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Localisation of atomic populations in the optical radiation field

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Abstract. **The possibility of two-dimensional spatial localisation of atomic populations under the influence of the travelling wave fields in the tripod-configuration of quantum states is studied for the first time. Three travelling waves propagating in the same plane at an angle of 120° to each other form a system of standing waves under the influence of which atomic populations are localised. The size of the region of spatial localisation of the populations, in principle, can be hundredths of a wavelength of optical radiation.**

Keywords: *spatial localisation of atomic populations, tripod-configuration of atomic levels.*

1. Introduction

Recent years have seen an increased interest in the study of spatial localisation of atomic populations in both one-dimensional and two-dimensional cases [1–9]. This interest stems from the possibility of obtaining narrow (much smaller than the optical radiation wavelength) spatial distributions of atoms, which are in certain quantum states.

The physical basis for the localisation of quantum-state populations of an atomic system is spatially inhomogeneous optical pumping, which determines the spatial distribution of the populations. In this case, a standing light wave, which is resonant with at least one transition of a multilevel system, is needed. The period of this wave determines the spatial period of changes in the populations. At the same time, the width of a single peak in the distribution of the populations depends on the intensity of the standing-wave field. Indeed, because the intensity in the standing-wave node is equal to zero, the optical pumping redistributes the populations from all the levels of the atomic system to the lower state, which is subjected to the action of the standing-wave field. On the other hand, if an atom is not exactly in the node, the optical pumping efficiency is reduced, because the standing-wave intensity increases. Thus, the higher the standing-wave intensity, the greater the spatial gradient of the field and, therefore, the smaller the spatial region near the node of the standing wave for which the optical pumping efficiency is high.

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It should be emphasised that the English scientific literature uses the term 'atom localisation' rather than the term 'localisation of atomic populations', which leads to a certain discrepancy between the physical phenomenon and its name. The fact is that in this case we are dealing with the localisation of the populations, i.e., the interaction of the atom with the optical radiation field leads to the spatial redistribution of the populations of the atomic states. Thus, the initial spatial distribution of atoms $w(x, t = 0)$, taking into account the populations of the quantum states $\rho_i(x, t = 0)$ before $[w_{\rm in}(x)]$ and after $[w_{\rm f}(x)]$ interaction, retains its form, while the spatial distributions of the populations after the interaction $\rho_i(x, t)$ will change:

$$w_{\rm in}(x) = w(x, t = 0) = \sum_i \rho_i(x, t = 0)$$
$$= \sum_i \rho_i(x, t) = w(x, t) = w_{\rm f}(x).$$

We assumed above that under the influence of optical radiation the atom does not change its translational state. In other words, the atom is considered to be so heavy that we can neglect the recoil energy $E_{\rm r} = \hbar^2 k^2/(2m)$, as compared with the kinetic energy of the atom $E = p^2/(2m)$ (here, m is the mass of the emitting atom; $k = 1/\lambdabar$ is the wavenumber of the photon emitted by the atom; $\lambdabar$ is the photon wavelength; $p = mv$ is the momentum of the atom; and v is the velocity of the atom).

Let us now take into account the fact that during absorption (or emission) of a single photon, the velocity of the atom changes by an amount equal to the recoil velocity $v_{\rm r} = \hbar k/m$. Then, the minimum size of the population localisation region, δx, can be defined as the distance travelled by an atom with velocity $v_{\rm r}$ during the lifetime of the excited state, $t \propto \gamma^{-1}$:

$$\delta x \geqslant v_{\rm r}\gamma^{-1} = \frac{1}{\pi}\frac{E_{\rm r}}{\hbar\gamma} \approx 3\times 10^{-3}\lambda,$$

where γ is the rate of spontaneous relaxation and λ is the wavelength of incident radiation; it is assumed that $E_{\rm r} \approx 10^{-2}\hbar\gamma$ for strong electric-dipole optical transitions in atoms. The effect of the recoil velocity on the finite width of the population localisation region should be accounted for in the case when it is needed to extract from the total ensemble only those atoms that are in a certain quantum state, because this extraction can also be realised by means of optical methods.

Note that the main problem here is the choice of the scheme of interaction of atoms with the laser field, because it is desirable to have only one excited upper level for efficient optical pumping to the lower levels of the system. As a result, all the above-considered schemes of atomic states, used to study the localisation of the populations, are based to some extent on a three-level Λ system. For example, the authors of papers [1–3] proposed one-dimensional localisation of the populations for the atoms with Λ configuration of the levels. In this case, the generalisation to the two-dimensional case is nontrivial due to the fact that in the equations for the density matrix elements one must explicitly take into account the polarisations of the light waves, i.e., the selection rules for the working transitions. Therefore, for the fields with both linear and circular polarisations to be applied to two-dimensional localisation of the populations, Ivanov and Rozhdestvensky [4] proposed the tripod-configuration of the atomic states.

In this paper, we have obtained for the first time the two-dimensional spatial localisation of the populations induced by the optical fields of the travelling waves in a four-level tripod-system. In this case, three travelling waves propagating in the same plane at an angle of 120° to each other produce standing waves which, interacting with the central transition of the tripod-system (Fig. 1a), ensure the spatial localisation of the atomic populations.

Note that the use of the travelling waves is very encouraging from the point of view of the practical implementation of the two-dimensional spatial localisation of the populations in the region, the size of which is much smaller than the wavelength of optical radiation λ; moreover, this effect can be of considerable interest for the modern nanotechnology.

2. Basic equations

Let us now consider in detail the energy level diagram of the atom in the tripod-configuration (Fig. 1a). In this case, the atomic system consists of three lower states with nonallowed optical transitions between them and one upper level. The optically allowed transitions $|n\rangle-|4\rangle$ ($n = 1-3$) are subjected to the action of the fields with Rabi frequencies g_1, g_2 and g_3 and detunings from the resonance frequencies of transitions Δ_1, Δ_2 and Δ_3, respectively. Figure 1b shows the orientation of the fields. It can be seen that the three travelling waves with the same Rabi frequency g_2, acting on the transition $|2\rangle-|4\rangle$, propagate in the xy plane at an angle of 120° to each other, while the wave fields with the Rabi frequencies g_1 and g_3, acting on the transitions $|1\rangle-|4\rangle$ and $|3\rangle-|4\rangle$, propagate in the negative and positive directions of the z axis, respectively. As a result, the expression for the total field in the tripod-system (Fig. 1) can be written in the form

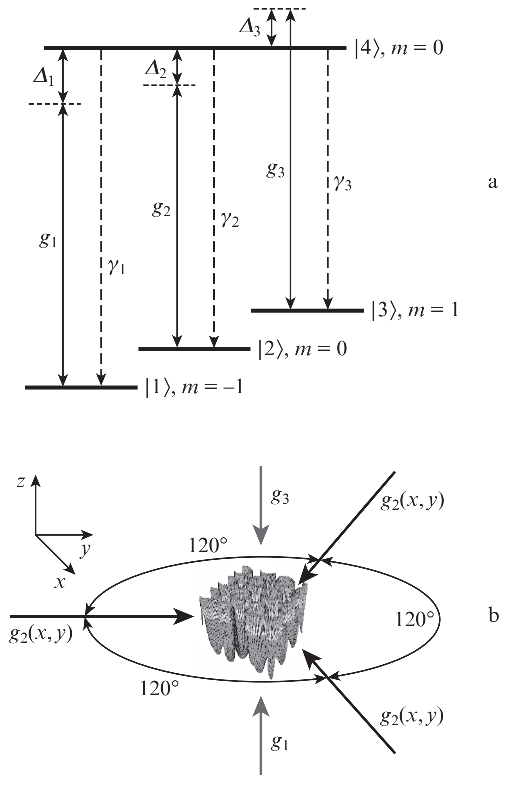

Figure 1. (a) Diagram of the quantum levels of the atom in the tripod-configuration and (b) mutual orientation of the optical fields.

$$\boldsymbol{E} = \boldsymbol{e}_2 E_2[\cos(\omega_2 t - \boldsymbol{k}_2^{(1)}\boldsymbol{r}) + \cos(\omega_2 t - \boldsymbol{k}_2^{(2)}\boldsymbol{r}) + \cos(\omega_2 t - \boldsymbol{k}_2^{(3)}\boldsymbol{r})] + \boldsymbol{e}_1 E_1 \cos(\omega_1 t + k_1 z) + \boldsymbol{e}_3 E_3 \cos(\omega_3 t + k_3 z). \quad (1)$$

The wave with frequency ω_1 and wavenumber k_1 propagates in the negative direction of the z axis, whereas the wave with ω_3 and k_3 – in the positive direction; in the xy plane, there are three waves having the same frequency ω_2 but different directions of the wave vectors: $(\widehat{k_2^{(i)}, k_2^{(j)}}) = 120°$ ($i \neq j$; $i, j = 1,2,3$). In this case, the unit vectors $\boldsymbol{e}_{1,3}$ specify positive and negative circular polarisations of the waves, respectively, and the vector $\boldsymbol{e}_2$ – linear polarisation.

The dynamics of the system can be described by using the equation for the density matrix elements $\tilde{\rho}_{ij}(x,y,t)$

$$\mathrm{i}\hbar\dot{\tilde{\rho}}_{ij} = [H, \tilde{\rho}]_{ij} + \mathrm{i}\Gamma_{ij}\tilde{\rho}_{ij} \quad (2)$$

with an interaction Hamiltonian in the form $H = H_0 + V$, where H_0 is the quantum state of the system in the absence of interaction, and

$$V = -\frac{1}{\hbar}\sum_{i=1}^{3}(\boldsymbol{d}_{i4}\boldsymbol{e}_i)E_i$$

determines the interaction with the optical radiation field (1) for the $|n\rangle-|4\rangle$ ($n = 1-3$) transitions with the matrix element of the dipole interaction operator d_{i4}.

In expression (2) the matrix elements Γ_{ij} specify the relaxation rates of the elements $\tilde{\rho}_{ij}(x,y,t)$. In this case, the relaxation rate of the diagonal matrix elements (i.e., populations) is determined by the natural width of the upper excited state of the system $2\gamma = \gamma_1 + \gamma_2 + \gamma_3$ (Fig. 1a), and the relaxation rates of the off-diagonal matrix elements Γ_{ij} ($i \neq j$) can, in addition to natural decay, be determined by collisions, finite spectral width of the exciting fields, etc.

As a result, the equations for the density matrix elements $\tilde{\rho}_{ij}(x,y,t)$ of the system of the levels in the tripod-configuration have the form

$$\mathrm{i}\dot{\rho}_{11} = g_1(\rho_{14} - \rho_{41}) + \mathrm{i}\gamma_1\rho_{44},$$

$$\mathrm{i}\dot{\rho}_{22} = g_2(\rho_{24} - \rho_{42}) + \mathrm{i}\gamma_2\rho_{44},$$

$$\mathrm{i}\dot{\rho}_{33} = g_3(\rho_{34} - \rho_{43}) + \mathrm{i}\gamma_3\rho_{44},$$

$$\mathrm{i}\dot{\rho}_{44} = g_1(\rho_{41} - \rho_{14}) + g_2(\rho_{42} - \rho_{24}) + g_3(\rho_{43} - \rho_{34}) - \mathrm{i}(\gamma_1 + \gamma_2 + \gamma_3)\rho_{44},$$

$$\mathrm{i}\dot{\rho}_{14} = g_1(\rho_{11} - \rho_{44}) + g_2\rho_{12} + g_3\rho_{13} + (\Delta_1 - \mathrm{i}\Gamma_{14})\rho_{14}, \quad (3)$$

$$\mathrm{i}\dot{\rho}_{24} = g_2(\rho_{22} - \rho_{44}) + g_1\rho_{21} + g_3\rho_{23} + (\Delta_2 - \mathrm{i}\Gamma_{24})\rho_{24},$$

$$\mathrm{i}\dot{\rho}_{34} = g_3(\rho_{33} - \rho_{44}) + g_1\rho_{31} + g_2\rho_{32} + (\Delta_3 - \mathrm{i}\Gamma_{34})\rho_{34},$$

$$\mathrm{i}\dot{\rho}_{12} = g_2\rho_{14} - g_1\rho_{42} + (\Delta_1 - \Delta_2)\rho_{12},$$

$$\mathrm{i}\dot{\rho}_{13} = g_3\rho_{14} - g_1\rho_{43} + (\Delta_1 - \Delta_3)\rho_{13},$$

$$\mathrm{i}\dot{\rho}_{23} = g_3\rho_{24} - g_2\rho_{43} + (\Delta_2 - \Delta_3)\rho_{23},$$

where $\rho_{ij} = \rho_{ij}^* \; (i \neq j)$.

In deriving the system of equations (3) we have neglected the terms containing the temporal oscillations at the doubled optical frequency (resonance approximation) and used the so-called rotating-wave approximation, which consists in the replacement of the off-diagonal matrix elements $\tilde{\rho}_{n4} = \rho_{n4}\exp(\mathrm{i}\Delta_n t)$ $(n = 1-3)$ for optical coherences and elements $\tilde{\rho}_{12} = \rho_{12}\exp[\mathrm{i}(\Delta_1 - \Delta_2)t]$, $\tilde{\rho}_{13} = \rho_{13}\exp[\mathrm{i}(\Delta_1 - \Delta_3)t]$, $\tilde{\rho}_{23} = \rho_{23}\exp[\mathrm{i}(\Delta_2 - \Delta_3)t]$ for low-frequency coherences. According to (1), on the central transition of the system (Fig. 1a) there are three fields with the same frequency but different directions of the wave vectors. In order to fix the direction of wave propagation with respect to the introduced coordinate system, we will use for the scalar products in (1) the expressions

$$\boldsymbol{k}_2^{(1)}\boldsymbol{r} = -k_2 y, \;\; \boldsymbol{k}_2^{(2)}\boldsymbol{r} = \frac{1}{2}k_2 y - \frac{\sqrt{3}}{2}k_2 x,$$
$$\boldsymbol{k}_2^{(3)}\boldsymbol{r} = \frac{1}{2}k_2 y + \frac{\sqrt{3}}{2}k_2 x. \quad (4)$$

Thus, expressions (4) clearly demonstrate that in the xy plane three multidirectional travelling waves yield a system of the standing waves, which provide the coordinate dependence of the populations of the quantum-system states.

3. Calculation results and discussion

The system of equations for the density matrix elements (3) completely determines the dynamics of the quantum system in the tripod-configuration in the field (1). To investigate the spatial dependence of the populations (i.e., diagonal elements of the density matrix), we use the steady-state solution of equations (3), which can be obtained by neglecting the time derivatives in the left-hand sides in comparison with terms in the right-hand sides containing the decay rates. Physically, this means that we consider the values of the density matrix elements for the times much greater than γ^{-1}, Γ_{ij}^{-1}. In this case, the expressions for the populations of the system states can be expressed as

$$\rho_{11} = \frac{D_1}{D}, \quad \rho_{22} = \frac{D_2}{D}, \quad \rho_{33} = \frac{D_3}{D}, \quad \rho_{44} = \frac{1}{D}, \quad (5)$$

$$D = 1 + D_1 + D_2 + D_3,$$

$$D_1 = 1 + \frac{3\gamma^2}{4g_1^2} + \frac{g_2(g_2A - g_1B)}{g_1\Delta_{12}} + \frac{g_3(g_3A - g_1C)}{g_1\Delta_{13}} - \frac{\Delta_1 A}{g_1},$$

$$D_2 = 1 + \frac{3\gamma^2}{4g_2^2} + \frac{g_1(g_2A - g_1B)}{g_2\Delta_{12}} + \frac{g_3(g_3A - g_2C)}{g_2\Delta_{23}} - \frac{\Delta_2 B}{g_2},$$

$$D_3 = 1 + \frac{3\gamma^2}{4g_3^2} + \frac{g_1(g_3A - g_1C)}{g_3\Delta_{13}} + \frac{g_2(g_3B - g_2C)}{g_3\Delta_{23}} - \frac{\Delta_3 C}{g_3}.$$

Here we introduce the following notations:

$$A = \frac{1}{3}\frac{\Delta_{13}(g_1^2 + g_2^2) + \Delta_{12}(g_1^2 + g_3^2) - \Delta_1\Delta_{12}\Delta_{13}}{g_1\Delta_{12}\Delta_{13}},$$

$$B = \frac{1}{3}\frac{\Delta_{12}(g_2^2 + g_3^2) - \Delta_{23}(g_1^2 + g_2^2) - \Delta_2\Delta_{12}\Delta_{23}}{g_2\Delta_{12}\Delta_{23}},$$

$$C = -\frac{1}{3}\frac{\Delta_{13}(g_2^2 + g_3^2) + \Delta_{23}(g_1^2 + g_3^2) + \Delta_3\Delta_{13}\Delta_{23}}{g_3\Delta_{13}\Delta_{23}},$$

$$\Delta_{ij} = \Delta_i - \Delta_j \; (i \neq j;\; i,j = 1,2,3).$$

In deriving expressions (5) we also assumed that $\Gamma_{14} = \Gamma_{24} = \Gamma_{34} = (\gamma_1 + \gamma_2 + \gamma_3)/2$, $\gamma_i = \gamma$ $(i = 1,2,3)$ and the transitions between the lower states of the system (Fig. 1a) are absent. We now consider the different cases of excitation of the system shown in Fig. 1a. As can be seen from expressions (5), the populations of the quantum-system states are strongly dependent on the difference between the detunings Δ_{ij}, which clearly suggests the existence of coherent population trapping (CPT) in such a system. Indeed, as is well known, the CPT occurrence in a three-level Λ system is conditioned by the equality of the vanishing difference between the frequency detunings of the exciting fields from the resonance values. The tripod-system (Fig. 1a) contains three different Λ systems, each of which has its own condition of the CPT existence. Thus, for the Λ system with the $|n\rangle - |4\rangle$ $(n = 1,2)$ transitions, the CPT existence will be conditioned by the equality $\Delta_{12} = 0$. In this case, the expressions for the populations of a four-level system have the form

$$\rho_{11} = \frac{g_2^2(x,y)}{g_1^2 + g_2^2(x,y)}, \quad \rho_{22} = \frac{g_1^2}{g_1^2 + g_2^2(x,y)},$$
$$\rho_{33} = 0, \quad \rho_{44} = 0. \quad (6)$$

For the Λ system with the $|n\rangle - |4\rangle$ $(n = 2,3)$ transitions, the CPT existence is conditioned by the equality $\Delta_{23} = 0$ and the expressions for the populations take the form

$$\rho_{11} = 0, \quad \rho_{22} = \frac{g_3^2}{g_3^2 + g_2^2(x,y)},$$
$$\rho_{33} = \frac{g_2^2(x,y)}{g_2^2 + g_3^2}, \quad \rho_{44} = 0. \quad (7)$$

Finally, when $\Delta_{13} = 0$, we obtain the expressions for the populations:

$$\rho_{11} = \frac{g_3^2}{g_1^2 + g_3^2}, \quad \rho_{22} = 0, \quad \rho_{33} = \frac{g_1^2}{g_1^2 + g_3^2}, \quad \rho_{44} = 0. \quad (8)$$

One can see from relations (6)–(8) that only in expressions (6) and (7) there is a dependence of the populations on the coordinates x, y; in the first case, the states $|1\rangle$ and $|2\rangle$ are populated and, in the second – the states $|2\rangle$ and $|3\rangle$. The corresponding spatial distributions are shown in Figs 2 and 3.

Figure 4 shows the spatial dependences of the populations of all the states of the system in the tripod-configuration. One can see that in our case, the two-dimensional (xy plane) localisation of the populations indeed takes place for all the states of the system. For the parameters selected, only the population ρ_{11} has a spatial dependence in the form of 'hills',

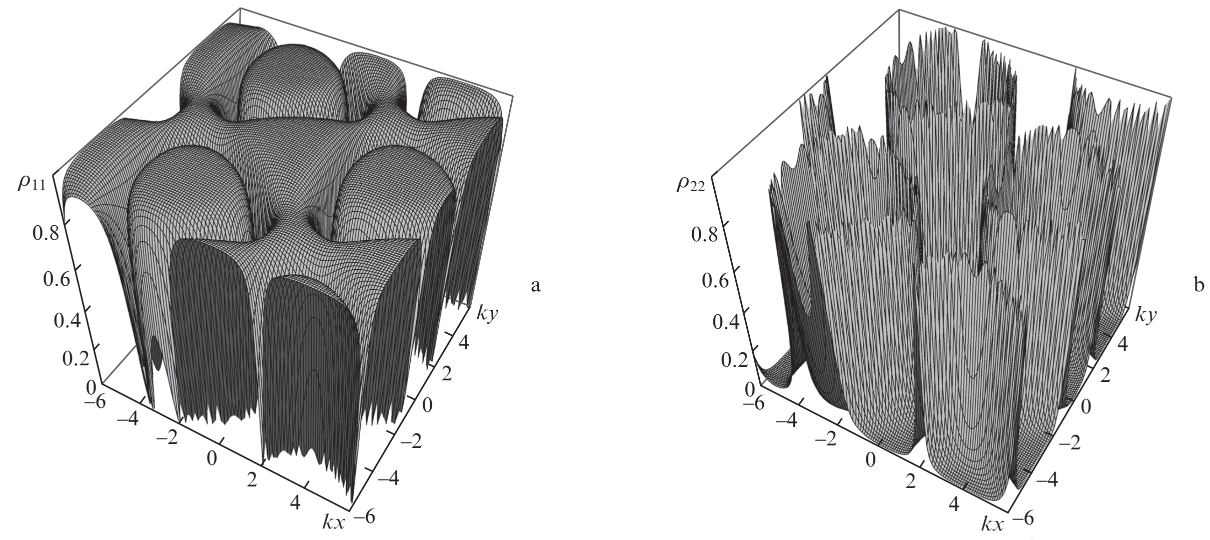

Figure 2. Coordinate dependences of the populations (a) ρ_{11} and (b) ρ_{22} of the tripod-system states in the case of CPT in the Λ system formed by the transitions $|n\rangle - |4\rangle$ ($n = 1, 2$). The interaction parameters and relaxation constants of the system are as follows: $\gamma_{1,2,3} = \gamma$, $\Gamma_{14} = \Gamma_{24} = \Gamma_{34} = 1.5\gamma$, $g_1 = \gamma$, $g_2 = 4\gamma$, $g_3 = 2\gamma$, $\Delta_1 = 4\gamma$, $\Delta_2 = 4\gamma$, $\Delta_3 = -2\gamma$.

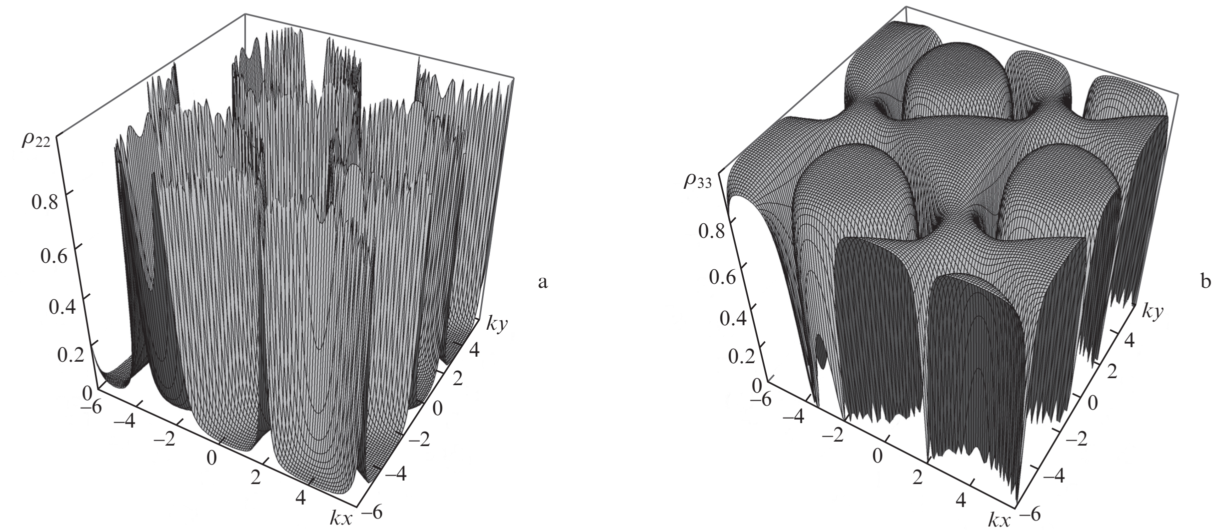

Figure 3. Coordinate dependences of the populations (a) ρ_{22} and (b) ρ_{33} of the tripod-system states in the case of CPT in the Λ system formed by the transitions $|n\rangle - |4\rangle$ ($n = 2, 3$). The interaction parameters and relaxation constants of the system are as follows: $\gamma_{1,2,3} = \gamma$, $\Gamma_{14} = \Gamma_{24} = \Gamma_{34} = 1.5\gamma$, $g_1 = \gamma$, $g_2 = 3\gamma$, $g_3 = 2\gamma$, $\Delta_1 = 4\gamma$, $\Delta_2 = -2\gamma$, $\Delta_3 = -2\gamma$.

whereas for other populations we observe ‘craters’. Thus, in certain points of the xy plane, the maximum values of the populations of the first two states may reach unity. This means that the entire quantum-system population is concentrated in all these points.

On the other hand, among the spatial structures presented in Fig. 4 the dependences of the populations ρ_{22}, ρ_{33} and ρ_{44} of the levels $|2\rangle$, $|3\rangle$ and $|4\rangle$ are of particular interest. Thus, the two-dimensional spatial dependences of the populations ρ_{33} and ρ_{44} of the third and fourth levels exhibit a double structure – the so-called double craters, in contrast to the usual crater (Fig. 4b). Of greatest interest here is the distribution of the population of the second level, ρ_{22}. The resulting craters have extremely narrow walls, which means a high degree of spatial localisation, namely, the size of the localisation region for the parameters used is 0.04 of the wavelength of incident radiation.

4. Conclusions

We have investigated the possibility of two-dimensional localisation of the populations in the tripod-system interacting with the field of the travelling light waves. It is found that the dependence of the populations on the spatial coordinates x and y is governed by the excitation of the central transition of the tripod-system by the field of multidirectional, linearly polarised travelling waves. The arising two-dimensional spatial distributions of the populations may have complex structures, such as double craters.

Note that for the two-dimensional localisation to be realised, the method for obtaining the spatial intensity dependence of the linearly polarised field, which is resonant with the central transition of the tripod-system, is not important. Therefore, the resulting two-dimensional distribution of the populations can be regarded as a kind of visualisation of the

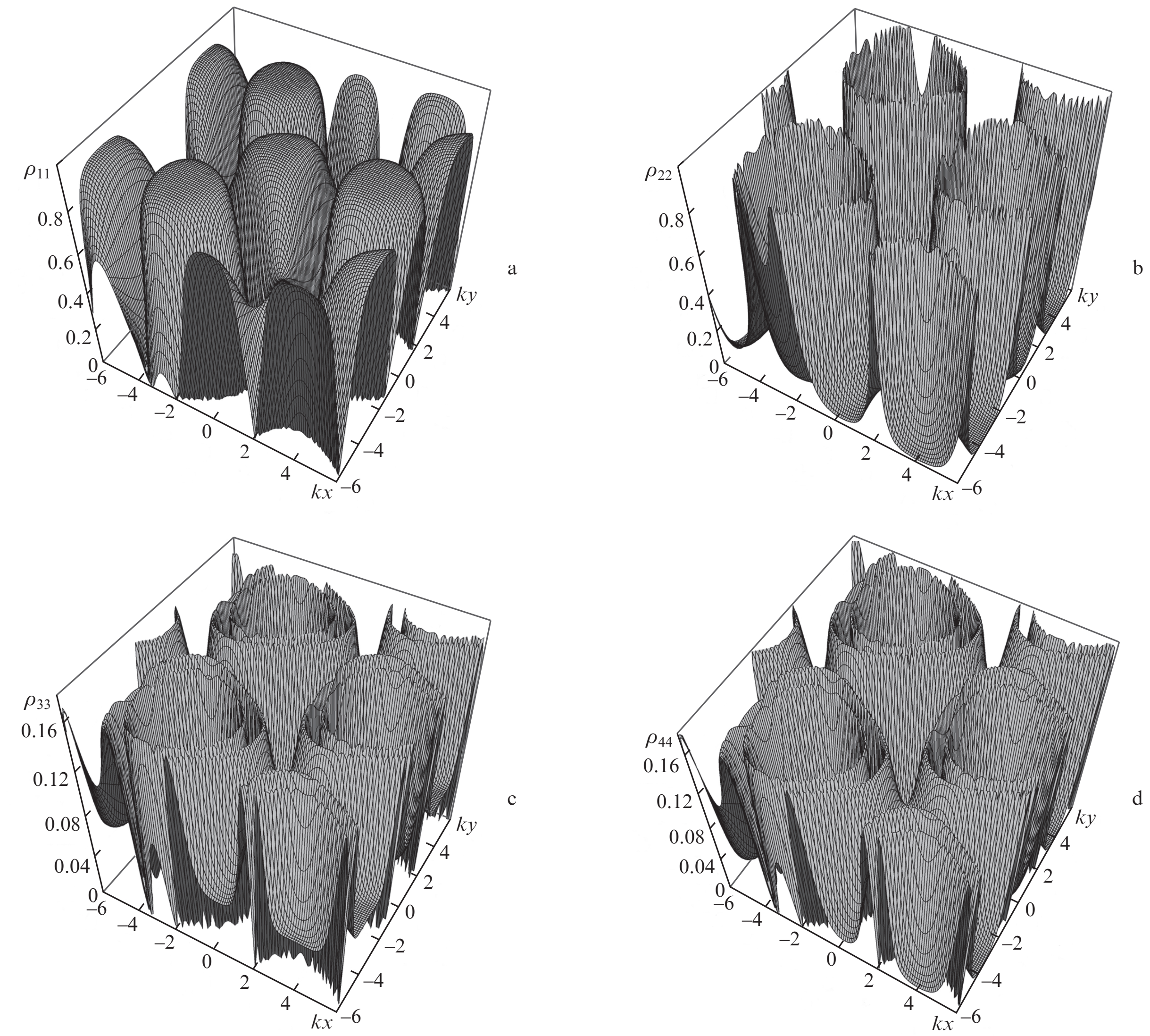

Figure 4. Coordinate dependences of the populations (a) ρ_{11}, (b) ρ_{22}, (c) ρ_{33} and (d) ρ_{44} of the tripod-system states. The interaction parameters and relaxation constants of the system are as follows: $\gamma_{1,2,3} = \gamma$, $\Gamma_{14} = \Gamma_{24} = \Gamma_{34} = 1.5\gamma$, $g_1 = \gamma$, $g_2 = 3\gamma$, $g_3 = 2\gamma$, $\Delta_1 = \gamma$, $\Delta_2 = 4\gamma$, $\Delta_3 = -2\gamma$.

spatial intensity distribution of the light field. Its characteristics are determined by the parameters under which we can observe the redistribution of the populations.

In our case, the speed of the response is equal to the time of action of optical pumping, $t \propto \gamma/g^2 = \gamma^{-1} \approx 10^{-7}$ s^{-1} ($g = \gamma$), and the minimum size of the localisation region is $\delta x \geqslant v_r\gamma^{-1} \approx 3\times10^{-3}\lambda \approx 2$ nm. In this case, the possibility of obtaining such a resolution is directly related to the values of the velocities of the atoms and can be realised in atomic ensembles cooled to the appropriate recoil energy temperature $T_r = E_r/k_B = \hbar^2k^2/(2mk_B)$.

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