximal possible efficiency of the abovementioned spatial differentiating filter. We define the energy efficiency as the quantity av in Eqn (11) that characterises the output signal amplitude. For the ideal filter $av = |H'(0)| = 1/\Delta_{\rm g}$ = 1.818 μm. The energy efficiency of the calculated diffraction structure is $av = |H'(0)| \approx 0.56$ μm. The smaller energy efficiency of the proposed diffraction structure is compensated for by its essentially smaller spatial dimensions (a few micrometres as compared with tens of centimetres for the Fourier correlator). Moreover, the presented maximal value of the energy effi-

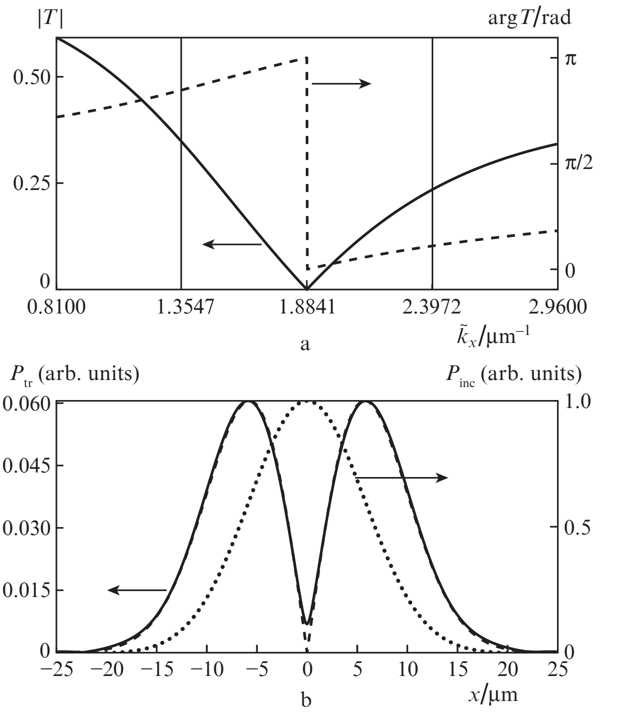

Figure 2. Modulus (solid curve) and argument (dashed curve) of the complex transmission coefficient as functions of $\tilde{k}_x$ (a), amplitudes of the transmitted (solid curve) and the incident (dotted curve) beams, and the absolute value of the analytically calculated derivative (dashed curve) (b) for the grating with the parameters $d = 0.49$ μm, $h = 0.46$ μm, $w = 0.295$ μm. Vertical lines show the interval of spatial frequencies used in the calculation.

ciency is not attainable in practice because of the Fresnel losses.

Now let us study the possibility of controlling the quality of differentiation by changing the geometric parameters of the diffraction grating. Figure 3 presents the parameter $|\nu|$ in Eqns (9) and (10) that determines the ratio of the differentiation precision and the transmitted signal energy, versus the step width w of the diffraction grating. In the course of calculating $|\nu|$ the values of k_{pole}, k_{x0}, and θ_u were recalculated for the changed structure geometry. According to Fig. 3, the minimum of $|\nu|$ is achieved at $w = 0.295$ μm. This value was used in the calculation of the dependences shown in Fig. 2.

To study the effect of the parameter $|\nu|$ on the quality of differentiation we modelled the optical beam diffraction

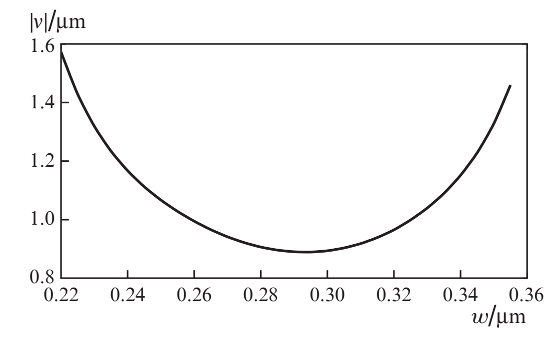

Figure 3. Parameter $|\nu|$ vs. step width w of the grating at $d = 0.49$ μm, $h = 0.46$ μm.

from the grating with the step width $w = 0.225$ μm, for which the parameter $|\nu|$ is greater than its minimal value by 60%. Figure 4 presents the transmission spectrum of such diffraction grating and the result of differentiation of the Gaussian beam with the parameters specified above. The zero in the transmission spectrum of the considered grating corresponds to the centre spatial frequency of the incident beam k_{x0} = 2.5647 μm^{-1} and the angle of incidence $\theta_u = 0.4493$ rad.

From the comparison of Figs 2b and 4b it is seen that the growth of the parameter $|\nu|$ gives rise to the increase in the

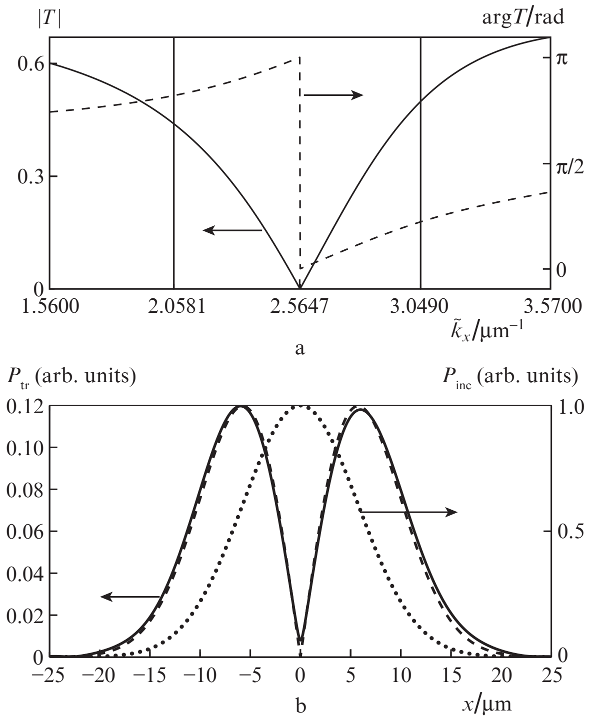

Figure 4. Same as in Fig. 2, but for $w = 0.225$ μm.

resulting beam amplitude (by two times in the considered case) with simultaneous worsening of the differentiation quality. Indeed, in Fig. 4b one can notice the deviations of the obtained beam shape from the module of the analytically calculated derivative beyond the central region, in which the beam energy is concentrated. In particular, the mean square deviation of the transmitted beam amplitude from the derivative, calculated analytically, increases from 0.0007 (Fig. 2, w = 0.295 μm) to 0.002 (Fig. 4, w = 0.225 μm). Nevertheless, the grating with the changed parameters (Fig. 4) still allows implementation of the signal differentiation with high accuracy. Thus, as a result of varying the step width w of the diffraction grating, it became possible to increase the amplitude of the resulting signal by two times with insignificant worsening of the differentiation quality. This result demonstrates wide capabilities of controlling the operation characteristics of the differentiator grating (the ratio of differentiation quality and energy efficiency) by changing the geometric parameters.

Note that the first derivative of the Gaussian beam coincides with the Hermite–Gaussian mode $H_1(x/\sigma)$ [$H_1 = 2\sqrt{2}\,x \times \exp(-x^2)$],which does not change its shape (to a scaling factor) in the course of free-space propagation [22]. Figure 5 shows the amplitude of the transformed beam at different distances z_{tr}. In the calculation we used the transmitted beam with the amplitude, presented in Fig. 4b. The dependences in Fig. 5 confirm that the transmitted beam retains its shape to a scaling factor.

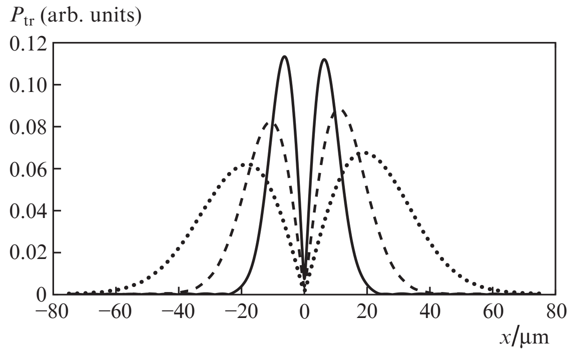

Figure 5. Amplitude of the transmitted beam at z_{tr} = 0 (solid curve), 500 μm (dashed curve) and 1000 μm (dotted curve).

5. Conclusion

We have obtained the general form of the transformation of a two-dimensional optical beam in the process of diffraction from a resonant diffraction grating. The possibility of implementing the operations of spatial differentiation and integration of the incident optical beam is shown. Within the framework of the electromagnetic theory the parameters of the diffraction grating that allows spatial differentiation in the transmission geometry are calculated. The possibility of controlling the ratio between the differentiation precision and the energy of the transmitted signal by changing the geometric parameters of the differentiating grating is demonstrated. As an important practical application of the differentiating grating, the conversion of the two-dimensional Gaussian beam into the Hermite–Gaussian mode is considered.

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