

NONLINEAR OPTICAL PHENOMENA

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Parametric generation of broadband biphotons in a periodic sequence of thin crystals

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Abstract. **Collinear parametric scattering of light under type-II phase matching is considered in a periodic sequence of nonlinear cells made of BBO crystals. The possibilities of reducing the duration of the wave packets of biphotons and increasing their intensity under monochromatic pumping are studied. The analysis is based on the account of dispersion of the refractive indices using Sellmeier equations. The obtained results allow one to conclude that at an appropriate choice of the crystal thicknesses and the number of cells, it is possible to significantly increase the flux of broadband biphotons and obtain extremely short quantum packets. It is shown that for sufficiently small thicknesses of individual crystals (layers), this system exhibits a fine structure in the spectrum of biphotons: with an increase in the number of cells (layers) the broadband spectrum is divided into a number of narrow bands and the number of these bands increases. When use is made of a nonlinear BBO crystal, this method allows one to obtain ultrashort packets of biphotons with duration $\tau_0 \approx 2$ fs. Compared with a different method of generation of broadband biphotons, which is based on the use of quasi-phase-matched periodically poled crystals, the method studied in this paper does not necessitate the need for the phase modulation of a biphoton to be converted into the amplitude modulation.**

Keywords: *parametric scattering, broadband biphoton, interference of biphotons, wave packet, fine structure of the spectrum.*

1. Introduction

In quantum optics and quantum information, one needs to control the temporal structure of biphoton wave packets. In some applications it is necessary to broaden the spectrum and to reduce the packet duration. This applies, for example, to use of biphotons for measuring group delays [1], for precision clock synchronisation [2] and for enhancing the maximum resolution in quantum-optical coherence tomography [3, 4] and nonlinear microscopy [5]. Katamadze and Kulik [6] have listed and briefly described the presently proposed and studied methods for controlling quantum states of biphotons in spontaneous parametric down-conversion (SPDC) in nonlinear crystals. In this regard, we mention here only three methods which are directly related to the study conducted below. The first method is based on using a thin crystal [7, 8]. Dauler et al. [7] and Katamadze et al. [8] showed that in the case of a BBO crystal, a biphoton packet can be compressed by reducing the crystal thickness up to 0.1 mm (at smaller thicknesses the intensity of scattered radiation is small and biphotons cannot be detected). The second method of generation of broadband biphotons [9–14] is based on the use of quasi-phase-matched periodically poled crystals. A periodic nonlinear-polarised superlattice allows the quasi-phase-matching condition to be satisfied. A change in the superlattice period at the crystal length leads to a broadening of the spectrum of biphotons (chirp).

The third way to reduce the duration of biphoton packets was proposed by Klyshko [15]. The intensity of radiation (flux of biphotons) decreases with decreasing crystal thickness. One can, as first pointed out by Klyshko, use sequentially placed thin crystals (or layers within a single crystal), in which, due to the interference between biphotons generated in some crystals, the intensity of scattered radiation increases, whereas the duration of biphoton packets does not increase and is equal to the duration of a packet generated in a single crystal. This method opens, in principle, ample opportunities for increasing the intensity of broadband biphotons.

Klyshko [15] and Belinsky and Klyshko [16] theoretically studied SPDC in sequentially placed nonlinear crystals and derived general formulas that allow one to study the properties of biphotons packets with arbitrary dispersion of the refractive index of the medium. However, a specific analysis was performed only for wave packets with a sufficiently long duration τ_0 when we can confine ourselves to terms of a first-order Taylor series expansion of the dispersion relations for the refractive indices of the spectral SPDC components.

The aim of this paper is to study the ultimate possibilities of reducing the duration of wave packets of biphotons and increasing their intensity, which open up when use is made of the method of generation of broadband biphotons, proposed by Klyshko. This problem cannot be solved by using the first order dispersion only and requires the use of exact dispersion relations. The analysis performed in this paper is based on the account of dispersion of the refractive indices using Sellmeier equations. We consider a specific structure on the basis of BBO crystals with type-II phase matching.

2. SPDC in a periodic set of crystals (qualitative examination)

Characteristic features of SPDC in the system in question have been studied in Refs [15, 16]. This section provides a qualitative description of generation of biphotons in a set of crystals, which allows the reader to understand the basic results relating to the case of biphotons packets of sufficiently

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long duration. Collinear scattering under type-II phase matching is analysed. We consider a special periodic structure of a nonlinear medium consisting of an arbitrary number of identical cells. Each cell contains two crystals. For simplicity, we assume that all the crystals in the set are uniaxial.

The pump field is considered classical and the change of its amplitude in SPDC is neglected. Inside the crystal the pump field can be represented as a plane monochromatic wave

$$E_p(z,t) = E_0 \exp[i(k_p z - \omega_p t)]. \quad (1)$$

The z axis is parallel to the wave vector of the pump and perpendicular to the boundaries of the crystals in the cell. The boundaries of the first crystal have coordinates $z = 0$ and $z = l_c$, where l_c is the crystal thickness. The nonlinear medium is assumed infinite in the transverse direction. The pump wave is linearly polarised, the polarisation vector being directed along the x axis.

Figure 1 shows the structure of a single cell. Both crystals in the cell have the same thickness l_c. The optical axis of the first crystal lies in the yz plane, and of the second – in the xz plane (the orientation of the second crystal axis is rotated through 90° around the z axis). The angles between the optical axes and the z axis are the same in all the crystals. This configuration of the cell was proposed in [15].

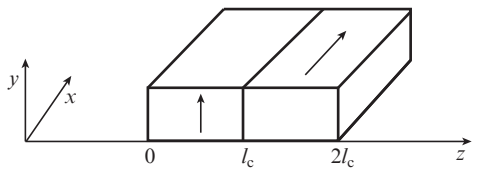

Figure 1. Cell structure. Arrows show the optical axes of the crystal.

In the case of type-II phase matching, biphoton packets are registered using the scheme shown in Fig. 2. At the exit from the crystal, the signal and idler waves have orthogonal polarisations. After passing through a half-wave plate, these polarisations are rotated through an angle of ±45° relative to the axis of the polarising beam splitter PBS. The combination of these two elements is equivalent to the beam splitter transmittance $T = 50\%$ for each photon, and so the operators $E_1^+(t)$ and $E_2^+(t)$ of the fields incident on the detectors D1 and D2 are related to the field operators of the ordinary and extraordinary waves, $E_o^+(t)$ and $E_e^+(t)$, at the output end of the crystal by the expressions:

$$E_{1,2}^+(t_{1,2}) = \frac{1}{\sqrt{2}}[E_o^+(t_{1,2}) \pm E_e^+(t_{1,2} + \tau)], \quad (2)$$

where t_1 and t_2 are the time moments of registration of the fields by detectors D1 and D2, respectively; and τ is the delay between the ordinary and extraordinary waves, which in the experiment is typically controlled by varying the number of quartz plates QP on the path of light beams.

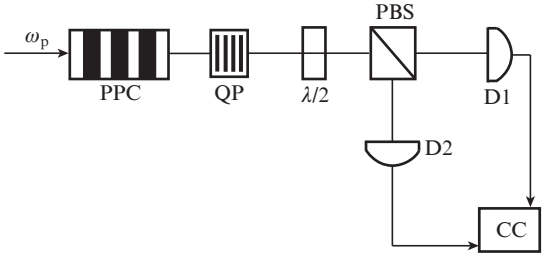

Figure 2. Schematic of registration of SPDC with type-II phase matching: (PPC) periodically placed nonlinear crystals; (QP) quartz plate; ($\lambda/2$) half-wave plate; (PBS) polarising beam splitter; (D1, D2) photodetectors; (CC) coincidence circuit.

The average rate of coincidences of photocounts $\langle R_c \rangle$ on detectors D1 and D2 is expressed by the second-order correlation function for the intensities $I_1(t_1) = E_1^-(t_1)\, E_1^+(t_1)$ and $I_2(t_2) = E_2^-(t_2)E_2^+(t_2)$ measured by detectors D1 and D2:

$$\langle R_c \rangle = \int_0^\infty dt_1 \int_0^\infty dt_2 \langle E_1^-(t_1) E_2^-(t_2) E_2^+(t_2) E_1^+(t_1) \rangle. \quad (3)$$

The temporal structure of biphoton packets can be determined experimentally in the form of a so-called anti-correlation dip (a V-shaped dip in the dependence of the rate of coincidences of photocounts on the delay time τ, arising due to correlation of orthogonally polarised photons) [17, 18].

2.1. A single crystal

Consider first spontaneous parametric scattering in a single crystal with a quadratic nonlinearity. The correlation function $F(t_1, t_2) = \langle E_1^+(t_1) E_2^+(t_2) \rangle$, which is called the amplitude of a biphoton state (biphoton amplitude) defines the correlation of the fields registered at time t_1 and t_2. In the first order of perturbations with respect to the value of the parametric amplification coefficient the biphoton amplitude has the form of a rectangular wave packet and is given by the formula

$$F(t_1, t_2) = W E_0 l_c \Pi(t_1 - t_2) \exp[-i\omega_p (t_1 - t_2)/2], \quad (4)$$

where W is a constant, which includes all the coefficients and slowly varying functions depending on the pump frequency ω_p; and $\Pi(x)$ is a rectangular function defined as follows:

$$\Pi(x) = \begin{cases} 1/\tau_0 \text{ for } 0 < x < \tau_0, \\ 0 \text{ outside this interval.} \end{cases} \quad (5)$$

The packet duration is

$$\tau_0 = A l_c, \quad (6)$$

where $A = 1/u_o - 1/u_e$; and u_o and u_e are the group velocities of ordinary and extraordinary waves in a crystal at a frequency $\omega_p/2$; in formula (4) it is assumed for simplicity that collinear phase matching at frequency ω_p is degenerate.

For a rectangular biphoton wave packet, taking into account relations (1) for the fields on detectors D1 and D2, we obtain the formula for the normalised coincidence rate $R_c = \langle R_c \rangle / R_{max}$ [R_{max} is the maximum value of $\langle R_c \rangle$]:

$$R_c(\tau) = \begin{cases} 2|\tau|/\tau_0 \text{ for } |\tau| < \tau_0/2, \\ 1 \text{ for } |\tau| > \tau_0/2. \end{cases} \quad (7)$$

The dependence of the coincidence rate on the delay time τ has a triangular dip. At the centre of the dip (at $\tau = 0$), the rate R_c vanishes in the case of degenerate phase matching.

2.2. A sequence of identical cells

If instead of a single crystal use is made of a single cell shown in Fig. 1, the biphoton amplitude transforms as follows:

$$F(t_1,t_2) = WE_0 l_c \Pi(t_1 - t_2 + \tau_0)\exp[-i\omega_p(t_1+t_2)/2]. \quad (8)$$

As can be seen from (8), one cell does not change the duration and shape of the biphoton packet, $\Pi(x)$. The second crystal in the cell shifts the centre of the biphoton packet (and a triangular dip) from position $\tau_1 = 0$ to a new position $\tau_2 = -\tau_0$. The biphoton is generated in the first crystal of the cell, where type-II phase-matching condition is fulfilled. In the second crystal, the pump wave is an ordinary wave, and the phase-matching condition is not met. The second crystal induces only an additional biphoton delay, which is opposite in sign to the delay in the first crystal. Changing the sign is due to the fact that an ordinary wave from the first crystal becomes extraordinary in the second one.

We now take three crystals. In this case, biphotons are generated in two crystals (first and third) and a biphoton packet can be given in the form:

$$F(t_1,t_2) = 2WE_0 l_c \Pi(t_1 - t_2)\exp[-i\omega_p(t_1-t_2)/2]. \quad (9)$$

In accordance with (9) the packet preserves its rectangular shape $\Pi(x)$, and its duration is still equal to τ_0, as in the case of a single crystal. The biphoton amplitude is doubled. The centre of the biphoton packet (and a triangular dip) in this case is at the same position $\tau_1 = \tau_0/2$, as in the case of a single crystal.

In the general case, with N identical cells, the duration of the rectangular biphoton packet remains constant, and the biphoton amplitude increases by N times. When using N identical crystals (or a single crystal of thickness Nl_c), the biphoton amplitude also increases by N times. However, the duration of a biphoton packet increases by N times (the spectrum width is reduced by N times).

3. Allowance for exact dispersion relations in the crystals of the cell

As noted above, the general formulas taking into account the refractive index dispersion of a medium were derived in [15]. In this study it was assumed that the second crystal in the cell plays only a passive role, compensating for dispersion in the first crystal. This assumption becomes invalid for very thin crystals ($l_c \approx 1$ μm). In this case, it is necessary to take into account an additional contribution, which arises due to SPDC occurring in the second crystal of the cell. Therefore, the formulas below are derived with allowance for SPDC in the second crystal of the cell.

In the spectral representation the annihilation operators $a_s(z,t)$ for the signal wave and $a_i(z,t)$ for the idler wave inside the crystal can be represented as

$$a_j(z,t) = \int d\omega_j a_j(\omega_j,z)\exp[i\omega_j t - ik_j(\omega_j)z], \quad (10)$$

where $k_j(\omega_j) = \omega_j n_j(\omega_j)/c$ are the wave vectors of the signal (j = s) and idler (j = i) waves; and $n_j(\omega_j)$ are the refractive indices taking into account the dispersion in the crystal.

In the Heisenberg representation the equations for the operators $a_j(z,\omega_j)$ have the form [19]

$$\frac{\partial a_s(z,\omega_s)}{\partial z} = WE_0 a_i^{\dagger}(z,\omega_i)\exp(i\Delta z), \quad (11a)$$

$$\frac{\partial a_i^{\dagger}(z,\omega_i)}{\partial z} = WE_0^{*} a_s(z,\omega_s)\exp(-i\Delta z), \quad (11b)$$

where the function Δ is introduced, which depends on the detuning of the wave vectors of the interacting waves:

$$\Delta = k_s(\omega_s) + k_i(\omega_i) - k_p. \quad (12)$$

With allowance for the phase-matching condition $\omega_p = \omega_s + \omega_i$, we will represent the frequency of the signal and idler waves in the form $\omega_s = \omega_0 + \Omega$ and $\omega_i = \omega_0 - \Omega$, where $\omega_0 = \omega_p/2$. Solving equation (11) in the first order of perturbation with respect to the parametric-gain coefficient $g = WE_0 l_c$, we find the biphoton amplitude produced by the first cell:

$$F(t_1,t_2) = WE_0 l_c \exp[i(\omega_p t - k_p l_c)]\int I_1(\Omega)\exp(-i\Omega t_-)d\Omega, \quad (13)$$

where

$$I_1(\Omega) = \frac{\exp[i\Delta(\Omega)l_c]-1}{i\Delta(\Omega)l_c}\exp[i\Delta(\Omega)l_c] + \frac{\exp[i\Delta_1(\Omega)l_c]-1}{i\Delta_1(\Omega)l_c}; \quad (14)$$

$t = (t_1 + t_2)/2$; $t_- = t_1 - t_2$.

Unlike (12), the function Δ_1 is defined as follows:

$$\Delta_1 = k_s(\omega_i) + k_i(\omega_s) - k_p. \quad (15)$$

This difference occurs due to the fact that the axis of the second crystal in the cell is rotated through 90°.

In the general case, when a biphotons packet is generated by a set of N cells, we obtain the formula for the biphoton amplitude $F_N(t_1,t_2)$:

$$F_N(t_1,t_2) = WE_0 l_c \exp[i(\omega_p t - k_p l_c)] \times \int I_1(\Omega)F_N(\Omega)\exp(-i\Omega t_-)d\Omega, \quad (16)$$

where

$$F_N(\Omega) = \frac{q^N - 1}{q-1}; \quad q = \exp\{i[\Delta(\Omega) + \Delta_1(\Omega)]l_c\}; \quad (17)$$

and $I_1(\Omega)$ is defined in (14).

Equations (14)–(17) for $F_N(t_1,t_2)$ differ, apart from the notations, from those derived in [15] by expression (14) for the shape of the spectrum $I_1(\Omega)$ in the first cell. The difference is due to the fact that Klyshko [15] made an assumption that the second crystal plays a passive role in the cell. This assumption becomes invalid for very thin crystals ($l_c \approx 1$ μm). In this case, the SPDC intensity in the second crystal cell is comparable with the SPDC intensity in the first crystal. As in [15], in deriving formulas (14)–(17) we neglect the reflection of the waves from the ends of the crystals. In all the cases considered below, SPDC in the second crystal of the cell has little effect on the spectrum of biphotons; therefore, we present below the results obtained for $I_1(\Omega)$ when instead of (14), use is made of a simpler formula

$$I_1(\Omega) = \frac{\exp[i\Delta(\Omega)l_c] - 1}{i\Delta(\Omega)l_c}\exp[i\Delta(\Omega)l_c]. \quad (18)$$

To calculate the coincidence rate $\langle R_c(\tau)\rangle$ we must take into account transformation (2) of the fields of the signal and idler waves on the beam splitter. As in [19], the dependence $\langle R_c(\tau)\rangle$ can be represented as the sum of two components:

$$\langle R_c(\tau)\rangle = 2R_0 - \rho(2\tau), \quad (19)$$

where

$$2R_0 = (WE_0l_c)^2\int G_N(\Omega)\,d\Omega; \quad (20)$$

$$\rho(2\tau) = (WE_0l_c)^2\int G_N(\Omega)\exp(-i2\Omega\tau)\,d\Omega. \quad (21)$$

The spectrum of the biphoton field intensity $G_N(\Omega)$ is given by

$$G_N(\Omega) = (1 - \Omega^2/\omega_0^2)\,|F_N(\Omega)|^2\,|I_1(\Omega)|^2. \quad (22)$$

Note that the factor $1 - \Omega^2/\omega_0^2$ in (22) arises from the fact that the spectral density includes the product of photon energies $\hbar^2\omega_s\omega_i = \hbar^2\omega_0^2(1 - \Omega^2/\omega_0^2)$. Given (17), the spectrum $G_N(\Omega)$ can be written in the form

$$G_N(\Omega) = G_1(\Omega)\frac{\sin^2[\Delta_\Sigma(\Omega)N/2]}{\sin^2[\Delta_\Sigma(\Omega)/2]}, \quad (23)$$

where

$$\Delta_\Sigma(\Omega) = \Delta(\Omega) + \Delta_1(\Omega);$$
$$G_1(\Omega) = (1 - \Omega^2/\omega_0^2)\,|I_1(\Omega)|^2. \quad (24)$$

In accordance with formulas (2), (3), (19)–(21), the coincidence rate $\langle R_c(\tau)\rangle$ is determined by the second-order time-correlation function [for intensities $I_1(t_1)$ and $I_2(t_2)$ registered by detectors D1 and D2] and is expressed in terms of the square of the modulus of the biphoton amplitude $|F(t_1, t_2)|^2$. It is this correlation function that sets the temporal profile and duration of a biphoton packet. According to (19)–(21), the coincidence rate $\langle R_c(\tau)\rangle$ in the centre of the dip (at $\tau = 0$) is reduced to zero. This takes place in the case of degenerate phase matching, which is considered in this paper. Under nondegenerate phase matching, the depth of the anticorrelation dip decreases.

The formulas obtained will be used for the analysis of SPDC in a periodic sequence of thin crystals, by taking into account the exact dispersion relations for $n_j(\omega_j)$ based on the Sellmeier equations.

4. Approximation of exact dispersion relations

As will be seen below, approximation of exact dispersion relations with allowance for the second-order terms ($\propto\Omega^2$) gives results which are in good agreement with those of exact dispersion relations almost in the entire region of the spectrum. Using this approximation, we give the following approximate formulas based on the Taylor series expansion with allowance for terms up to the second order in Ω:

$$\Delta^a(\Omega) = A\Omega + B\Omega^2, \quad (25a)$$

$$\Delta_1^a(\Omega) = k_{p1} - k_p - A\Omega + B\Omega^2, \quad (25b)$$

$$\Delta_\Sigma^a(\Omega) = \Delta^a(\Omega) + \Delta_1^a(\Omega) = k_{p1} - k_p + 2B\Omega^2. \quad (25c)$$

Here, the superscript 'a' means that for the function $\Delta_j(\Omega)$ use is made of the quadratic approximation in Ω; and k_p is the wave vector of the pump field in the first crystal of the cell, and k_{p1} – in the second. The difference between k_p and k_{p1} arises from the fact that in the first crystal the pump wave is extraordinary, and in the second – ordinary.

Taking into account (23) and (25) we obtain a spectrum in the form

$$G_N(\Omega) = G_1(\omega)\frac{\sin^2[(\varphi + B\Omega^2l_c)N]}{\sin^2(\varphi + B\Omega^2l_c)}, \quad (26)$$

where $\varphi = (k_{p1} - k_p)l_c$. Analysis of formula (26) shows that the second cofactor plays a role of a frequency filter with a bandwidth

$$\delta = \sqrt{2\pi/(Bl_c)}. \quad (27)$$

At a sufficiently large number of cells,

$$N > \tau_0^2/(2\pi Bl_c), \quad (28)$$

the filter bandwidth is divided into a number of narrow bands with a width

$$\delta_N = \sqrt{2\pi/(Bl_cN)}. \quad (29)$$

5. Results of numerical studies

Dispersion of the refractive indices will be taken into account in accordance with the Sellmeier equations. As a nonlinear crystal, use is made of a BBO crystal with type-II phase-matching (e–oe). The numerical results presented below are obtained for crystals cut so that the wave vector of the pump (z axis) is directed at an angle $\theta_0 = 49°$ to the crystallographic axis; the pump wavelength (in vacuum) λ_p is assumed equal to 351 nm. This case under study corresponds to the conditions of the experiments from [7]. Detailed theoretical analysis for a single crystal was carried out in [20].

In the first crystal of the cell, the signal wave is ordinary and its refractive index is $n_s(\omega_s) = n_o(\lambda_s)$. For the idler wave, $n_i(\omega_i) = n_e(\lambda_i)$ and

$$n_i^2(\omega_i) = \frac{n_e^2(\lambda_i)}{1 - \gamma(\lambda_i)\cos^2\theta_0}, \quad (30)$$

where $\gamma(\lambda) \equiv 1 - n_e^2(\lambda)/n_o^2(\lambda)$. The values of the refractive indices for the ordinary and extraordinary waves, $n_o(\lambda)$ and $n_e(\lambda)$, are calculated using Sellmeier equations for the BBO crystal.

Let us first compare dispersion dependences calculated by using the Sellmeier equations and their approximation with allowance for the second-order terms ($B\Omega^2$). Figure 3 shows the exact dependence $\Delta(\omega)$ and its approximation $\Delta^a(\omega) = A\omega + B\omega^2$. One can see that the exact dependence and its approximation calculated using approximate formulas (25) are in good agreement with each other in almost the entire range of changes in ω (except narrow gaps near the boundaries of the transparency region).

In approximating exact dispersion relations, we used the following parameters: $A = 2.5 \times 10^{-10}$ s m^{-1} and $B = 0.82 \times$

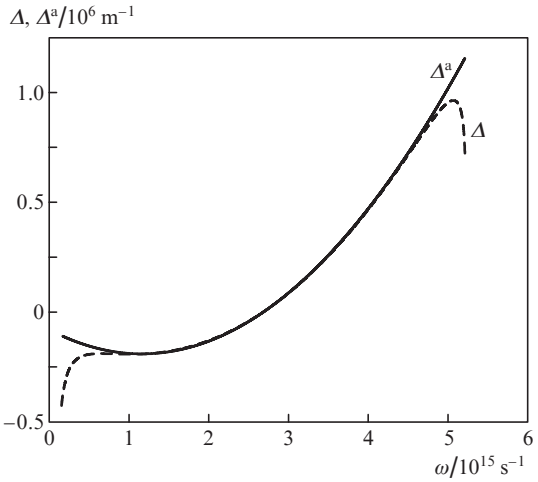

Figure 3. Dispersion dependence $\Delta(\omega)$ calculated in accordance with Sellmeier equations and its approximation $\Delta^a(\omega) = A\omega + B\omega^2$.

10^{-25} s m^{-2}. Parameter $k_{p1} - k_p$, included in formula (25) for $\Delta_1(\Omega)$ and $\Delta_\Sigma(\Omega)$, is equal to 1.3840×10^6 m^{-1}.

5.1. Possibility of increasing the flux of biphotons

Let us analyse the possibilities of the thus generated broadband biphotons in several BBO crystals with $l_c = 0.1$ mm. We present the results of calculations. Below, instead of the conventional coincidence rate $\langle R_c \rangle$ we will use the relative rate $\langle R_c(\tau) \rangle / R_{ref}$, where R_{ref} is the maximum rate of coincidences, observed in the case of a single crystal of thickness 0.1 mm. Figure 4 shows the relative rates of coincidences for a single crystal and a stack of 20 cells. The packet duration is $\tau_0 =$ 25 fs, which agrees well with the experimental data [7]. For 20 cells the coincidence rate increases by 250 times, whereas the packet duration remains the same.

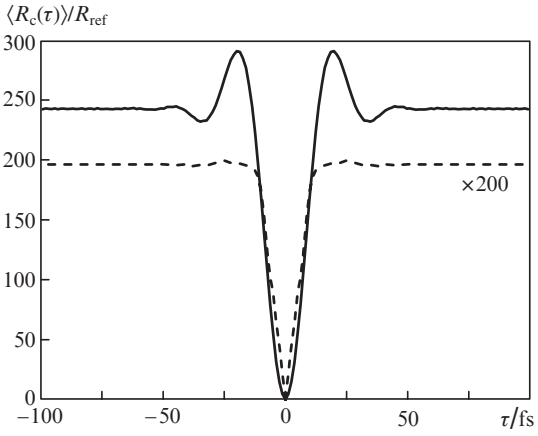

Figure 4. Relative rate of coincidences of photocounts $\langle R_c(\tau) \rangle / R_{ref}$ (form of an anticorrelation dip) at $N = 20$ (solid line) and 1 (dashed curve); $l_c = 0.1$ mm.

From our results it follows that for 0.1-mm-thick crystals it is possible to increase the flux of biphotons with increasing number N of cells in the system. In this case, the spectral width and the duration of a wave packet are the same as in the case of a single crystal.

5.2. Possibility of broadening the spectrum of biphotons

Let us analyse the possibilities of broadening the spectrum of biphotons generated in a periodic sequence containing N cells. Figure 5 shows the spectrum $G_1(\omega)$ in the case of a single cell with $l_c = 9.08$ μm, calculated by formulas (18) and (24) (dashed curve) and with allowance for SPDC in the second crystal of the cell (solid curve). The crystal thickness $l_c =$ 9.08 μm is selected to satisfy the condition $\varphi = (k_p - k_{p1})l_c = 4\pi$. One can see that taking into account SPCD in the second crystal of the cell has little effect on the spectrum of a generated biphoton. This also applies to other spectra discussed below; therefore, below we present the results obtained by using formulas (18) and (24).

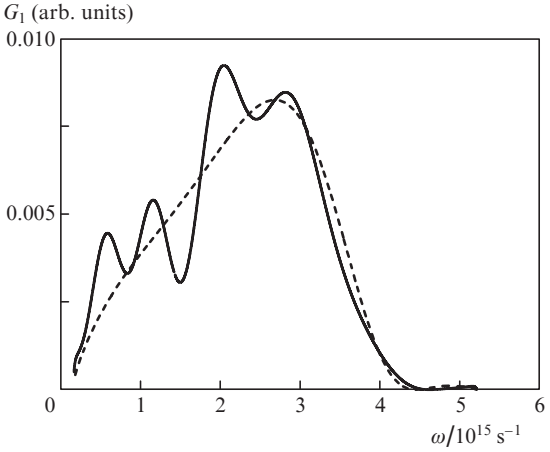

Figure 5. Spectrum of a biphoton packet $G_1(\omega) = |I_1(\omega)|^2$ in the case of a single cell at $l_c = 9.08$ μm. The dashed curve shows the $G_1(\omega)$ spectrum by neglecting SPDC in the second crystal cell.

In accordance with the results of experimental studies carried out in [7], a biphoton packet can be registered at a coincidence rate for which $\langle R_c(0) \rangle / R_{ref} \geqslant 1$. Such rates of coincidences at crystal thicknesses $l_c \leqslant 10$ μm are obtained when the number of cells is $N \geqslant 5$.

In the case of five cells with crystals having thickness $l_c =$ 9.08 μm, the spectrum $G_N(\omega)$ is shown in Fig. 6a. For comparison, Fig. 6c shows the spectrum obtained for a single 0.1-mm-thick crystal. It is seen that the use of five cells formed from 9.08-μm-thick crystals allows a four-fold broadening of the maximum width of the spectrum observed in [7] for a single 0.1-mm-thick crystal. The flux of biphotons is the same in both cases.

In using 100 cells formed from 4.505-μm-thick crystals, the biphoton spectrum $G_N(\omega)$ is shown in Fig. 6b. In this case, the flux of biphotons increases by about five times (in comparison with the flux in the case of a single crystal with $l_c = 0.1$ mm). One can see that when there are many cells, the biphoton spectrum has a fine structure and exhibits a set of narrow bands throughout its entire width; the number of bands, into which the spectrum of a broadband biphoton is divided, increases with increasing number of cells.

6. Discussion of the results

As we have already mentioned in Introduction, one of the ways to generate broadband biphotons is based on the use of

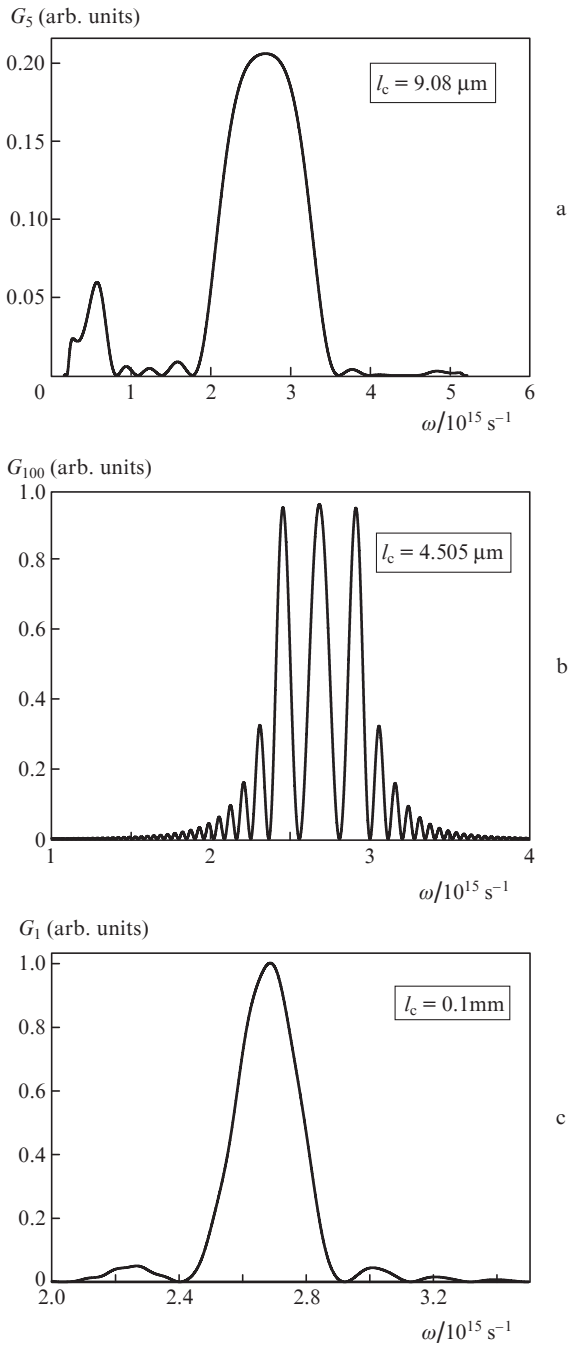

Figure 6. Spectra of a biphoton packet $G_N(\omega)$ in the case of (a) five and (b) 100 cells and in the case of (c) a single crystal for different values of l_c.

thin crystals. Previous experimental studies [7, 8] showed that for BBO crystals there exists a maximum thickness $l_c = 0.1$ mm, caused by a decrease in the flux of generated biphotons. Our studies have shown that when the BBO crystal thickness is no less than 0.05 mm, the method of generation of broadband biphotons, proposed by Klyshko, should work well. At thicknesses $l_c \geqslant 0.05$ mm, the flux of generated biphotons can by markedly increased by using a sufficiently large number N of cells. In this case, the width of the wave spectrum and duration of the packet remain the same as in the case of a single crystal ($\tau_0 \approx 10$ fs).

For smaller crystal thicknesses ($l_c \leqslant 0.05$ mm), application of the method proposed by Klyshko allows the spectrum of biphotons to be further broadened and their duration to be reduced. As shown by the above calculations, one can generate packets 2 fs in duration. For sufficiently small thicknesses of individual crystals (layers) in the system under study there appears a fine structure in the spectrum of biphotons: the broadband spectrum is divided into a number of narrow bands with increasing number of cells (layers) and the number of these bands increases. Of interest are the experimental studies of this structure in the spectrum of broadband biphotons.

Currently, as noted above, the spectrum broadening and the biphoton packet compression can be achieved by using quasi-phase-matched periodically poled nonlinear crystals in which the optical axes of adjacent layers are antiparallel and coplanar [9–14]. This method is similar to that proposed by Klyshko. Nevertheless, these methods essentially differ. In the case of layers with antiparallel axes in the cell, the spectrum of a biphoton packet is broadened in the presence of a chirp (with changing the modulation period of the nonlinear polarisation in the crystal thickness). Therefore, there appears only a phase modulation of a biphoton packet, while its duration remains the same (the packet duration is determined by the crystal thickness). The packet duration can be reduced in two stages: at the first stage a biphoton is generated in a quasi-phase-matched periodically poled nonlinear crystal, and at the second – the packet is compressed as a result of conversion of phase modulation into amplitude modulation during the passage of one of biphoton components (for example, the signal wave) through an additional medium with group velocity dispersion. This two-stage conversion of biphotons has been implemented only in [14], where the authors compressed a biphoton packet down to 100 fs. The method proposed by Klyshko has in this respect a significant advantage: an ultrashort biphoton wave packet is generated in SPDC in a stack of nonlinear crystals, and the second stage is not necessary. The studies carried out in this paper show that this method allows one to generate packets with a duration $\tau_0 \approx 2$ fs, i.e., two orders of magnitude smaller than in [14].

Note that our analysis applies to the case of monochromatic pumping when biphotons effectively interact with the pump field throughout the crystal. In another limiting case, when pumping is performed by an ultrashort pulse and the length of its coherence is less than the crystal thickness l_c, the flux of emitted biphotons is generated only by a part of the crystal. For this case our analysis proves incorrect and the problem of the duration reduction and the increase in the flux of biphotons may have specific characteristics. These characteristics are expected to be considered in a separate publication.

7. Conclusions

Almost 20 years ago, Klyshko [15] proposed to compensate for a decrease in the biphoton flux with decreasing nonlinear crystal thickness by a special periodic multilayer structure. In this paper, we have calculated the spectra of broadband biphotons with allowance for exact dispersion in a BBO crystal. Our results have shown that the implementation of the method proposed by Klyshko will make a significant progress in solving problems that require an increase in the intensity of ultrashort biphotons and in research aimed at the biphoton spectrum broadening and at generating extremely short quantum packets.

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