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Reflection of a TE-polarised Gaussian beam from a layered structure under conditions of resonance excitation of waveguide modes

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Abstract. **The problem of reflection of a TE-polarised Gaussian light beam from a layered structure under conditions of resonance excitation of waveguide modes using a total internal reflection prism is considered. Using the spectral approach we have derived the analytic expressions for the mode propagation lengths, widths and depths of *m*-lines (sharp and narrow dips in the angular dependence of the specular reflection coefficient), depending on the structure parameters. It is shown that in the case of weak coupling, when the propagation lengths l_m of the waveguide modes are mainly determined by the extinction coefficient in the film, the depth of *m*-lines grows with the mode number *m*. In the case of strong coupling, when l_m is determined mainly by the radiation of modes into the prism, the depth of *m*-lines decreases with increasing *m*. The change in the TE-polarised Gaussian beam shape after its reflection from the layered structure is studied, which is determined by the energy transfer from the incident beam into waveguide modes that propagate along the structure by the distance l_m, are radiated in the direction of specular reflection and interfere with a part of the beam reflected from the working face of the prism. It is shown that this interference can lead to the field intensity oscillations near *m*-lines. The analysis of different methods for determining the parameters of thin-film structures is presented, including the measurement of mode angles θ_m and the reflected beam shape. The methods are based on simultaneous excitation of a few waveguide modes in the film with a strongly focused monochromatic Gaussian beam, the waist width of which is much smaller than the propagation length of the modes. As an example of using these methods, the refractive index and the thickness of silicon monoxide film on silica substrate at the wavelength 633 nm are determined.**

Keywords: *Gaussian beam, layered waveguide structures, thin films, refractive index measurement, resonance excitation of waveguide modes.*

1. Introduction

The method of resonance excitation of waveguide modes in a film in the geometry of frustrated total internal reflection (FTIR) is widely used in the studies of thin film structures [1 – 9]. The essence of the method is that the studied film is brought into contact with the working face of the measuring prism with a high refractive index N_p and is illuminated from the prism side with a monochromatic light beam. For the rays incident on the prism – film interface at the angles θ_m, for which the matching condition $N_p \sin\theta_m = \beta_m$ holds, where $m = 0, 1, 2, \ldots$, and β_m is the effective refractive index for the mode with number m, the condition of total internal reflection is violated, and the light can penetrate into the film, exciting the appropriate waveguide mode in it. In this case, in the angular dependence of the coefficient $R(\theta)$ of specular reflection of the light beam from the working face of the prism one can observe sharp and narrow minima, referred to as dark m-lines. If two mode angles θ_m are known, then, provided that N_p is known too, one can calculate β_m and by solving the system of dispersion equations for the waveguide modes determine two unknown parameters: the refractive index n_f and the thickness H_f of the film. Recently, the papers have appeared that essentially extend the area of application of this method. Thus, the possibility to determine the extinction coefficient of thin films from the experimentally measured width of m-lines was demonstrated [10, 11].

Practically, as a rule, the method of resonance excitation of waveguide modes is implemented by varying the angle of incidence of the collimated laser beam on the working face of the FTIR prism. Using this principle the instruments produced by Metricon Corp. and Sairon Technology Inc. [12] operate. However, such an approach requires the use of high-precision angle-measuring instruments and does not allow the measurement of the extinction coefficient in weakly absorbing films when the width of m-lines is smaller than the angular divergence of the laser beam.

In the present paper we consider the techniques for measuring the thickness H_f, the refractive index n_f and the extinction coefficient m_f of thin films within a wide range of wavelengths (from 400 to 1100 nm), free of the abovementioned drawbacks. The methods are based on simultaneous excitation of several waveguide modes in the film with a strongly focused monochromatic light beam, the lateral dimension of which in the waist is much smaller than the mode propagation length. The values of H_f, n_f and m_f are found by measuring the mode angles θ_m and by analysing the shape of the beam reflected from the layered structure. The methods do not imply angular scanning and are applicable to a wide class of thin film structures.

The diffraction of a transform-limited light beam from a layered waveguide structure has been considered previously [2, 8, 13, 14]. However, the results presented in these papers are not sufficient for the analysis of the changed shape of the beam reflected from the structure. Therefore, below we present a rigorous theoretical consideration of the problem of

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2. Reflection of a TE-polarised Gaussian beam from a layered waveguide structure

Consider a monochromatic Gaussian beam with the wavelength λ, incident at the angle θ_0 on a layered structure from the side of the measuring prism having the refractive index N_{p} (Fig. 1). Below we assume that the absorption in the prism material is absent, and the permittivity is given by the expression $\varepsilon_{\mathrm{p}} = N_{\mathrm{p}}^2$. The film with the thickness H_{f} and the refractive index n_{f} is deposited on the substrate with the refractive index n_{s}. The gap between the film and the prism has the thickness H_{i} and can be filled with air or an immersion liquid with a refractive index n_{i}. The permittivities of the immersion layer, the film and the substrate are determined by the expressions $\varepsilon_{\mathrm{i}} = (n_{\mathrm{i}} + \mathrm{i}m_{\mathrm{i}})^2$, $\varepsilon_{\mathrm{f}} = (n_{\mathrm{f}} + \mathrm{i}m_{\mathrm{f}})^2$ and $\varepsilon_{\mathrm{s}} = (n_{\mathrm{s}} + \mathrm{i}m_{\mathrm{s}})^2$, where m_{i}, m_{f} and m_{s} are extinction coefficients of the appropriate media.

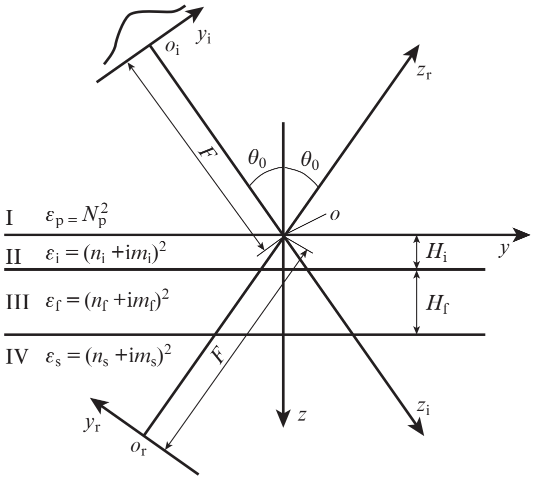

Figure 1. Reflection of a Gaussian light beam from a layered structure: (I) measuring prism; (II) immersion liquid (or air); (III) film; (IV) substrate; o_{i}, o and o_{r} are the origins of Cartesian rectangular coordinate systems, related to the incident beam, the layered structure and the reflected beam, respectively (the distances from o_{i} and o_{r} to o are equal to F).

In the coordinate system $(x_{\mathrm{i}}, y_{\mathrm{i}}, z_{\mathrm{i}})$, related to the incident beam, the TE-polarised two-dimensional Gaussian beam has the form [15]

$$\begin{gathered} \boldsymbol{e}_{\mathrm{i}}(y_{\mathrm{i}}, z_{\mathrm{i}}, t) = \boldsymbol{e}_{\mathrm{i}}(y_{\mathrm{i}}, z_{\mathrm{i}})\exp(-\mathrm{i}\omega t) + \mathrm{c.c.}, \\ \boldsymbol{e}_{\mathrm{i}}(y_{\mathrm{i}}, z_{\mathrm{i}}) = \boldsymbol{x}_{\mathrm{i}}\frac{w_0}{2\sqrt{\pi}}\int_{-\infty}^{+\infty}\exp\left(-\frac{k_{\mathrm{i}y}^2 w_0^2}{4} + \mathrm{i}k_{\mathrm{i}y}y_{\mathrm{i}} + \mathrm{i}k_{\mathrm{i}z}z_{\mathrm{i}}\right)\mathrm{d}k_{\mathrm{i}y}, \end{gathered} \tag{1}$$

where $2w_0$ is the beam diameter in the waist located in the plane $z_{\mathrm{i}} = 0$; and $k_{\mathrm{i}y}$ and $k_{\mathrm{i}z} = \sqrt{k^2\varepsilon_{\mathrm{p}} - k_{\mathrm{i}y}^2}$ are the projections of the wave vectors $\boldsymbol{k}_{\mathrm{i}} = (k_{\mathrm{i}y}, k_{\mathrm{i}z})$ and $k = \omega/c = 2\pi/\lambda$ of the plane waves belonging to the beam Fourier decomposition onto the axes y_{i} and z_{i}, respectively. Equation (1) determines the decomposition of the Gaussian beam in terms of plane waves. Performing the coordinate transformation to the axes (x, y, z) related to the layered structure $x_{\mathrm{i}} = x$, $y_{\mathrm{i}} = y\cos\theta_0 - z\sin\theta_0$, $z_{\mathrm{i}} = y\sin\theta_0 + z\cos\theta_0 + F$ and introducing the notations $k_y = k_{\mathrm{i}y}\cos\theta_0 + k_{\mathrm{i}z}\sin\theta_0$, $k_z = -k_{\mathrm{i}y}\sin\theta_0 + k_{\mathrm{i}z}\cos\theta_0$, we express the incident Gaussian beam (1) as

$$\boldsymbol{e}_{\mathrm{i}}(y, z) = \boldsymbol{x}\frac{w_0}{2\sqrt{\pi}}\int_{-\infty}^{+\infty}\exp\left(-\frac{k_{\mathrm{i}y}^2 w_0^2}{4} + \mathrm{i}k_{\mathrm{i}z}F + \mathrm{i}k_y y + \mathrm{i}k_z z\right)\mathrm{d}k_{\mathrm{i}y}. \tag{2}$$

Then, the light beam reflected from the layered structure can be presented in the form

$$\begin{gathered} \boldsymbol{e}_{\mathrm{r}}(y, z) = \boldsymbol{x}\frac{w_0}{2\sqrt{\pi}} \\ \times\int_{-\infty}^{+\infty} R_{\mathrm{a}}^{\mathrm{s}}(k_{\mathrm{i}y})\exp\left(-\frac{k_{\mathrm{i}y}^2 w_0^2}{4} + \mathrm{i}k_{\mathrm{i}z}F + \mathrm{i}k_y y - \mathrm{i}k_z z\right)\mathrm{d}k_{\mathrm{i}y}, \end{gathered} \tag{3}$$

where the amplitude reflection coefficient $R_{\mathrm{a}}^{\mathrm{s}}(k_{\mathrm{i}y})$ for plane waves from the incident beam Fourier spectrum is given by the expression (see Ref. [11])

$$\begin{aligned} R_{\mathrm{a}}^{\mathrm{s}}(k_{\mathrm{i}y}) = {} & \frac{k_z - \mathrm{i}\gamma^{\mathrm{II}}}{k_z + \mathrm{i}\gamma^{\mathrm{II}}}\left\{1 + \frac{4\mathrm{i}k_z\gamma^{\mathrm{II}}}{k^2(\varepsilon_{\mathrm{p}} - \varepsilon_{\mathrm{i}})}\right. \\ & \times\left(\frac{\gamma^{\mathrm{II}} - \gamma^{\mathrm{III}}}{\gamma^{\mathrm{II}} + \gamma^{\mathrm{III}}} + \frac{\gamma^{\mathrm{III}} - \gamma^{\mathrm{IV}}}{\gamma^{\mathrm{III}} + \gamma^{\mathrm{IV}}}\exp(-2\gamma^{\mathrm{III}}H_{\mathrm{f}})\right)\exp(-2\gamma^{\mathrm{II}}H_{\mathrm{i}}) \\ & \times\left[1 + \frac{\gamma^{\mathrm{II}} - \gamma^{\mathrm{III}}}{\gamma^{\mathrm{II}} + \gamma^{\mathrm{III}}}\frac{\gamma^{\mathrm{III}} - \gamma^{\mathrm{IV}}}{\gamma^{\mathrm{III}} + \gamma^{\mathrm{IV}}}\exp(-2\gamma^{\mathrm{III}}H_{\mathrm{f}}) + \frac{k_z - \mathrm{i}\gamma^{\mathrm{II}}}{k_z + \mathrm{i}\gamma^{\mathrm{II}}}\right. \\ & \left.\left.\times\left(\frac{\gamma^{\mathrm{II}} - \gamma^{\mathrm{III}}}{\gamma^{\mathrm{II}} + \gamma^{\mathrm{III}}} + \frac{\gamma^{\mathrm{III}} - \gamma^{\mathrm{IV}}}{\gamma^{\mathrm{III}} + \gamma^{\mathrm{IV}}}\exp(-2\gamma^{\mathrm{III}}H_{\mathrm{f}})\right)\exp(-2\gamma^{\mathrm{II}}H_{\mathrm{i}})\right]^{-1}\right\}. \end{aligned} \tag{4}$$

Here, $\gamma^{\mathrm{II}} = (k_y^2 - k^2\varepsilon_{\mathrm{i}})^{1/2}$, $\mathrm{Re}\gamma^{\mathrm{II}} \geqslant 0$, $\mathrm{Im}\gamma^{\mathrm{II}} \leqslant 0$; $\gamma^{\mathrm{III}} = (k_y^2 - k^2\varepsilon_{\mathrm{f}})^{1/2}$, $\mathrm{Re}\gamma^{\mathrm{III}} \geqslant 0$, $\mathrm{Im}\gamma^{\mathrm{III}} \leqslant 0$; and $\gamma^{\mathrm{IV}} = (k_y^2 - k^2\varepsilon_{\mathrm{s}})^{1/2}$, $\mathrm{Re}\gamma^{\mathrm{IV}} \geqslant 0$, $\mathrm{Im}\gamma^{\mathrm{IV}} \leqslant 0$. The first term in the braces in Eqn (4) describes the reflection of a plane light wave from the prism boundary, when the sample (film on substrate) is removed. The rest terms are related to the excitation of waveguide modes in the film and with radiation of these modes in the direction of specular reflection, when the sample is in optical contact with the prism.

Figure 2 presents the dependences of the reflection coefficient $R_{\mathrm{s}}(\theta) = |R_{\mathrm{a}}^{\mathrm{s}}(k_{\mathrm{i}y})|^2$ on the angle of incidence θ of the plane TE wave with the wave vector $\boldsymbol{k}_{\mathrm{i}} = (k_{\mathrm{i}y}, k_{\mathrm{i}z})$ onto the prism–film interface, the angle θ being expressed as follows:

$$\theta = \arcsin\left(\frac{1}{N_{\mathrm{p}}}\right)\left[\frac{k_{\mathrm{i}y}}{k}\cos\theta_0 + \sqrt{N_{\mathrm{p}}^2 - \left(\frac{k_{\mathrm{i}y}}{k}\right)^2}\sin\theta_0\right].$$

It is seen that in the dependence $R_{\mathrm{s}}(\theta)$ one can observe narrow minima (m-lines) caused by the resonance excitation of TE waveguide modes in the film. To each mode with the number m there corresponds the angle θ_m, at which the resonance excitation of this mode by the incident plane wave occurs. The m-line widths $\delta\theta_m$ are determined by the extinction coefficients of the modes in the waveguide and usually amount to 0.01° – 0.5°. The sharp bend of the curve $R_{\mathrm{s}}(\theta)$ at $\theta = 42.5°$ ('a knee') arises when $N_{\mathrm{p}}\sin\theta = n_{\mathrm{s}}$ and is caused by tunnelling of light into the substrate.

Performing the coordinate transformation to the system $(x_{\mathrm{r}}, y_{\mathrm{r}}, z_{\mathrm{r}})$ related to the reflected beam $x = x_{\mathrm{r}}$, $y = -y_{\mathrm{r}}\cos\theta_0 + (z_{\mathrm{r}} - F)\sin\theta_0$ and $z = -y_{\mathrm{r}}\sin\theta_0 - (z_{\mathrm{r}} - F)\cos\theta_0$, we obtain for the beam reflected from the layered structure

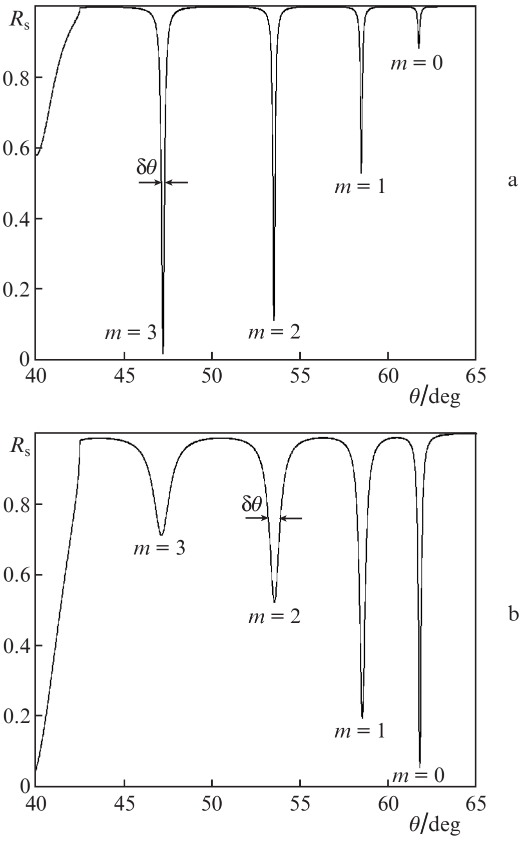

Figure 2. Dependences of the specular reflection coefficient $R_{\rm s} = |R_{\rm a}^{\rm s}(k_{{\rm i}y})|^2$ on the angle of incidence θ under illumination of the film–prism interface with a plane TE-polarised electromagnetic wave with $\lambda = 632.8$ nm at $H_{\rm i}$ = (a) 150 and (b) 60 nm. The parameters of the layered structure are $N_{\rm p} = 2.15675$, $\varepsilon_{\rm i} = 1$, $\varepsilon_{\rm f} = (1.92 + {\rm i}0.001)^2$, $H_{\rm f} = 1000$ nm, $n_{\rm s} = 1.45705$, $m_{\rm s} = 0$.

$$\boldsymbol{e}_{\rm r}(y_{\rm r},z_{\rm r}) = \boldsymbol{x}_{\rm r}\frac{w_0}{2\sqrt{\pi}}\int_{-\infty}^{+\infty} R_{\rm a}^{\rm s}(k_{{\rm i}y})$$
$$\times\exp\left(-\frac{k_{{\rm i}y}^2 w_0^2}{4} - {\rm i}k_{{\rm i}y}y_{\rm r} + {\rm i}k_{{\rm i}z}z_{\rm r}\right){\rm d}k_{{\rm i}y}. \quad (5)$$

Equations (4) and (5) are exact. They are valid at any relation between the width $2w_0$ of the TE-polarised Gaussian beam in the waist and the propagation lengths l_m of the waveguide modes.

For simplicity, assume that $\varepsilon_{\rm i} = \varepsilon_{\rm s}$ and $\gamma^{\rm II} = \gamma^{\rm IV}$. Let us also consider the immersion layer and the substrate to be non-absorbing, i.e., $m_{\rm i} = m_{\rm s} = 0$. The absorption is present only in the film having the complex permittivity $\varepsilon_{\rm f} = \varepsilon_{\rm f}' + {\rm i}\varepsilon_{\rm f}''$. Then, with the small width of m-lines (see Fig. 2) taken into account, expanding the function $R_{\rm a}^{\rm s}(k_{{\rm i}y})$ in a Taylor series at $k_{{\rm i}y}^m = kN_{\rm p}\sin(\theta_m - \theta_0)$ with respect to the small parameter $k_{{\rm i}y} - k_{{\rm i}y}^m$, we obtain

$$R_{\rm a}^{\rm s}(k_{{\rm i}y}) \approx \frac{k_{mz} - {\rm i}\gamma_m^{\rm II}}{k_{mz} + {\rm i}\gamma_m^{\rm II}}\left[1 + \frac{{\rm i}A_m/l_m\cos\theta_0}{(k_{{\rm i}y} - k_{{\rm i}y}^m) - {\rm i}/l_m\cos\theta_0}\right], \quad (6)$$

where

$$A_m = \frac{2a_m\exp(-2\gamma_m^{\rm II}H_{\rm i})}{b_m\varepsilon_{\rm f}'' + a_m\exp(-2\gamma_m^{\rm II}H_{\rm i})};\ l_m = \frac{c_m}{b_m\varepsilon_{\rm f}'' + a_m\exp(-2\gamma_m^{\rm II}H_{\rm i})}. \quad (7)$$

The expressions for the coefficients a_m, b_m and c_m that enter Eqn (7) and depend on the parameters of the layered structure are presented in the Appendix. The first term in the denominator of Eqn (7) for l_m describes the mode attenuation due to the light absorption in the film, while the second term is responsible for the attenuation due to the mode radiation into the prism. The mode propagation length is seen to decrease both with increasing extinction coefficient of the film and with narrowing gap $H_{\rm i}$ between the film and the prism.

Let us assume that $z_{\rm r} \ll w_0(kw_0N_{\rm p})^3$, then in Eqn (5) we can restrict ourselves to the expansion $k_{{\rm i}z} \approx kN_{\rm p} - k_{{\rm i}y}^2/(2kN_{\rm p})$. Then, substituting Eqns (6) and (7) into Eqn (5) and performing the integration, we obtain for the reflected beam

$$\boldsymbol{e}_{\rm r}(y_{\rm r},z_{\rm r}) = \boldsymbol{x}_{\rm r}\frac{k_{mz} - {\rm i}\gamma_m^{\rm II}}{k_{mz} + {\rm i}\gamma_m^{\rm II}}\frac{w_0}{w(z_{\rm r})}\exp\left[{\rm i}kN_{\rm p}z_{\rm r} - \left(\frac{y_{\rm r}}{w(z_{\rm r})}\right)^2\right]$$
$$\times\left\{1 - \sqrt{\pi}A_m\frac{w(z_{\rm r})/\cos\theta_0}{2l_m}\right.$$
$$\times\exp\left[\left(\frac{w(z_{\rm r})/\cos\theta_0}{2l_m}(1 - {\rm i}k_{{\rm i}y}^m l_m\cos\theta_0) + \frac{y_{\rm r}}{w(z_{\rm r})}\right)^2\right]$$
$$\left.\times{\rm erfc}\left[\frac{w(z_{\rm r})/\cos\theta_0}{2l_m}(1 - {\rm i}k_{{\rm i}y}^m l_m\cos\theta_0) + \frac{y_{\rm r}}{w(z_{\rm r})}\right]\right\}. \quad (8)$$

Here

$$w(z_{\rm r}) = w_0\sqrt{1 + \frac{{\rm i}2z_{\rm r}}{kN_{\rm p}w_0^2}}$$

is the complex radius of the Gaussian beam at the distance $z_{\rm r}$ from the waist, located in the plane $z_{\rm r} = 0$; and

$${\rm erfc}z = \frac{2}{\sqrt{\pi}}\int_z^{\infty}\exp(-t^2){\rm d}t$$

is the error integral [16]. Equation (8) describes the shape of the beam reflected from the layered structure under the conditions of resonance excitation of the waveguide mode with the number m.

3. Analysis of the beam shape after reflection from the layered structure

Consider different limiting cases. First, let us analyse the case of a 'wide' Gaussian beam, when $2w_0 \gg l_m$. Then, the angular divergence of the incident beam $\Delta\theta \approx \lambda/(2w_0)$ is much smaller than the angular width $\delta\theta_m$ of the resonance, related to the excitation of the mth waveguide mode (Fig. 2). Using the asymptotic expression $\sqrt{\pi}z\exp(z^2){\rm erfc}z \approx 1$, valid if $z \to \infty$, $0 < \arg z < \pi/2$ [16], with the relation $k_{{\rm i}y}^m = kN_{\rm p}\sin(\theta_m - \theta_0) \approx kN_{\rm p}(\theta_m - \theta_0)$ taken into account, we obtain from Eqn (8) that

$$|\boldsymbol{e}_{\rm r}(y_{\rm r},z_{\rm r})| \approx \left|\frac{k_{mz} - {\rm i}\gamma_m^{\rm II}}{k_{mz} + {\rm i}\gamma_m^{\rm II}}\frac{w_0}{w(z_{\rm r})}\exp\left[-\left(\frac{y_{\rm r}}{w(z_{\rm r})}\right)^2\right]\right|$$
$$\times\left|1 - \frac{A_m}{1 - {\rm i}kN_{\rm p}(\theta_m - \theta_0)l_m\cos\theta_0}\right|. \quad (9)$$

From Eqn (9) it is seen that a wide Gaussian beam is reflected from the layered structure without changing its shape. In this case the m-line angular width depends on the waveguide mode propagation length and is determined by the relation

$$\delta\theta = \frac{2}{kN_{\rm p}l_m\cos\theta_0}. \quad (10)$$

Increasing the film extinction coefficient or decreasing the gap between the film and the prism leads to the reduction of the waveguide mode propagation length l_m [see Eqn (7)] and, according to Eqn (10), to the mode resonance broadening. If the angle of incidence θ_0 of the 'wide' Gaussian beam onto the layered structure is equal to the mode angle θ_m, then from Eqns (9) and (7) we find

$$|\boldsymbol{e}_{\rm r}(y_{\rm r},z_{\rm r})| = \left|\frac{k_{mz}-{\rm i}\gamma_m^{\rm II}}{k_{mz}+{\rm i}\gamma_m^{\rm II}}\frac{w_0}{w(z_{\rm r})}\exp\left[-\left(\frac{y_{\rm r}}{w(z_{\rm r})}\right)^2\right]\right| \times\left|1-\frac{2a_m\exp(-2\gamma_m^{\rm II}H_{\rm i})}{b_m\varepsilon_{\rm f}''+a_m\exp(-2\gamma_m^{\rm II}H_{\rm i})}\right|, \qquad (11)$$

where a_m and b_m are defined in the Appendix. Consider the case of weak coupling, when $b_m\varepsilon_{\rm f}'' \gg a_m\exp(-2\gamma_m^{\rm II}H_{\rm i})$, i.e., the length of the waveguide mode propagation is mainly determined by the absorption in the film. In this case Eqn (11) yields

$$|\boldsymbol{e}_{\rm r}(y_{\rm r},z_{\rm r})| = \left|\frac{k_{mz}-{\rm i}\gamma_m^{\rm II}}{k_{mz}+{\rm i}\gamma_m^{\rm II}}\frac{w_0}{w(z_{\rm r})}\exp\left[-\left(\frac{y_{\rm r}}{w(z_{\rm r})}\right)^2\right]\right| \times\left|1-\frac{2a_m\exp(-2\gamma_m^{\rm II}H_{\rm i})}{b_m\varepsilon_{\rm f}''}\right|. \qquad (12)$$

From Eqn (12) it is easily seen that in the case of weak coupling the deepest m-lines are those with small $\gamma_m^{\rm II}$, i.e., correspond to high-order modes with minimal localisation inside the film (see Fig. 2a).

In the case of strong coupling, when $b_m\varepsilon_{\rm f}'' \ll a_m \times \exp(-2\gamma_m^{\rm II}H_{\rm i})$ and the attenuation of waveguide modes is mainly determined by the radiation into the prism, Eqn (11) yields

$$|\boldsymbol{e}_{\rm r}(y_{\rm r},z_{\rm r})| = \left|\frac{k_{mz}-{\rm i}\gamma_m^{\rm II}}{k_{mz}+{\rm i}\gamma_m^{\rm II}}\frac{w_0}{w(z_{\rm r})}\exp\left[-\left(\frac{y_{\rm r}}{w(z_{\rm r})}\right)^2\right]\right| \times\left|1-\frac{2b_m\varepsilon_{\rm f}''\exp(2\gamma_m^{\rm II}H_{\rm i})}{a_m}\right|. \qquad (13)$$

From Eqn (13) it is seen that in the case of strong coupling the deepest m-lines are those with large $\gamma_m^{\rm II}$, i.e., correspond to the low-order modes strongly localised within the film (see Fig. 2b). In the general case, when for lower modes the weak coupling case is implemented, while for higher-order modes the coupling is strong, the depth of m-lines at first increases and then decreases with increasing mode number.

Now let us analyse the case of a 'narrow' Gaussian beam having the waist diameter much smaller than the waveguide mode propagation length, $2w_0 \ll l_m$. In this case the angular divergence of the incident beam is much greater than the angular width of the m-line. Then, using the asymptotic representation $\operatorname{erfc} z \approx 2$, valid at $z \to \infty$, $\pi/2 < \arg z < \pi$ [16], we obtain from Eqn (8):

$$|\boldsymbol{e}_{\rm r}(y_{\rm r},z_{\rm r})| \approx \left|\left|\frac{k_{mz}-{\rm i}\gamma_m^{\rm II}}{k_{mz}+{\rm i}\gamma_m^{\rm II}}\frac{w_0}{w(z_{\rm r})}\right|\exp\left[-\left(\frac{y_{\rm r}}{w(z_{\rm r})}\right)^2\right] - 2\sqrt{\pi}A_m\frac{w(z_{\rm r})/\cos\theta_0}{2l_m}\exp\left\{\left[\frac{w(z_{\rm r})/\cos\theta_0}{2l_m}(1-{\rm i}k_{{\rm i}y}^m l_m\cos\theta_0)\right]^2 + \frac{y_{\rm r}/\cos\theta_0}{l_m}(1-{\rm i}k_{{\rm i}y}^m l_m\cos\theta_0)\right\}\right|. \qquad (14)$$

The first term in the right-hand side of Eqn (14) describes the Gaussian beam reflected from the prism boundary, and the second term is related to the excitation of the waveguide mode in the layered structure. From Eqn (14) it follows that the amplitude of the waveguide mode is damped and oscillates following the law

$$\exp\left(\frac{y_{\rm r}/\cos\theta_0}{l_m}\right)\cos[ky_{\rm r}N_{\rm p}\sin(\theta_m-\theta_0)] \quad \text{at } y_{\rm r}\to-\infty.$$

Figure 3 presents the theoretically calculated intensity distribution $|\boldsymbol{e}_{\rm r}(y_{\rm r}, z_{\rm r} = {\rm const})|^2$ over the cross section of the reflected beam in the case when the 'narrow' Gaussian beam is incident on the layered structure under the conditions of simultaneous excitation of three waveguide modes with m = 0, 1, 2. The calculations were carried out using Eqns (4) and (5).

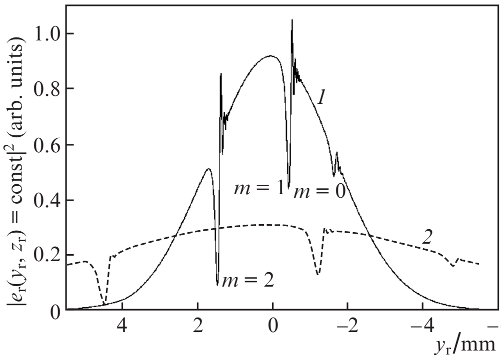

Figure 3. Field intensity distribution $|\boldsymbol{e}_{\rm r}(y_{\rm r}, z_{\rm r} = {\rm const})|^2$ in the cross section of the reflected beam, calculated using Eqns (4) and (5) in the case of a Gaussian beam with λ = 632.8 nm, $2w_0$ =2.83 μm incident on the layered structure at θ_0 = 45.62°, F = 0 when $z_{\rm r}$ = (*1*) 5 and (*2*) 15 cm. The parameters of the prism and the waveguide structure are $N_{\rm p}$ = 2.15675, $\varepsilon_{\rm i}$ = 1, $H_{\rm i}$ = 160 nm, $\varepsilon_{\rm f} = (1.6 + {\rm i}0.001)^2$, $H_{\rm f}$ = 1400 nm, $n_{\rm s}$ = 1.45705, $m_{\rm s}$ = 0.

As seen from Fig. 3, in the specularly reflected beam there are three m-lines, corresponding to the mode structure of the waveguide. To the right of them, in the direction of waveguide mode propagation, the field intensity oscillations are observed. The cause of these oscillations is that the incident Gaussian beam excites the waveguide mode with the number m that propagates along the layered structure by the distance l_m and is radiated in the direction of specular reflection into the prism, where it interferes with the part of the beam reflected from the working face of the prism. Note that in the far-field zone (at $z_{\rm r} \to \infty$) the amplitude of oscillations decreases in correspondence with Eqn (14). The field intensity oscillations in the cross section of the beam specularly reflected from the layered structure were observed by us experimentally (Fig. 4).

4. Measurement of the thin film structure parameters

The measurement of optical parameters and thickness of thin films using the method of resonance excitation of waveguide modes was implemented by means of the spectroscopic prism coupling device designed by us earlier [17]. This device allows determination of the refractive index, extinction coefficient and thin film thickness within a wide (400–1100 nm) wave-

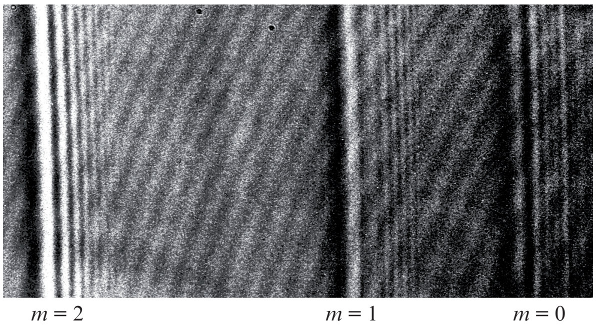

Figure 4. Field intensity oscillations in the cross section of the specularly reflected beam near m-lines under the illumination of Al_2O_3 film with $H_f \approx 1.6$ μm on a silica substrate with the focused light beam at a wavelength of 589.3 nm.

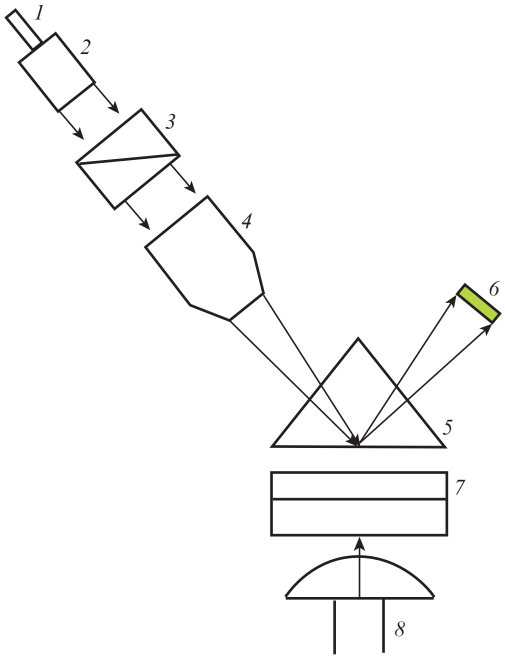

Figure 5. Schematic diagram of the prism coupling device for measuring the parameters of thin films using the method of resonance excitation of waveguide modes:
(*1*) optical fibre; (*2*) collimator; (*3*) Glan polarising prism; (*4*) microscope objective; (*5*) measuring prism; (*6*) silicon CCD matrix; (*7*) sample film on substrate; (*8*) pneumatic pusher.

length range. A schematic diagram of the device is presented in Fig. 5.

By means of an optical fibre (*1*), collimator (*2*), polarising prism (*3*) and microscope objective (*4*), the monochromatic light beam with TE or TM polarisation and the wavelength λ (e.g., from a monochromator or a spectral lamp) is focused on the hypotenuse face of the measuring prism (*5*), against which the studied thin-film structure (*7*) is pressed by means of a pneumatic pusher (*8*). The beam reflected from the region of the optical contact between the prism and the film is recorded by the silicon CCD matrix (*6*) (the sensitivity range 400–1100 nm), from which the image is transmitted to the PC monitor (not shown in Fig. 5) via the USB cable. The CCD matrix measures the intensity distribution in the cross section of the reflected beam $I(N_{\rm pix})$ under the conditions of resonance excitation of waveguide modes, and $I_0(N_{\rm pix})$ when the sample is removed. Here $N_{\rm pix}$ is the number of the pixel in the row of the CCD matrix, corresponding to a certain value of the coordinate $y_{\rm r}$ (see Fig.1). Then the ratio $I(N_{\rm pix})/I_0(N_{\rm pix})$ is calculated, from which the values of $N_{\rm pix}^m$ corresponding to the positions of m-lines are determined. A certain mode angle θ_m corresponds to each pixel $N_{\rm pix}^m$. Indeed, in the far-field zone of the beam, reflected from the layered structure, when $2z_{\rm r}/(kN_{\rm p}w_0^2) \gg 1$, using the asymptotic expression $\sqrt{\pi}\,z\exp(z^2)\,{\rm erfc}\,z \approx 1$ we find from Eqn (8) that

$$|\boldsymbol{e}_{\rm r}(y_{\rm r}, z_{\rm r})| \approx \left| \frac{k_{mz} - {\rm i}\gamma_m^{\rm II}}{k_{mz} + {\rm i}\gamma_m^{\rm II}} \frac{w_0}{w(z_{\rm r})} \exp\left[-\left(\frac{y_{\rm r}}{w(z_{\rm r})}\right)^2\right]\right|$$

$$\times\left|1 - \frac{A_m}{1 - {\rm i}l_m \cos\theta_0 kN_{\rm p}[\sin(\theta_m - \theta_0) + y_{\rm r}/z_{\rm r}]}\right|. \qquad (15)$$

This expression is valid for $z \to \infty$, $0 < \arg z < \pi/2$. From Eqn (15) it is easy to see that the m-line minimum at the CCD matrix is achieved at $y_{\rm r} = z_{\rm r}\sin(\theta_0 - \theta_m)$. Therefore, having measured $N_{\rm pix}^m$, one can determine the mode angles θ_m and calculate the effective refractive indices of the waveguide modes $\beta_m = N_{\rm p}\sin\theta_m$. The relation between $N_{\rm pix}^m$ and θ_m is determined by the calibration curve and is established in the course of the device tuning by measuring test thin film structures with known position of m-lines. If at least two mode angles θ_m are determined, then from the found values of β_m by solving the system of nonlinear dispersion equations for the waveguide modes [18] the refractive index $n_{\rm f}(\lambda)$ and the thickness $H_{\rm f}$ of the film are calculated.

Figure 6 shows the light intensity distribution on the CCD matrix when the silicon monoxide SiO film with the thickness $H_{\rm f} \approx 1$ μm, deposited on silica substrate, is pressed to the measuring prism, and when the film is removed.

Figure 7a presents the measured dependences $I(N_{\rm pix})$ and $I_0(N_{\rm pix})$ in the central row of the CCD matrix, which correspond to Fig. 6, while Fig. 7b illustrates the ratio $I(N_{\rm pix})/I_0(N_{\rm pix})$.

As follows from Figs 6 and 7, four m-lines corresponding to the excitation of TE waveguide modes with $m = 0, 1, 2, 3$ by the focused light beam are observed in the film. From the experimentally measured position of m-lines the film parameters $n_{\rm f}(\lambda = 633\ {\rm nm}) \approx 1.9298$, $H_{\rm f} \approx 1.015$ μm were calculated. Let us estimate the precision of measuring the thin-film structure parameters by means of the proposed method. The precision of measuring the refractive index $n_{\rm f}$ and the film thickness $H_{\rm f}$ depends on the CCD matrix resolution, as well as on the precision of determining the refractive index and the refracting angles of the measuring prism. For example, when using the matrix with 4096×3288 pixels and the objective with the numerical amplitude 0.4, the angular range of which amounts to $\sim 20°$, the resolution of one pixel corresponds to $20°/4096 = 18''$. If the prism refractive index is determined using a goniometer with the error $\pm 5 \times 10^{-5}$, and its refracting angles are determined with the error $\pm 5''$, then the absolute error of measuring $n_{\rm f}$ by means of the prism coupling device equals $\pm 2 \times 10^{-4}$. This corresponds to the typical error of the refractive index measurements using refractometers of the Abbe type.

The more advanced technique allows simultaneous determination of the refractive index $n_{\rm f}$, the extinction coefficient $m_{\rm f}$ and the thickness $H_{\rm f}$ of the film. The technique consists in the following: using rigorous expressions (4) and (5) the ratio $R_{\rm thr}(N_{\rm pix}) = I(N_{\rm pix})/I_0(N_{\rm pix})$ is calculated, which is then com-

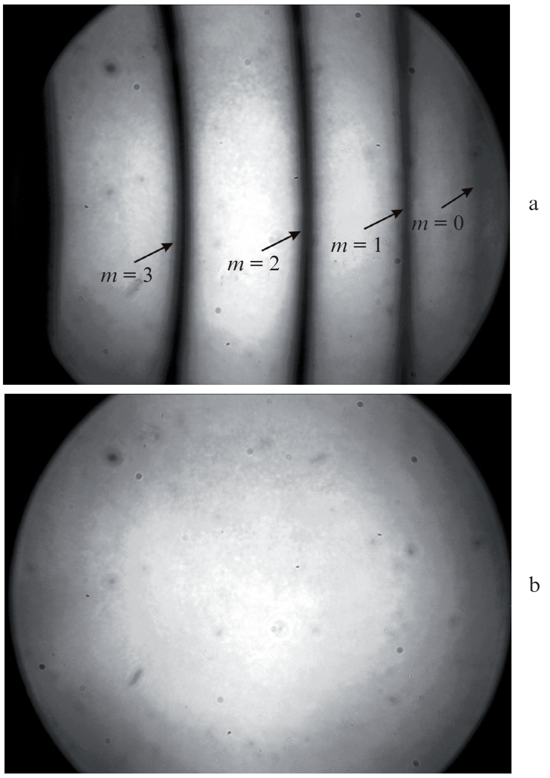

Figure 6. Light intensity distribution on the CCD matrix under the illumination of the SiO film on a silica substrate with a focused TE-polarised light beam with λ = 633 nm; (a) the film is in optical contact with the prism, (b) the film is removed.

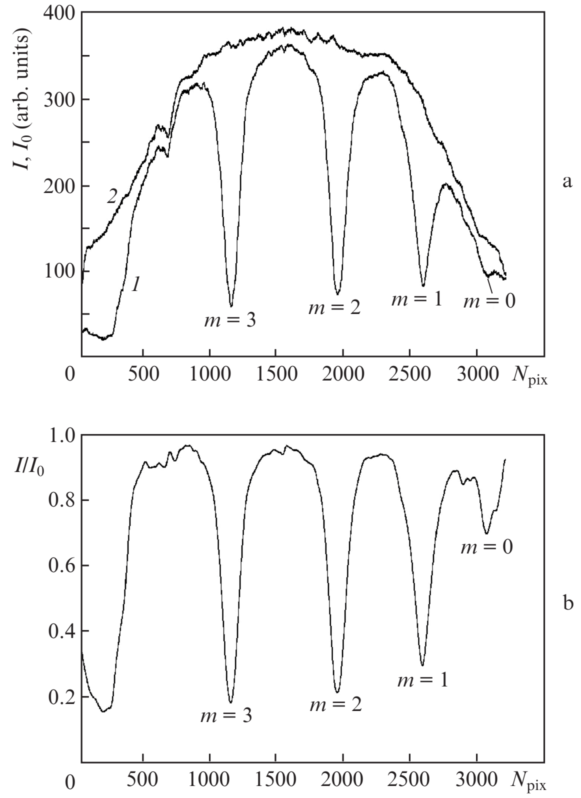

Figure 7. (a) Intensity distributions (1) $I(N_{\text{pix}})$ and (2) $I_0(N_{\text{pix}})$ over the central row of the CCD matrix, corresponding to Fig. 6, and (b) the ratio I/I_0 vs. N_{pix}.

pared with the corresponding dependence $R_{\text{exp}}(N_{\text{pix}})$ measured experimentally. By varying n_{f}, m_{f} and H_{f} the best fit between $R_{\text{thr}}(N_{\text{pix}})$ and $R_{\text{exp}}(N_{\text{pix}})$ is achieved and, thus, the desired film parameters are determined. This technique is more labour-consuming, because it requires significant computational resources. However, potentially it is applicable to the study not only of homogeneous films, but also of films with the depth-modulated permittivity. The use of this technique for determining the parameters of thin film structures will be described elsewhere.

5. Conclusions

The problem of reflection of a TE-polarised Gaussian light beam from a layered structure under the conditions of resonance excitation of waveguide modes is considered. The expressions for the waveguide mode propagation length as a function of the extinction coefficient in the thin film and the thickness of the gap between the prism and the film are derived. The change in the specularly reflected beam shape caused by the energy transfer from the incident beam into the waveguide mode and from the waveguide mode into the reflected beam is studied. Different methods of measuring the parameters of thin film structures, based on measuring the mode angles θ_m and analysing the reflected beam shape are analysed. These methods are based on simultaneous excitation of several waveguide modes in the film by a focused light beam, having the width much smaller than the propagation length of waveguide modes. Using the designed prism coupling device the optical constants of silicon monoxide SiO film at the wavelength 633 nm are determined. The device includes a silicon CCD matrix, which allows the measurements at any wavelength within the range 400–1100 nm. When using the InGaAs CCD matrix, the operating range of the prism coupling device can be extended up to 1700 nm.

The reflection of a Gaussian light beam from a layered structure is considered for TE polarisation. All the results obtained are qualitatively valid also in the case of TM polarisation of the incident beam.

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Appendix

The expressions for a_m, b_m and c_m that enter Eqn (7) and depend on the parameters of the layered structure are found from Eqn (4). They have the form:

$$a_m = \frac{2k_{mz}\gamma_m^{\text{II}}}{k^2(\varepsilon_{\text{p}} - \varepsilon_{\text{i}})},$$

$$b_m = \frac{[|\gamma_m^{\text{III}}|^2 - (\gamma_m^{\text{II}})^2]\cot(|\gamma_m^{\text{III}}|H_{\text{f}}) + 2\gamma_m^{\text{II}}|\gamma_m^{\text{III}}|}{k^2(\varepsilon'_{\text{f}} - \varepsilon_{\text{i}})^2}\,\frac{\gamma_m^{\text{II}}}{|\gamma_m^{\text{III}}|}$$
$$+ \frac{\gamma_m^{\text{II}} H_{\text{f}}}{(\varepsilon'_{\text{f}} - \varepsilon_{\text{i}})\sin^2(|\gamma_m^{\text{III}}|H_{\text{f}})}, \qquad \text{(A1)}$$

$$c_m = \frac{2k_{my}}{k^2(\varepsilon'_{\text{f}} - \varepsilon_{\text{i}})}\times$$

$$\times\left[2+\cot(|\gamma_m^{\mathrm{III}}|\,H_{\mathrm{f}})\left(\frac{|\gamma_m^{\mathrm{III}}|}{\gamma_m^{\mathrm{II}}}-\frac{\gamma_m^{\mathrm{II}}}{|\gamma_m^{\mathrm{III}}|}+\frac{2\gamma_m^{\mathrm{II}}H_{\mathrm{f}}}{\sin(2\,|\gamma_m^{\mathrm{III}}|\,H_{\mathrm{f}})}\right)\right],$$

where $k_{my} = kN_{\mathrm{p}}\sin\theta_m$; $\gamma_m^{\mathrm{II}} = \left|k_{my}^2 - k^2\varepsilon_{\mathrm{i}}\right|^{1/2}$; and $|\gamma_m^{\mathrm{III}}| = |k_{my}^2 - k^2\varepsilon'_{\mathrm{f}}|^{1/2}$. Note that a_m, b_m and c_m do not depend on the extinction coefficient in the film and the thickness of the gap between the film and the prism.

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