

PACS numbers: 42.50.Md; 89.70.–a
DOI: 10.1070/QE2015v045n07ABEH015389

Two-frequency information recording in a three-level system using stimulated photon echo

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Abstract. **The process of recording and reproducing information in a three-level system using stimulated photon echo is studied as a function of the amount of information embedded in the first and second two-frequency object laser pulses. It is shown that two-frequency information recording leads to an increase in the power of the stimulated photon echo response on one frequency transition and to its reduction on the other.**

Keywords: *stimulated photon echo, three-level system, two-frequency information recording, information encoding, amount of information.*

1. Introduction

Photon echo can serve as a way of storing, converting and reproducing a spatiotemporal structure of excitation pulses, called echo-holography. The formation of echo holograms is significantly affected by random and relaxation processes, degeneration of resonance levels, as well as external spatially nonuniform electric fields. This enables the possibility of converting the spatiotemporal structure of echo hologram responses, which can be used in real-time data processing systems [1–3].

If a resonant medium consists of multi-level optical centres, which interact with a sequence of laser pulses having different frequencies, these centres can behave as multilevel quantum gates performing logical operations. Along with logical operations, it is possible to change the real-time scale and the sequence of events, information about which is embedded in the spatiotemporal structure of the object pulse [4, 5]. Thus, in recording an echo hologram, one more dimension, i.e., the frequency one (colour echo holography [6]), is added.

A colour echo hologram can be recorded at Pr^{3+} levels in a LaF_3 matrix, where long-lived photon echo was detected on $^3H_4 - {}^3P_0$ ($\lambda = 4777$ Å) and $^3H_4 - {}^1D_2$ ($\lambda = 5925$ Å) transitions [7, 8]. Thus, a multi-frequency excitation character, i.e., multichannelling (due to different frequencies) of information recording and storing, can be implemented.

It should be noted that in the case of two-frequency excitation of a three-level system, two modes of formation of stimulated-photon-echo (SFE) responses are possible. The first mode is implemented in the presence of inhomogeneous broadening correlation on different transitions in a three-level system, whereas the second mode is realised in the absence of such a correlation. In the first case, the recording information channels are coupled, and, in the second case, they are independent [9].

If information is embedded into an object dual-frequency pulse by means of its encoding in the temporal shape of the pulse, the amount of information to be recorded depends on the spectrum of the object pulse and the width of inhomogeneously broadened lines of resonance transitions. Therefore, there must be a dependence of the SPE response power on the amount of recorded information.

In this paper we study the formation of SPE as a function of the amount of information encoded in the temporal shape of the two-frequency object laser pulse.

2. Basic equations

Consider the scheme of SPE excitation in a three-level system shown in Figure 1, where the object pulse is the first one.

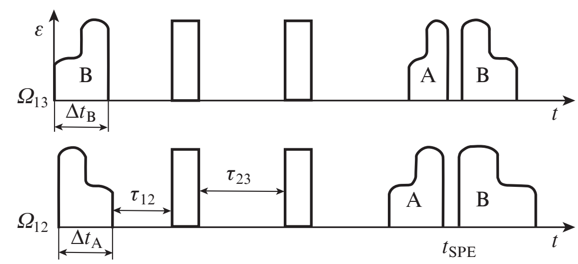

Figure 1. Scheme of SPE excitation in the case of two-frequency information recording in a three-level system [$\varepsilon(t)$ is the envelope of the electric field intensity of laser pulses].

In the first excitation mode, information contained in two-frequency object pulses A and B is reproduced on each frequency transition. In the second case, only information embedded in pulse A will be reproduced on the 1–2 transition, whereas information contained in pulse B will be reproduced on the 1–3 transition.

We consider the first excitation mode in the presence of inhomogeneous broadening correlation on transitions 1–2 and 1–3. Let us find the evolution operator of a three-level system upon its resonant excitation by laser pulses of duration Δt_η, simultaneously acting and having different carrier frequencies.

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Received 23 January 2014; revision received 12 November 2014
Kvantovaya Elektronika **45** (7) 668–671 (2015)
Translated by I.A. Ulitkin

Neglecting relaxation processes during the laser pulse action, we write for the wave function the Schrödinger equation

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t} = (H_0 + V^{(\eta)}(\boldsymbol{r},t))\psi,$$

where H_0 is the initial Hamiltonian of the system, and $V^{(\eta)}(\boldsymbol{r},t)$ is the interaction operator of the ηth laser pulse with a quantum system. Using the rotating coordinate system with the transformation

$$\tilde{\psi} = \exp(\mathrm{i}At)\psi,$$

where A is the transition operator, we obtain

$$\frac{\partial\tilde{\psi}}{\partial t} = -\frac{\mathrm{i}}{\hbar}B_\eta(t)\tilde{\psi},$$

where $B_\eta(t) = \tilde{H}_0 - \hbar A + \tilde{V}(t)$; $\tilde{H}_0(t) = \exp(\mathrm{i}At)H_0\exp(-\mathrm{i}At)$; and $\tilde{V}^{(\eta)} = \exp(\mathrm{i}At)V_\eta\exp(-\mathrm{i}At)$.

Let us represent $\tilde{\psi}$ in the form $\tilde{\psi}(t) = U(t)\tilde{\psi}(0)$, where the evolution operator $U(t)$ satisfies the equation

$$\frac{\mathrm{d}U}{\mathrm{d}t} = -\frac{\mathrm{i}}{\hbar}B_\eta(t)\,U = -\frac{\mathrm{i}}{\hbar}(B'_\eta + B''_\eta)\,U. \quad (1)$$

Here $B'_\eta = \tilde{H}_0 - \hbar A$; $B''_\eta = \tilde{V}_\eta(t)$; $\tilde{H}_0 = \exp(\mathrm{i}At)H_0\exp(-\mathrm{i}At)$; $\tilde{V}_\eta = \exp(\mathrm{i}At)V_\eta\exp(-\mathrm{i}At)$; $U(0) = I$ is the initial condition; and I is the identity matrix. The solution to equation (1) has the form

$$U = U_1(B''_\eta)\,U_2(B''_\eta). \quad (2)$$

Substituting (2) into (1) we obtain

$$\frac{\mathrm{d}U_1}{\mathrm{d}t} = -\frac{\mathrm{i}}{\hbar}B'_\eta U_1, \quad (3)$$

$$\frac{\mathrm{d}U_2}{\mathrm{d}t} = -\frac{\mathrm{i}}{\hbar}U_1^{-1}B''_\eta\,U_1U_2 = QU_2\,, \quad (4)$$

where

$$Q = -\frac{\mathrm{i}}{\hbar}U_1^{-1}B''_\eta\,U_1;$$

$$B''_\eta = P_{12}\mu_\eta + P_{13}\varphi_\eta + P_{21}\mu^*_\eta + P_{31}\varphi^*_\eta;$$

$$B'_\eta = P_{22}\Delta_{12} + p_{33}\Delta_{13};$$

$$\mu_\eta = -\,d_{12}\frac{1}{2}\varepsilon_{12}^{(\eta)}(t)\exp(-\mathrm{i}\boldsymbol{k}_{12}^{(\eta)}\boldsymbol{r}) = A_{12}^{(\eta)}S_{12}^{(\eta)};$$

$$S_{12}^{(\eta)}(t) = \varepsilon_{12}^{(\eta)}(t)\exp(-\mathrm{i}\boldsymbol{k}_{12}^{(\eta)}\boldsymbol{r});$$

$$S_{13}^{(\eta)}(t) = \varepsilon_{13}^{(\eta)}(t)\exp(-\mathrm{i}\boldsymbol{k}_{13}^{(\eta)}\boldsymbol{r});$$

$$\varphi_\eta = -\,d_{13}\frac{1}{2}\varepsilon_{13}^{(\eta)}(t)\exp(-\mathrm{i}\boldsymbol{k}_{13}^{(\eta)}\boldsymbol{r}) = A_{13}^{(\eta)}S_{13}^{(\eta)};$$

$\Delta_{13} = \Omega_{13} - \omega_{13}$; and

$$A_{1i}^{(\eta)} = d_{1i}\frac{1}{2}\,(i = 2,3).$$

The solution to equation (3) at an instant of time t_η after the action of a laser pulse of duration Δt_η will have the form

$$U_1 = \exp\left(-\frac{\mathrm{i}}{\hbar}B'_\eta\,\Delta t_\eta\right),$$

and the solution to (4) can be formally written as

$$U_2 = T\exp\left(-\frac{\mathrm{i}}{\hbar}\int_{t_\eta-\Delta t_\eta/2}^{t_\eta+\Delta t_\eta/2}U_1^{-1}B''_\eta\,U_1\mathrm{d}t\right), \quad (5)$$

where T is the Dyson chronological operator. Using a chronological ordering procedure, we obtain [10]

$$U_2 = \sum_{n=0}^{\infty}\frac{1}{n!}\int_{t_\eta-\Delta t_\eta/2}^{t_\eta+\Delta t_\eta/2}\mathrm{d}t_1\int_{t_\eta-\Delta t_\eta/2}^{t_\eta+\Delta t_\eta/2}\mathrm{d}t_2\ldots\int_{t_\eta-\Delta t_\eta/2}^{t_\eta+\Delta t_\eta/2}\mathrm{d}t_n$$
$$\times TQ(t_1)Q(t_2)\ldots Q(t_n) = I + \int_{t_\eta-\Delta t_\eta/2}^{t_\eta+\Delta t_\eta/2}Q(t_1)\mathrm{d}t_1$$
$$+\frac{1}{2!}\int_{t_\eta-\Delta t_\eta/2}^{t_\eta+\Delta t_\eta/2}Q(t_1)\mathrm{d}t_1\int_{t_\eta-\Delta t_\eta/2}^{t_\eta+\Delta t_\eta/2}Q(t_2)\mathrm{d}t_2 + \ldots. \quad (6)$$

In some cases it is possible to sum up this series and get an approximate solution to the problem. Knowing the evolution operator, we can determine the density matrix after the action of the ηth laser pulse:

$$\rho(t_\eta + \Delta t_\eta) = U(\Delta t_\eta)\rho(t_\eta)\,U^+(\Delta t_\eta). \quad (7)$$

In the case under study, the Hamiltonian of the system can be represented as:

$$\tilde{H}_0 = \hbar\Delta_{12}\begin{pmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & \Gamma\end{pmatrix},$$

where $\Gamma = \Omega_{13}/\Omega_{12}$ is the equidistant parameter of the spectrum of the system; Ω_{ij} is the frequency of the i–j transition; $\Delta_{12} = \Omega_{12} - \omega_{12}$; and ω_{12} is the frequency of the laser radiation resonant to the 1–2 transition.

The matrix operator of interaction with the ηth laser pulse can be written as

$$V^{(\eta)} = P_{12}V_{12}^{(\eta)}\exp(-\mathrm{i}\omega_{12}t) + P_{21}V_{21}^{(\eta)}\exp(\mathrm{i}\omega_{12}t)$$
$$+ P_{13}V_{13}^{(\eta)}\exp(-\mathrm{i}\omega_{13}t) + P_{31}V_{31}^{(\eta)}\exp(\mathrm{i}\omega_{13}t), \quad (8)$$

where

$$V_{ij}^{(\eta)} = -\frac{1}{2}d_{ij}\varepsilon_{ij}^{(\eta)}(t)\exp(\mathrm{i}\omega_{ij}t - \mathrm{i}\boldsymbol{k}_{ij}^{(\eta)}\boldsymbol{r});$$

$\boldsymbol{r}$ is the radius vector of the optical centre; d_{ij} is dipole moment of the i–j transition; $\boldsymbol{k}_{ij}^{(\eta)}$ is the wave vector; and $\varepsilon_{ij}^{(\eta)}(t)$ is the envelope of the electric field intensity of the ηth laser pulse.

The electric field intensity of the response is

$$\boldsymbol{E}(\boldsymbol{r},t') = \frac{1}{\hbar^3c^2R_0}\sum_j\int\langle\ddot{\boldsymbol{d}}_j\rangle\times\boldsymbol{n}\times\boldsymbol{n}g(\Delta_{12})\mathrm{d}\Delta_{12}, \quad (9)$$

where $\boldsymbol{n}$ is the unit vector in the direction of observation; $g(\Delta_{12})$ is the frequency distribution function of an inhomogeneously broadened line of the resonance transition;

$t' = t - \boldsymbol{R}_0\boldsymbol{n}/c + \boldsymbol{r}_j\boldsymbol{n}/c$; $\boldsymbol{R}_0$ is the radius vector of the observation point; $\boldsymbol{r}_j$ is the radius vector of the jth optical centre;

$$\langle \boldsymbol{d}_j(t') \rangle = \mathrm{Sp}(\rho \boldsymbol{d}_j(t')) = \boldsymbol{d}_{21}\rho_{12}^{(3)} + \boldsymbol{d}_{31}\rho_{13}^{(3)} + \text{c.c.};$$

and the matrix elements of the density matrix after exposure to three exciting laser pulses are obtained from (7) with (5) and (6) taken into account.

The spatiotemporal structure of the SPE response on the 1–2 transition is expressed as

$$E \approx \int_V \int_{-\infty}^{+\infty} g(\Delta_{12})\,\mathrm{d}\Delta_{12} \sin\theta_1 \sin\theta_2 \sin\theta_3$$
$$\times \Big\{ \tilde{S}_{13}^{(1)*}(\Gamma\Delta_{12}) \tilde{S}_{13}^{(2)}(\Gamma\Delta_{12}) \tilde{S}_{12}^{(3)}(\Delta_{12})$$
$$\times \exp\{\mathrm{i}(\Delta_{12})[(t - \tau_{12} - \tau_{23}) - \Gamma\tau_{12}]\} + \tilde{S}_{12}^{(1)*}(\Delta_{12}) \quad (10)$$
$$\times \tilde{S}_{12}^{(2)}(\Delta_{12}) \tilde{S}_{12}^{(3)}(\Delta_{12}) \exp\{\mathrm{i}(\Delta_{12})[(t - \tau_{12} - \tau_{23}) - (\Delta_{12})\tau_{12}]\} \Big\} \mathrm{d}V,$$

where θ is the area of the ηth pulse; $S^{(\eta)}(t) = \varepsilon_\eta(t)\exp(-\mathrm{i}\boldsymbol{k}_\eta \boldsymbol{r})$; $\boldsymbol{k}_\eta$ is the wave vector of the ηth laser pulse; and

$$\tilde{S}^{(\eta)}(\Delta) = \int_{t_\eta - \Delta t_\eta/2}^{t_\eta + \Delta t_\eta/2} S^{(\eta)}(t)\exp(-\,\mathrm{i}\Delta t)$$

is the spectrum of the envelope of the ηth pulse.

A similar expression for the spatiotemporal structure of the SPE response on the 1–3 transition reads as:

$$E \approx \int_V \int_{-\infty}^{+\infty} g(\Delta_{12})\,\mathrm{d}\Delta_{12} \sin\theta_1 \sin\theta_2 \sin\theta_3$$
$$\times \tilde{S}_{13}^{(3)}(\Gamma\Delta_{12}) \Big\{ \tilde{S}_{12}^{(1)*}(\Delta_{12}) \tilde{S}_{12}^{(2)}(\Delta_{12})$$
$$\times \exp\Big\{\mathrm{i}(\Delta_{12})\Big[(t - \tau_{12} - \tau_{23})\Gamma - \frac{\tau_{12}}{\Gamma}\Big]\Big\} + \tilde{S}_{13}^{(1)*}(\Delta_{12}\Gamma)$$
$$\times \tilde{S}_{13}^{(2)}(\Delta_{12}\Gamma) \exp\{\mathrm{i}\Gamma(\Delta_{12})[(t - \tau_{12} - \tau_{23}) - (\tau_{12}]\} \Big\} \mathrm{d}V. \quad (11)$$

The intensity of the SPE response is given by the expression

$$I \sim EE^*, \quad (12)$$

and the response power is defined as $W = |E^2|/\Delta t$, where Δt is the response duration.

3. Investigation of the SPE response power in a three-level system as a function of the amount of recorded information

Consider the reproducibility of information on transitions 1–2 and 1–3 in a three-level system upon two-frequency excitation (see Fig. 1). To describe the amount of recorded information encoded in the temporal shape of an object laser pulse, use is made of differential information entropy of the Fourier spectrum of the object laser pulse, because in a resonant medium information is transferred in q bits, which are distributed in the region of an inhomogeneously broadened line of the resonance transition [11].

We write the Fourier components of the electric field intensity of the ηth object laser pulse with a duration of Δt_η in the form

$$E(\omega') = \int_{t_\eta}^{t_\eta + \Delta t_\eta} \varepsilon_\eta(t)\exp(-\mathrm{i}\omega' t)\,\mathrm{d}t, \quad (13)$$

where ω' is the frequency of the Fourier spectrum, and $\varepsilon_\eta(t)$ is the temporal shape of the ηth object laser pulse.

The amplitude of the Fourier components of the electric field of the object pulse is

$$A(\omega') = |E(\omega')| = \sqrt{\mathrm{Re}(E(\omega'))^2 + \mathrm{Im}(E(\omega'))^2}, \quad (14)$$

and differential information entropy has the form

$$J_\mathrm{c}' = J_\mathrm{c} - J_{\mathrm{c}0},$$

where

$$J_\mathrm{c} = -\int_{-\infty}^{\infty} p(\omega')\log_2 p(\omega')\,\mathrm{d}\omega'; \quad (15)$$

$$p(\omega') = \frac{A(\omega')}{\int_{-\infty}^{\infty} A(\omega')\,\mathrm{d}\omega'}; \quad (16)$$

and $J_{\mathrm{c}0}$ is determined similarly to expression (15) for the object pulse envelope $\varepsilon_\eta(t)$ of rectangular shape.

Figures 2–4 show the results of a numerical calculation of the SPE intensity at different encodings of information in the temporal shape of the object laser pulse in a $LaF_3:Pr^{3+}$ crystal on ${}^3H_4 - {}^3P_0$ and ${}^3H_4 - {}^1D_2$ transitions and at different excitation scenarios. The duration of object pulses was set equal to 1.5 ns.

Figure 2 shows the result of a numerical calculation of expressions (13)–(15) in the case of information encoding in the temporal shape of a two-frequency object laser pulse. Figure 2a shows the temporal shape of the SPE response on the 1–2 transition, and Fig. 2b – on the 1–3 transition in a three-level system with two-frequency information recording.

It follows from Fig. 2a that on the 1–2 transition the real-time scale tends to increase for pulse B, and on the 1–3 transition it decreases for pulse A (Fig. 2b).

Figure 3 shows the SPE response power in two-frequency information recording in a three-level system as a function of the amount of information in the first and second object laser pulses on the 1–3 transition.

Similar dependences for the response on the 1–2 transition are shown in Fig. 4.

Figures 3 and 4 show that in the case of two-frequency information recording in a three-level system the SPE response power depends on the amount of recorded information, the form of this dependence being different on different transitions. This is due to a different nonuniform width of the resonance transitions, different frequency of transitions and different forms of the Fourier spectrum of the two-frequency object laser pulse.

4. Conclusions

1. It is shown that in the case of two-frequency information recording in a three-level system, one can observe a change in the duration of stimulated photon echo responses.

2. In the case of two-frequency information recording in a three-level system, the power of the SPE response depends on the amount of information recorded.

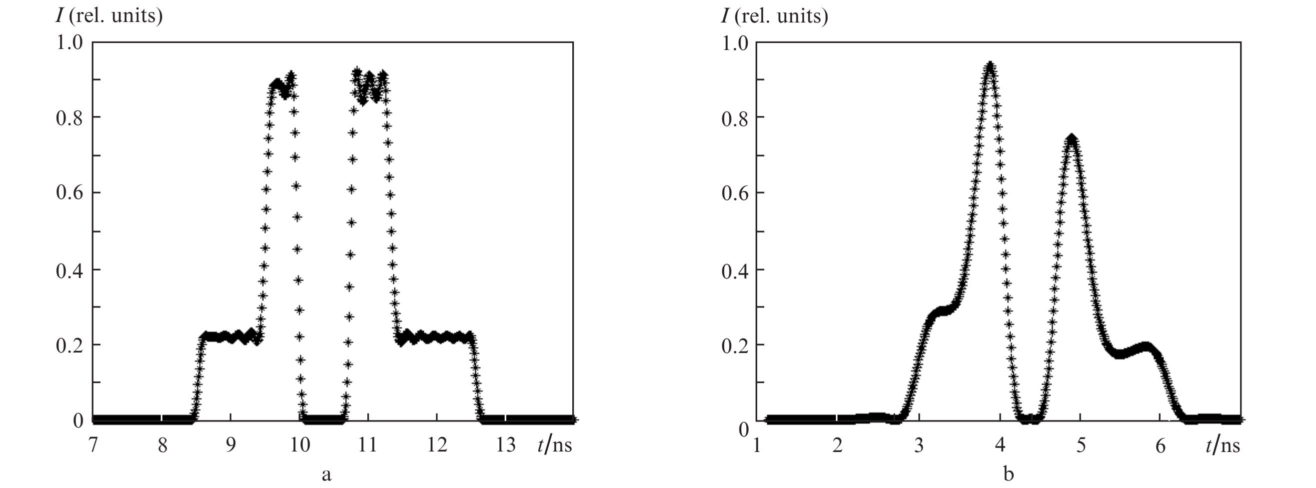

Figure 2. Temporary shape of the SPE response at $\Gamma = 1.26$ on the transitions (a) 1–2 and (b) 1–3 in the case of two-frequency information recording in a three-level system (the sample size, $L = 0.01$ m; the width of the inhomogeneously broadened line, $\sigma = 0.5$ cm^{-1}; θ_A, $\theta_B \ll \pi$).

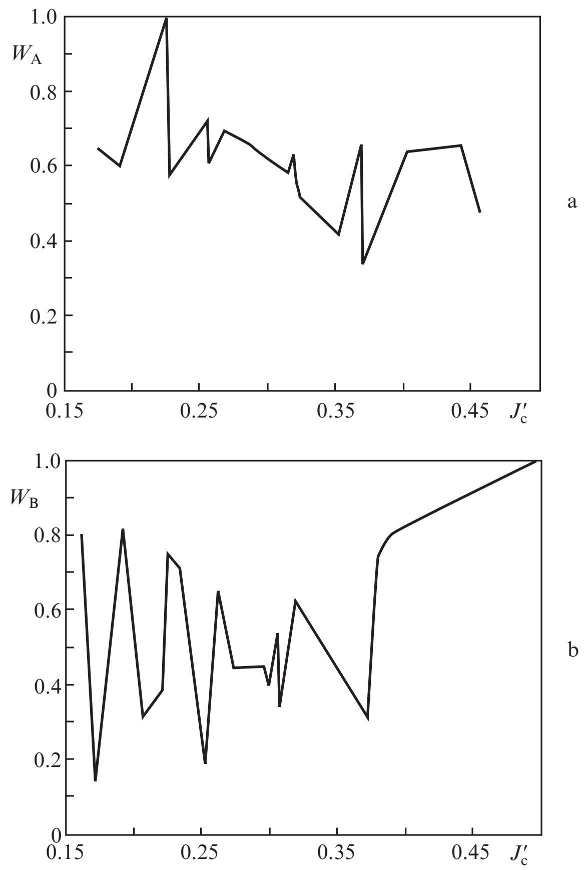

Figure 3. SPE response power in the case of two-frequency information recording in a three-level system on the 1–3 transition as a function of the amount of recorded information for object laser pulses (a) A and (b) B.

3. On different resonance transitions in a three-level system, the dependence of the SPE response power on the amount of information recorded has a different shape.

Acknowledgements. The work was supported by a grant allocated to the Kazan Federal University to perform State Tasks in the field of scientific research.

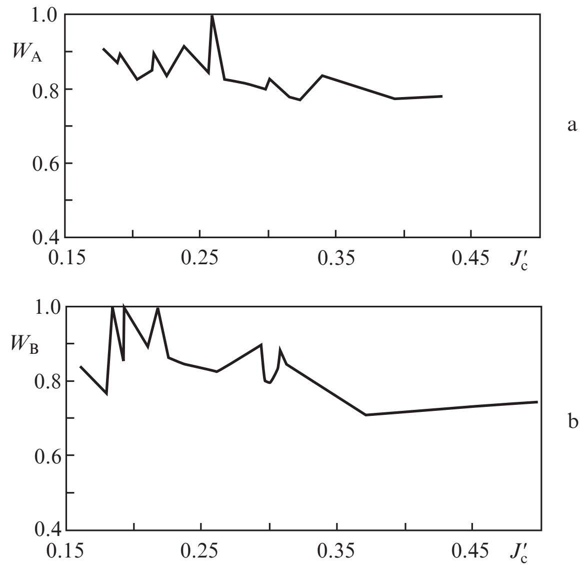

Figure 4. SPE response power in the case of two-frequency information recording in a three-level system on the 1–2 transition as a function of the amount of recorded information for object laser pulses (a) A and (b) B.

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