

# Anomalous high noise levels in a fibre Bragg grating semiconductor laser

V.D. Kurnosov, K.V. Kurnosov

**Abstract.** Taking into account gain nonlinearity allows one to obtain anomalously high noise levels in a fibre Bragg grating laser diode. This paper examines the effect of the gain nonlinearity due to spectral hole burning on noise characteristics.

**Keywords:** anomalously high noise level, gain nonlinearity, spectral hole burning.

## 1. Introduction

As shown in experimental studies of the low-frequency amplitude noise in a fibre Bragg grating (FBG) laser diode (LD) [1], there is an anomalously high noise level in the mode switching region of the LD, where lasing stops to be single-frequency. According to a theoretical study of the low-frequency noise in an FBG LD [2], systems of rate equations fail to properly describe experimental data for both single- and double-mode operation. Calculation results were inconsistent with the increase in the noise caused by switching from one laser mode to another, which was accounted for by the fact that one cannot obtain simultaneous lasing in two modes with roughly equal amplitudes.

In this paper, we demonstrate that an anomalous noise level sets in if calculations take into account nonlinear interaction between optical fields (spectral hole burning).

Nonlinear interaction between optical fields and LD single-frequency operation self-stabilisation have been the subject of extensive studies [3–13]. Anomalous interaction between spectral modes in LDs was first considered by Bogatov et al. [3]. As shown in that study, it can be understood by taking into account that the beating between two modes results in carrier density modulation (diffraction grating formation), which leads to high-power mode scattering. The carrier density modulation entails dielectric permittivity modulation, which in turn leads to additional (induced) gain. Issues pertaining to single-frequency lasing self-stabilisation were addressed by Bogatov et al. [4] and Kazarinov et al. [5], and four-wave mixing was considered by Agrawal [6].

It is worth noting a number of reports by Yamada et al. [7–13], who investigated gain nonlinearity in the case of spectral hole burning. Their results will be used in the calculations below.

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## 2. Basic relations

As in a previous study [2], consider first a system of rate equations for the photon density in the  $i$ th mode ( $S_{li}$ ) and the carrier density in the active region ( $n_a$ ):

$$\frac{dS_{li}}{dt} = \left( F_{li} G_i - \frac{1}{\tau_i} \right) S_{li} + \beta F_{li} R_{sp} + F_i(t), \quad (1)$$

$$\frac{dn_a}{dt} = \frac{I}{eV_a} - A_n n_a - R_{sp} - \sum_i G_i S_{li} + F_c(t), \quad (2)$$

where  $F_{li}$  is the relative fraction of the photon density in the active region;  $I$  is the pump current; the coefficient  $\beta$  takes into account the spontaneous emission contribution to the lasing mode;  $F_i(t)$  and  $F_c(t)$  are Langevin noise operators;  $A_n n_a$  is the nonradiative carrier recombination rate;  $V_a$  is the volume of the active region;  $\tau_i$  is the photon lifetime in the  $i$ th mode; and  $R_{sp}$  is the spontaneous carrier recombination rate.

The gain coefficient can be written in the form (see Ref. [2])

$$G_i = \bar{G}_i(T, \bar{n}_a) = Q_i \{ \bar{n}_a [D_{i0}(T, \bar{n}_a) + A_{Tqi} P] - \bar{n}_{a0} \} - \delta g_i, \quad (3)$$

where  $Q_i = (c_0 \Gamma_a / n_{1i}) (dg/dn_a)$ ;  $\Gamma_a$  is the optical limiting coefficient;  $dg/dn_a$  is the differential gain coefficient;  $\delta g_i$  is the additional gain coefficient taking into account spectral hole burning; and  $n_{1i}$  is the refractive index of the active region at the frequency of the  $i$ th mode.

Consider interaction between two modes ( $i = p$  and  $q$ ), taking the photon density in mode  $p$  to be much greater than that in mode  $q$ :  $S_p \gg S_q$ . As shown earlier [7–13], the additional gain coefficient for mode  $q$  is proportional to the photon density in mode  $p$ , and vice versa. Because of this, we will consider the additional gain coefficient only for mode  $q$ :  $\delta g_p = 0$  (other nonlinearities of the gain coefficient for modes  $p$  and  $q$  from Refs [7–13] will be neglected).

The additional gain coefficient for mode  $q$  can be written in the form

$$\delta g_q = A_q \bar{S}_p, \quad (4)$$

where

$$A_q = D_{qp} + H_{qp}; \quad (5)$$

$$D_{qp} = \frac{4}{3} \frac{B}{(2\pi c \tau_{in})^2 (\lambda_p - \lambda_q)^2 / \lambda_q^2 + 1}; \quad (6)$$

$$B = \frac{9}{2} \frac{\pi c \Gamma_a G_q \tau_{in}^2}{\varepsilon_0 n_1^2 \hbar \lambda_q} |R_{cv}|^2; \quad (7)$$

$$H_{qp} = \frac{3\lambda_q^2 Q_q G_q}{8\pi c} \frac{\alpha_H}{\lambda_p - \lambda_q}; \quad (8)$$

$\bar{S}_p$  is the steady-state photon density in mode  $p$ ;  $\tau_{in}$  is the intraband carrier relaxation time;  $\varepsilon_0$  is the electric constant;  $R_{cv}^2$  is the dipole moment; and  $\alpha_H$  is the Henry factor [14]. For the GaAs/AlGaAs system, we take  $R_{cv}^2 = 2.8 \times 10^{-57} \text{ K}^2 \text{ m}^2$  [7].

In contrast to relations (6)–(10) in Ref. [7], Eqns (4)–(8) contain the gain coefficient  $G_q$  instead of  $\xi a(N - N_g)$ . The Henry factor of the GaAs/AlGaAs system was reported to lie in the range  $0 \leq \alpha_H \leq 3.5$  [7]. In our calculations, we take  $\alpha_H = 3$  and  $\tau_{in} = 0.1 \times 10^{-13} \text{ s}$ .

As in a previous study [2], we identified the modes capable of propagating through the FBG LD at a given pump current. In the first calculation step, the additional gain coefficient was taken to be zero and we determined the photon densities in modes  $p$  and  $q$ . In the second calculation step, we recalculated the photon densities in modes  $p$  and  $q$ , taking into account the additional gain coefficient for mode  $q$ . If the calculated photon density in mode  $q$  exceeded that in mode  $p$ , mode  $q$  was assigned the number  $p$ . Taking into account the maximum photon density obtained in mode  $p$ , we performed the next additional gain coefficient calculation step for mode  $q$  and calculated the photon densities in modes  $p$  and  $q$ . The procedure was repeated until the difference in the photon density in the modes between neighbouring steps relative to the absolute value exceeded  $10^{-3}$ .

The relative intensity noise (RIN) per unit bandwidth is given by [2]

$$\text{RIN} = 10 \lg \frac{\langle \Delta S_{1p}(\omega) \Delta S_{1p}^*(\omega) \rangle}{\bar{S}_{1p}^2} = 10 \lg \left[ \frac{F_p^2 |T_p|^2 + F_c^2 |T_c|^2 + 2F_p F_c \text{Re}(T_p T_c^*)}{\bar{S}_{1p}^2} \right], \quad (9)$$

where the superscript  $*$  denotes the complex conjugate.

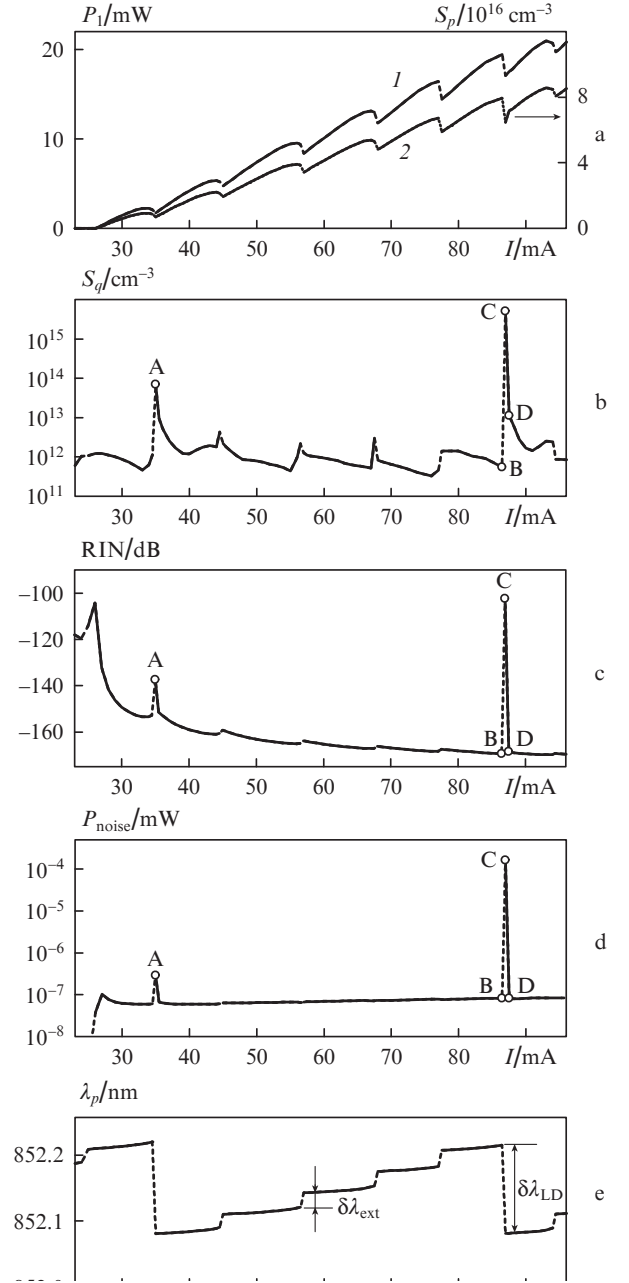
The rms noise power per unit bandwidth is defined as

$$P_{\text{noise}} = \sqrt{\langle \Delta P_1^2 \rangle} = A_{\text{power}} \sqrt{\langle \Delta S_{1p}(\omega) \Delta S_{1p}^*(\omega) \rangle}. \quad (10)$$

The coefficients in (9) and (10) are defined by Eqns (27)–(29) in Ref. [2].

Figure 1 presents characteristics of an FBG LD calculated with allowance for the additional gain coefficient, as defined by (4), at  $\alpha_H = 3$ , a thermal resistance of the LD  $R_T = 38 \text{ K W}^{-1}$  and a frequency  $f = 60 \text{ kHz}$ . The calculation took into account Eqn (8) in Ref. [2] and was performed for the system of rate equations (1) and (2).

Comparison of Figs 1c and 1d in this study and Figs 2e and 2f in Ref. [2] demonstrates that the curves differ drastically in shape. At pump currents of 35 (point A) and 87 mA (point C), there are sharp peaks in RIN (Fig. 1c) and  $P_{\text{noise}}$  (Fig. 1d). Figure 1e shows the emission wavelength of mode  $p$  as a function of pump current. It is seen that the anomalous increase in noise in Figs 1c and 1d is associated with LD mode switching (Fig. 1e) and a sharp rise in photon density in mode  $q$  (Fig. 1b). It is worth noting that mode switching in the external cavity by  $\delta\lambda_{\text{ext}}$  (Fig. 1e) is not accompanied by any



**Figure 1.** Calculated (a) ( $I$ ) output power ( $P_1$ ), (2) photon density in mode  $p$  ( $S_p$ ), (b) photon density in mode  $q$  ( $S_q$ ), (c) relative intensity noise (RIN) per unit bandwidth, (d)  $P_{\text{noise}}$  per unit bandwidth and (e) emission wavelength of mode  $p$  as functions of pump current at  $\alpha_H = 3$ . The calculation took into account Eqn (8) in Ref. [2] and was performed for two modes in the system of Eqns (1) and (2).  $\delta\lambda_{\text{ext}}$  and  $\delta\lambda_{\text{LD}}$  are mode spacings.

increase in noise level relative to the noise in Figs 2e and 2f in Ref. [2].

Thus, the anomalous noise level at pump currents of 35 and 87 mA coincides with FBG LD operation at more than one frequency.

The effect of  $\alpha_H$  on the spectral characteristics of single-frequency quantum-well heterostructure lasers was investigated by Konyaev et al. [15]. According to their results, in lasers with  $\alpha_H \leq 2.5$  single-frequency operation sets in at threshold. Raising the injection current leads to well-defined self-stabilisation of the dominant longitudinal mode, and its

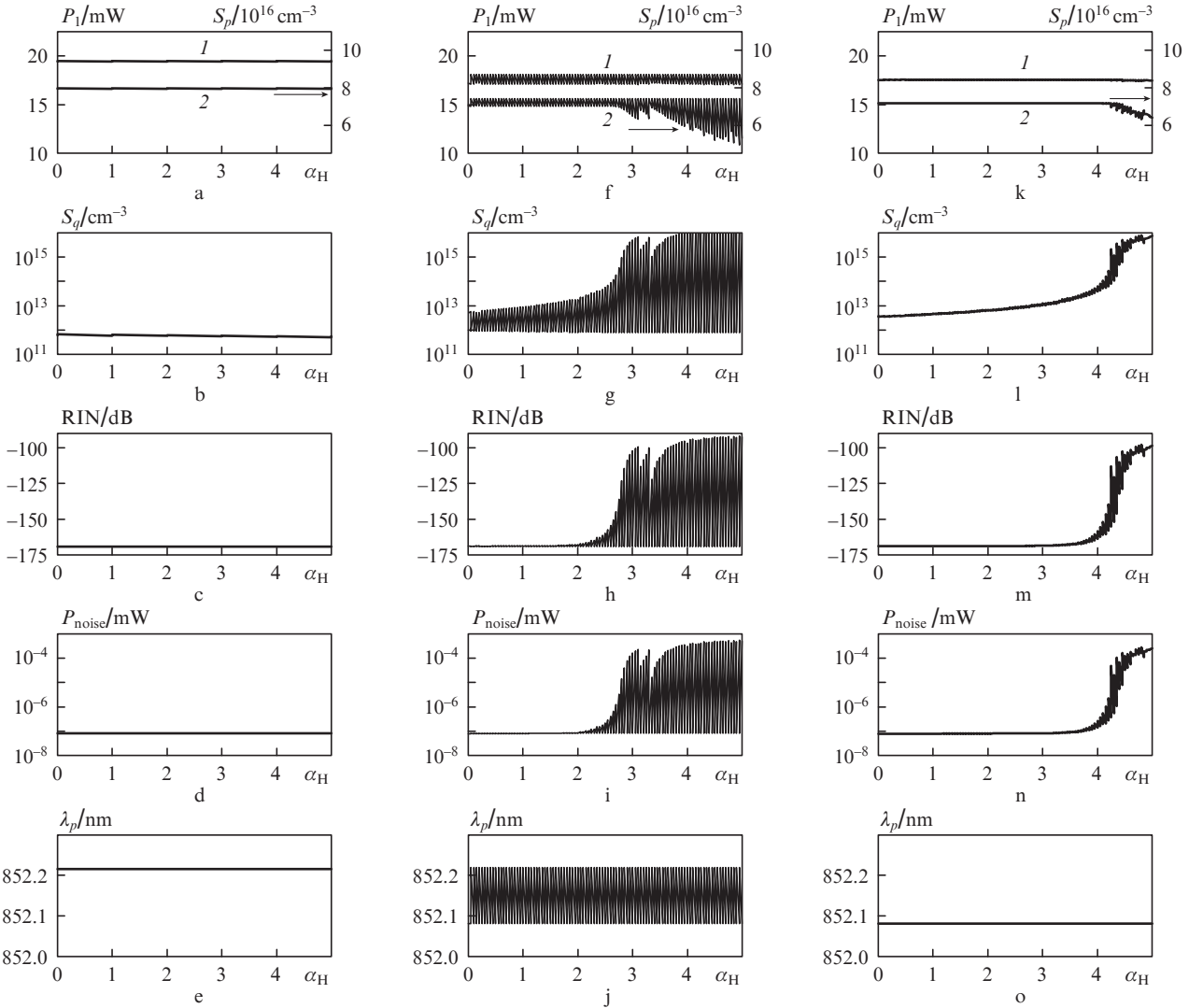
intensity rises linearly, whereas the intensity of neighbouring modes drops. At moderate  $\alpha_H$  values ( $2.5 \leq \alpha_H \leq 3$ ), single-frequency operation is unstable slightly above threshold. It sets in with increasing injection current, but self-stabilisation is poorly defined or missing. In lasers with  $\alpha_H \geq 3$ , only multi-mode operation is usually observed.

Let us analyse the effect of  $\alpha_H$  on the noise characteristics of an FBG LD in the range  $0 \leq \alpha_H \leq 5$ . In the case  $\alpha_H = 3$ , an anomalous noise value is observed at a pump current of 87 mA (Fig. 1, point C). Let us choose three points in Fig. 1 for analysis. Point B corresponds to a pump current of 86.5 mA; point C corresponds to 87 mA; and point D, to 87.5 mA (to the right of the anomalous noise point). Figure 2 shows the output power  $P_1$ , photon densities in modes  $p$  and  $q$ , relative intensity noise RIN, noise power  $P_{\text{noise}}$  and emission wavelength as functions of  $\alpha_H$ . It is seen in Fig. 2 that, at a pump current of 86.5 mA (point B), these characteristics are independent of  $\alpha_H$ . At a current of 87 mA, the relative intensity noise varies little in the range  $0 \leq \alpha_H \leq 2.5$ . Anomalous noise emerges in the range  $2.5 \leq \alpha_H \leq 5$ . The photon density in mode  $q$  (Fig. 2g) increases nonlinearly with  $\alpha_H$  in the range

$0 \leq \alpha_H \leq 5$ , leading to a nonlinear rise in RIN and  $P_{\text{noise}}$  (Figs 2h, 2i). Analysis of the noise in Fig. 2 at a current of 87 mA indicates that the noise is due to laser wavelength instability and switching from one laser mode to another (Fig. 2j). This is mode switching noise [16].

At a pump current of 87.5 mA, anomalous noise is observed in the range  $3.5 \leq \alpha_H \leq 5$  and has a different nature. It is seen that, in this range, the RIN and  $P_{\text{noise}}$  (Figs 2m, 2n) increase nonlinearly, whereas the emission wavelength is independent of  $\alpha_H$  (Fig. 2o). The photon density in mode  $q$  rises nonlinearly (Fig. 2l), and that in mode  $p$  decreases nonlinearly [Fig. 2k, curve (2)]. At the same time, the output power of the FBG LD remains essentially unchanged [Fig. 2k, curve (1)]. The noise in question is due to the photon density redistribution between modes  $q$  and  $p$  [16], i.e. anomalously high noise may occur at a constant output power.

It is worth noting that, at a pump current of 86.5 mA, there is a stable emission wavelength of 852.22 nm (Fig. 2e). At a pump current of 87 mA, the wavelength is unstable (Fig. 2j). At a current of 87.5 mA, stable lasing at a wave-



**Figure 2.** Calculated (a, f, k) (1) output power ( $P_1$ ), (2) photon density in mode  $p$  ( $S_p$ ), (b, g, l) photon density in mode  $q$  ( $S_q$ ), (c, h, m) relative intensity noise (RIN) per unit bandwidth, (d, i, n)  $P_{\text{noise}}$  per unit bandwidth and (e, j, o) emission wavelength  $\lambda_p$  as functions of Henry factor  $\alpha_H$  for two modes in the system of Eqns (1) and (2) at pump currents of (a–e) 86.5, (f–j) 87 and (k–o) 87.5 mA.

length of 852.08 nm is observed (Fig. 2o), in spite of the anomalous noise in the range  $3.5 \leq \alpha_H \leq 5$ .

It is of interest to analyse analogous characteristics for the system of rate equations (1) and (2b) in Ref. [2]. Characteristically, the rate equation for carrier density in this system contains no optical limiting coefficient:

$$\frac{dn_a}{dt} = \frac{I}{eV_a} - A_n n_a - R_{sp} - \frac{1}{\Gamma_a} \sum G_i S_{li} + F_c(t). \quad (11)$$

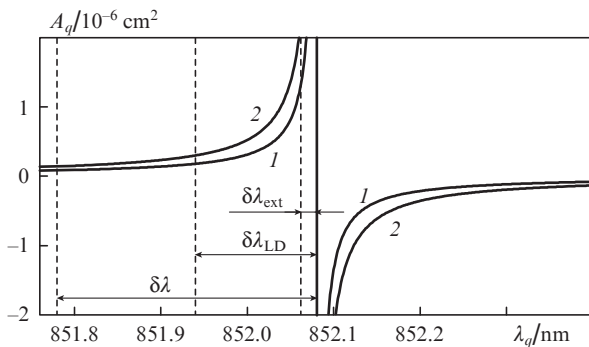
Calculation results for the system of Eqns (1) and (11) are essentially identical to the data in Figs 2a–2d in Ref. [2]. The reason for this is that the photon density in mode  $p$  for the system of Eqns (1) and (11) is almost two orders of magnitude lower than that for the system of Eqns (1) and (2). This leads to a reduction in the contribution of the additional gain coefficient for mode  $q$  by two orders of magnitude, so changes in characteristics are insignificant.

### 3. Discussion

Thus, the system of Eqns (1) and (2) better describes the experimental noise data reported by Zholnerov et al. [1] than does the system of Eqns (1) and (11).

An FBG LD may have anomalously high noise levels due to both laser operation instability and intensity redistribution between modes (at a constant output power).

Figure 3 illustrates the effect of  $\lambda_q$  on  $A_q$  calculated using Eqn (5) at  $\alpha_H = 3$  and 5. The vertical line is located at  $\lambda_p = 852.08$  nm. In identifying the modes capable of propagating through an FBG LD, the mode spacing is determined by the intermodal separation of the external cavity, which is 0.018 nm (0.14 nm for the LD) [17]. Calculations were made for 18 modes to the left and right of the Bragg frequency of the fibre grating (a total of 36 modes). The variation of  $|\lambda_q - \lambda_p|$  in a wide  $\alpha_H$  range is characterised by a large change in additional gain coefficient, which complicates calculations for  $\alpha_H \geq 5$ . In Refs [7–13],  $|\lambda_q - \lambda_p|$  was taken to be 0.3 nm (Fig. 3), which considerably exceeds the mode spacing of the external cavity used in the calculations in this study.



**Figure 3.**  $A_q$  as a function of emission wavelength  $\lambda_q$  at  $\alpha_H = (1) 3$  and  $(2) 5$ ;  $\delta\lambda$  is the mode spacing used in calculations in Refs [7–13].

### 4. Conclusions

1. Given that the gain coefficient is nonlinear, anomalously high noise levels can be obtained in an FBG LD.

2. An anomalously high noise level is only possible in the system of rate equations (1) and (2), and not in the system of Eqns (1) and (11).

3. In FBG LD fabrication, heterostructures with the lowest possible Henry factor  $\alpha_H$  should be used in order to reduce the noise level.

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