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Raman scattering of a photon with frequency doubling by a channelled positron

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Abstract. **We have analysed the possibility of appearance of anti-Stokes lines in the spectrum of Raman scattering of a photon by a 'quasi-bound' charged particle in the regime of planar (axial) channelling. It is shown that radiation may emerge at the frequency, which is a combination of the incident photon frequency ω_0 and transition frequency ω_i in the transverse quantised motion of a channelled particle: $\omega = \omega_0 \pm 2\gamma^2\omega_i$, where γ is the relativistic (Lorentz) factor of a channelled particle.**

Keywords: *Raman scattering, channelling.*

1. Introduction

Consider the motion of a charged particle (positron) having the energy E and momentum $\boldsymbol{p}$ directed at a small angle θ to the crystallographic plane in a single crystal. If this angle is less than the so-called Lindhardt angle θ_L [1], the particle in the single crystal moves in the channelling regime. In the longitudinal direction, the potential responsible for planar channelling is constant (the continuous potential of a plane is characterised by a lack of dependence on the longitudinal coordinate [2]), so that the longitudinal momentum of the channelled particle is conserved:

$$E_1 + \hbar\omega_1 = E_2 + \hbar\omega_2, \tag{1}$$

$$p_{1z} + \hbar k_{1z} = p_{2z} + \hbar k_{2z} \ \ (\omega_{1,2} = ck_{1,2}). \tag{2}$$

The transverse component of the momentum of a positively charged particle during such a motion in the potential of crystallographic planes is quantised: $p_n = p\theta_n$ (Fig.1) [2, 3].

It should be noted that the quantised energy of a channelled particle in the co-moving coordinate system (CCS) that is moving with velocity $V = E_1/p_1$ depends on the energy of a channelled particle, which is due to the fact that the potential $U_{\rm pl}$ created by the crystallographic planes depends on the particle energy: $U_{\rm pl} = \gamma U(x)$, where $U(x)$ is the potential of the planes in the laboratory frame and γ is the relativistic factor of an electron.

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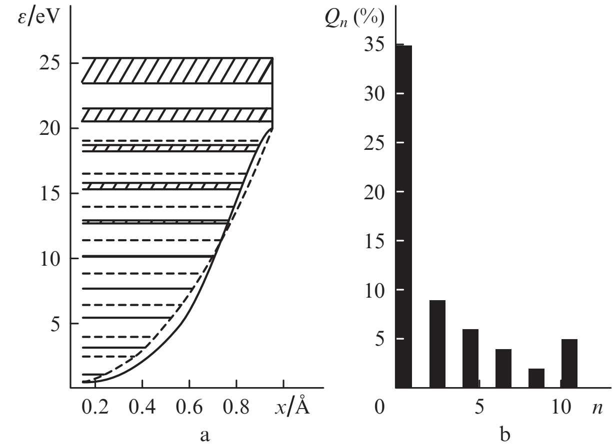

Figure 1. (a) Energy bands (levels) and (b) coefficients of population levels for the positron energy E_1 = 28 MeV in Si at a zero angle of incidence relative to the (110) plane. The dashed curve is a parabolic potential, dashed horizontal lines show the level positions in it.

The channelled particle may undergo transitions between the zones of transverse motion, located inside the well [2, 3].

The transverse motion of a channelled particle is characterised by a strongly expressed quantised band energy spectrum. The bands lying deep in the wells are very narrow, and so it is virtually possible to speak about discrete levels in the well. Thus, in the case of a channelled particle moving in the regime of planar channelling, we have to deal (in the CCS) with a 'one-dimensional' atom, the emission spectrum of which is significantly affected by the Doppler effect. This suggests that various effects known in atomic physics may take place for the channelled particles; particularly, Raman scattering (generation of multiple harmonics) may occur in the laser beam interacting with a photon.

2. Kinematics of Raman scattering of a photon by a 'bound' channelled positron

Main characteristics of Raman scattering can be obtained by analysing the laws of conservation of energy and longitudinal momentum. Assume that a positron with the momentum $\boldsymbol{p}_1$ and energy E_1 and a photon with the momentum $\boldsymbol{k}_1$ and energy ω_1 fall onto a single crystal (we assume that $h = c = 1$ unless otherwise stated). As a result of the interaction of the photon with the channelled positron, a photon with the momentum $\boldsymbol{k}_2$ and energy E_2 is emitted, while the momentum and energy of the positron in the final state take the values $\boldsymbol{p}_2$

and E_2. Note that, if this the reaction takes place in a constant arbitrary field, the energy of system (1) is conserved.

In the longitudinal direction, the potential responsible for the channelling of a positively charged particle is constant and the system possesses a certain longitudinal momentum (2) [the photon momentum in a medium is equal to $k_{1,2}n(\omega)$, where $n(\omega)$ is the refractive index of the crystal, which for simplicity we assume to be equal to unity].

Let us analyse relation (2), which describes the law of conservation of the longitudinal momentum. In accordance with the kinematics of motion of a channelled ultra-relativistic positron

$$p_{1zn} = \sqrt{p_1^2 - 2E_1\varepsilon_n(p_1)}, \quad p_{2zm} = \sqrt{p_2^2 - 2E_2\varepsilon_m(p_2)}, \qquad (3)$$

where $\varepsilon_n(p_1)$ and $\varepsilon_m(p_2)$ are the quantised energies of transverse motion in the initial and final states.

By using (3), we can rewrite equation (2) as

$$\omega_1\cos\vartheta_0 + \sqrt{E_1^2 - m_{\rm p}^2 - 2E_1\varepsilon_n(E_1)}$$
$$= \omega_2\cos\vartheta + \sqrt{E_2^2 - m_{\rm p}^2 - 2E_2\varepsilon_m(E_2)}, \qquad (4)$$

where ϑ_0 and ϑ are the angles between the photon propagation direction and the longitudinal axis before and after scattering, respectively; and $m_{\rm p}$ is the rest mass of the positron.

Consider the most interesting case of Raman scattering under the condition of planar channelled motion of positively charged particles (positrons) with the energies E_1, $E_2 \gg \omega_1, \omega_2$.

Because the total energy of a channelled particle is much larger than the quantised energy of its transverse motion, the square roots in relation (4) can be expanded into a Taylor series. After the expansion we obtain

$$(\omega_2 - \omega_1)\beta + (\omega_1\cos\vartheta_0 - \omega_2\cos\vartheta)$$
$$-\beta[\varepsilon_n(E_1) - \varepsilon_m(E_2)] = 0, \qquad (5)$$

where $\beta = V/c$.

Expression (5) can be significantly simplified in the case of 'forward' Raman scattering, when $\cos\vartheta_0 = 1$ (i.e. $\vartheta_0 = 0$) and $\cos\vartheta = 1$ (i.e. $\vartheta = 0$):

$$(\omega_2 - \omega_1)(1 - \beta) = \beta(\varepsilon_m - \varepsilon_n). \qquad (6)$$

For the ultra-relativistic case, relation (6) appears as

$$\omega_2 - \omega_1 = \frac{\beta}{1-\beta}(\varepsilon_m - \varepsilon_n) = 2\gamma^2(\varepsilon_m - \varepsilon_n). \qquad (7)$$

If $\omega_2 = 2\omega_1$, we may expect the appearance of the second-harmonic generation under the condition

$$\omega_1 = 2\gamma^2(\varepsilon_m - \varepsilon_n). \qquad (8)$$

Suppose that the channelled positron can undergo transitions between the discrete levels of the transverse motion. Then, the frequency shift and its dependence on the channelled particle energy in the simplest cases of the well shape approximation can be found explicitly. For simplicity, assume that the continuous potential of the plane represents the Kronig–Penney model of a rectangular well. Consequently,

$$\varepsilon_n = \frac{\pi^2}{2m_{\rm p}d^2}n^2, \qquad (9)$$

where $n = 1, 2, 3, \ldots$; and d is the distance between the planes.

The frequency shift in the transitions between the adjacent levels with the fixed values (n, m) is defined by

$$\Delta\omega = 2\gamma^2(\varepsilon_m - \varepsilon_n) = \frac{\pi^2}{m_{\rm p}^3 d^2}(m_{\rm p}^2 - n^2)E^2. \qquad (10)$$

Thus, the frequency shift increases quadratically with increasing energy of the channelled particle, and for $\varepsilon_m - \varepsilon_n \ll m_{\rm p}$ the condition $\Delta\omega \ll E$ is always fulfilled.

In the transitions between the adjacent levels ($m = n + 1$)

$$\Delta\omega = \frac{\pi^2}{m_{\rm p}^3 d^2}(2n+1)E^2. \qquad (11)$$

The estimate of the anti-Stokes component ($n = 1$) gives $\Delta\omega \sim 5\gamma^2$ (in eV).

3. Differential cross section of Raman scattering of a photon by a channelled bound positron

Assume that a positron with the momentum $\boldsymbol{p}_1$ and energy E_1 falls onto a single crystal. As a result of interaction of the photon with the channelled positron, a photon with the momentum $\boldsymbol{k}_2$ and energy ω_2 is emitted, while the momentum and energy of the positron in the final state take the values $\boldsymbol{p}_2$ and E_2.

In a coordinate system, in which the initial longitudinal momentum of the positron is zero, the positron can be considered as a one-dimensional quantised (in the case of axial channelling – as a two-dimensional) object [2, 3]. In this coordinate system we are dealing with a nonrelativistic object, for which the amplitude (cross section) of scattering of the photon by a bound particle is well known [4, 5].

In this regard, it is sufficient to transform the amplitude (cross section) of scattering into the laboratory coordinate system by means of the simple rules {see, for example [6] (§3, pp 88–90)}, taking herewith into account that the quantum object associated with the channelled positron possesses a one-dimensional (planar channelling) or two-dimensional (axial channelling) momentum.

For example, the amplitude of forward elastic scattering of a photon on a channelled particle is

$$f(\omega) = \frac{\sqrt{1-\beta^2}}{|1 - \beta\cos\theta|}F(\omega'), \qquad (12)$$

where $F(\omega')$ is the scattering amplitude in the rest system (in the CCS) of the channelled particle and $\omega' = (1 - \beta\cos\theta)\gamma\omega$ is the photon frequency in the CCS.

Consider scattering of the photon with the momentum $\boldsymbol{k}_1$, energy ω_1 and polarisation $\boldsymbol{e}_1$ by a positron having the energy ε_{1n} and situated in the state with the wave function $\psi_1^{(+)}(x) = u_{1n}(x)\exp(-{\rm i}\varepsilon_{1n}t)$. As a result of the scattering, a photon with the momentum $\boldsymbol{k}_2$, energy ω_2 and polarisation $\boldsymbol{e}_2$ arises, and the positron passes over into the state with the energy ε_{2m} and the wave function $\psi_2^{(+)}(x) = u_{2m}(x)\exp(-{\rm i}\varepsilon_{2m}t)$.

Using the standard technique of Feynman diagrams (see [4], § 59), consider the nonrelativistic case, when the photon energies are small compared to the rest energy of the positron, $\omega_1 \ll m_{\rm p}$, $\omega_2 \ll m_{\rm p}$, and the energy values ε_{m2} and ε_{n1} of transverse motion of the channelled positron differ little from m_p:

$$|\varepsilon_{n1} - m_p| \ll m_p, \quad |\varepsilon_{m2} - m_p| \ll m_p. \tag{13}$$

These assumptions allow significant simplification of the expression for the matrix element of the transition:

$$W = -2\pi\alpha \exp[i(k_1 - k_2)x]\sqrt{\omega_1\omega_2} \times \sum_s \left(\frac{\langle 2|\boldsymbol{x}\boldsymbol{e}_2^*|s\rangle\langle s|\boldsymbol{x}\boldsymbol{e}_1|1\rangle}{\varepsilon_1 - \varepsilon_s + \omega_1} + \frac{\langle 2|\boldsymbol{x}\boldsymbol{e}_1|s\rangle\langle s|\boldsymbol{x}\boldsymbol{e}_2^*|1\rangle}{\varepsilon_1 - \varepsilon_s - \omega_2} \right), \tag{14}$$

where α is the fine structure constant.

The differential cross section of Raman scattering is related to the matrix element W by the expression

$$d\sigma = 2\pi\,|W|^2\,\delta(\varepsilon_1 + \omega_1 - \varepsilon_2 - \omega_2)\frac{d^3k_2}{(2\pi)^3}. \tag{15}$$

After eliminating the δ-function by integration over $d\omega_2$, we find

$$d\sigma = \omega_1\omega_2^3 d\Omega_2 \times \left| \sum_s \left(\frac{\langle 2|\boldsymbol{Q}\boldsymbol{e}_2^*|s\rangle\langle s|\boldsymbol{Q}\boldsymbol{e}_1|1\rangle}{\varepsilon_1 - \varepsilon_s + \omega_1} + \frac{\langle 2|\boldsymbol{Q}\boldsymbol{e}_1|s\rangle\langle s|\boldsymbol{Q}\boldsymbol{e}_2^*|1\rangle}{\varepsilon_1 - \varepsilon_s - \omega_2} \right) \right|^2, \tag{16}$$

where $\boldsymbol{Q}$ is the dipole moment of the undulator and Ω_2 is the solid angle of photon scattering.

4. Resonant scattering of a photon by a channelled positron

The expression for the matrix element (14) contains a sum over all excited states of the undulator in the CCS. If the photon energy ω_1 is equal to the energy difference between one of the excited states and the ground state of the undulator, i.e. $\omega_1 = \varepsilon_s - \varepsilon_1$, the scattering cross section tends to infinity, which indicates the inapplicability of the obtained expression at $\omega_1 = \varepsilon_s - \varepsilon_1$. This case corresponds to the resonance. The reason for the inapplicability of formula (14) in the vicinity of the resonance is that we have considered $\psi_s^{(+)}(x)$ as the wave functions of the stationary states, containing the time as $\exp(-i\varepsilon_{1s}t)$.

Meanwhile, the band nature of the transverse motion energy should be taken into account, along with the fact that the excited states are approximately stationary. Such states can be described as the states with complex energy, herewith the wave functions would contain the time as $\exp[-i(\varepsilon_s - i\Gamma_s/2)t]$, where Γ_s is a real positive value (the level width). Consequently, at the frequencies close to the resonance, we can drop all the terms except the resonance ones in expression (14), and replace ε_s by $\varepsilon_s - i\Gamma_s/2$ in this expression. Thus, we obtain the expression for the scattering amplitude:

$$W = 2\pi\alpha\sqrt{\omega_1\omega_2}\sum_s \left(\frac{\langle 2|\boldsymbol{x}\boldsymbol{e}_2^*|s\rangle\langle s|\boldsymbol{x}\boldsymbol{e}_1|1\rangle}{\varepsilon_s - \varepsilon_1 - \omega_1 - i\Gamma_s/2} \right), \tag{17}$$

where the summation is spread over all states with the energy ε_s. Accordingly, the differential scattering cross section can be represented in the form

$$d\sigma = \omega_1\omega_2^3 d\Omega_2 \left| \sum_s \left[\frac{\langle 2|\boldsymbol{Q}\boldsymbol{e}_2^*|s\rangle\langle s|\boldsymbol{Q}\boldsymbol{e}_1|1\rangle}{(\varepsilon_s - \varepsilon_1 - \omega_1)^2 + \Gamma_s^2/4} \right] \right|^2. \tag{18}$$

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